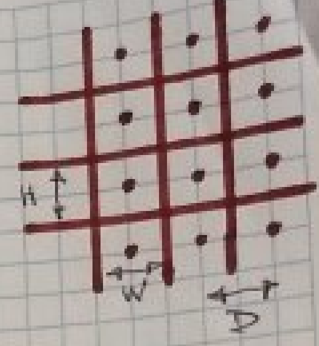
Quiz No. 1



Rectangular grid:

- Height = H
- width = W

Square grid:

- H = W
- Area, $A = H \times W = W^2$ (i)

Q1. Area $A = 4\ m^2$

What is W? $4 = W^2$ From (i)

$W = \sqrt{4} = 2\ m$ (& $H = 2m$)

Distance between the points at the center of each square (D) is the same as the distance between the top-left corner of each square, which is the distance between grid lines. $\therefore D = W = \underline{2\ m}$

Q2. Area $A = 2\ m^2$

What is W? $2 = W^2$ From (i)

$W = \sqrt{2} = 1.4\ m$ (& $H = 1.4m$)

As above, $D = W = \underline{1.4m}$

This means that a square grid pattern with $2\ m^2$/person is not good enough for people to maintain a 1.5 m distance from each other (for a large room where tricks at corners & edges have less effect).

Q3. $D = 1.5m \Rightarrow W = 1.5m$ (& $H = 1.5m$)

$\therefore$ Area $A = W^2 = 1.5^2 = \underline{2.25\ m^2}$

Question 4

If you have a requirement that a large number of points must be spaced 1.5m or more apart, can you find any arrangement/layout of those points such that the average *area-per-person* is 2 square metres or less?

Answer

Let’s assume each person can be represented by a point.

Figure 1 shows person A at the center of a circle with radius 1.5m. No other people are permitted to be within the circle otherwise they will be closer than 1.5m. However, since we are looking to pack people as close together as possible (while still keeping individuals 1.5m apart from each other) then we will place person B on the circumference of person A’s circle.

Figure 1 also shows that person B is at the center of another circle with radius 1.5m. No other people are permitted to be within person B’s circle nor person A’s circle. So let’s place person C at a point where person A’s circle intersects person B’s circle. You can see that A-B-C now form an equilateral triangle where each side is 1.5m.

Figure 2 shows an extension of the same persons A, B and C in Figure 1. Here a circle has been drawn around person C. Where person C’s circle intersects person A’s circle I have placed person D, and so on. So as we move anti-clockwise around the circumference of person A’s circle, we have 6 people - B, C, D, E, F and G. Each of those 6 people are 1.5m away from person A. Also, each of those 6 people are 1.5m away from their neighbour. For example, person B is 1.5m from both person C and person G; and person D is 1.5m from both person E and person C.

Figure 2 also shows the placement of some of the of the other people who can be put 1.5m from people B to G, that is people H, I ,J, K, L and M.

Figure 3 is almost identical to figure 2, except I have removed the circles and drawn a straight 1.5m line between people and their neighbours. You can see that this is a pattern of (tessellated) equilateral triangles with 1.5m between people.

Figure 4 is identical to figure 3 but with boundaries between people shown as a brick-layout (which I didn’t notice until someone pointed it out).

Figure 5 is an equilateral triangle with sides of length 1.5m. I have divided it into 2 right angle triangles to help calculate the area and hence the area-per-person. The next page shows the area-per-person for figure 3 (and 1, 2 and 4) is 2.0m^2.

Q4.

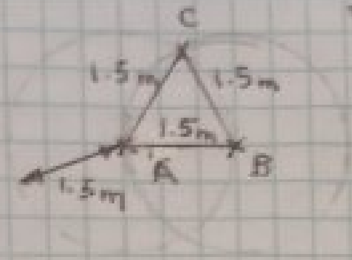
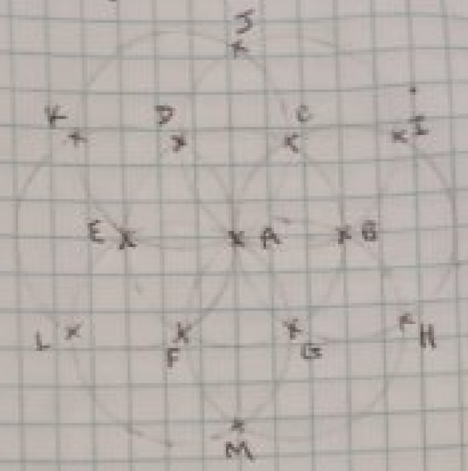


Fig 1.

Fig 2.



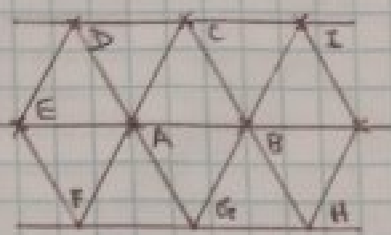


Fig 3.

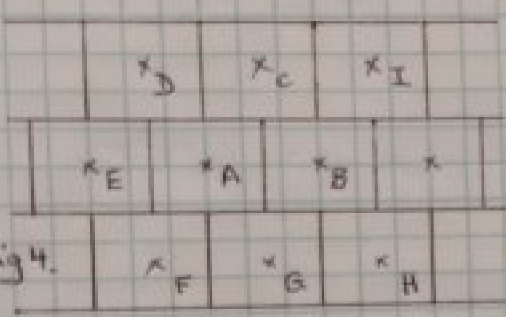
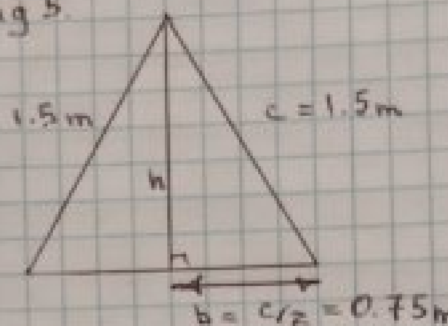


Fig 4.

Fig 5.



From Fig 5, Area of equalateral triangle (EQ Δ) =

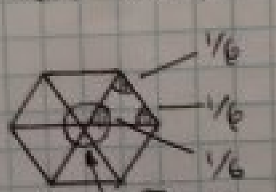
2x Area of right angle triangle (RA Δ).

Pythagoras Th: $h^2 + b^2 = c^2$; $h^2 + \left(\frac{c}{2}\right)^2 = c^2$

$h^2 = c^2 - \frac{c^2}{4} = \frac{3}{4}c^2$; $h = \frac{\sqrt{3}}{2}c$

Area RA Δ $= \frac{1}{2} \cdot b \cdot h = \frac{1}{2}\left(\frac{c}{2}\right)\left(\frac{\sqrt{3}}{2}\right)c = \frac{\sqrt{3}}{8} c^2$

Area EQ Δ $= 2 \times$ (Area RA Δ) $= \frac{\sqrt{3}}{4}c^2 = \frac{9}{16}\sqrt{3}$ (for $c = 1.5 = 3/2$)
(in m^2)



A 1/6th of a person is in each vertex (corner) of one EQ Δ.

$\therefore 3 \times 1/6 = \frac{1}{2}$ person in each EQ Δ.

$\Rightarrow$ 1/2 person in area $\frac{9}{16}\sqrt{3}$ m^2

$\Rightarrow$ 1 " " " $\frac{9}{8}\sqrt{3} = 1.95\ m^2$

Hence if you pack people into a room with arrangement as per Figures 1, 2, 3 and 4, then you can achieve a density of 1 person per 2.0 m^2 (rounded to 2 significant figures).