1. General Equations

Magnetic circuit:

$$\mathcal{F} = \mathcal{R}\phi$$

$\mathcal{F}$ is the magnetomotive force, or mmf
$\mathcal{R}$ is the reluctance of the magnetic circuit

$$\mathcal{F} = Ni$$

N is the number of turns of the coil
i is the current flowing through the coil

$$\mathcal{R} = \frac{l_c}{A\mu}$$

l_c is the average length of the magnetic core
A is the cross section area of the magnetic core
μ is the magnetic permeability ($\mu = \mu_0\mu_r$, where $\mu_0 = 4\pi\times10^{-7}$ H/m is the permeability of free space and μ_r is relative permeability).

$$\phi = BA$$

B is the flux density
A cross section area of the magnetic core

Flux linkage:

$$\lambda = \sum_{Nc}\phi_t$$

λ is the flux linkage of a winding
ϕ_t is the flux passing through each turn of the winding
Nc is the total number of turns in a winding

Transformer action:

$$e = -\frac{d\lambda}{dt}$$

where e is the induced voltage

Motor action:

$$\vec{F} = i(\vec{l}\times\vec{B})$$

i is the current in a conductor
l is a vector describing the length of conductor
B is the flux density of the field

Generator action:

$$e = (\vec{v}\times\vec{B})\cdot\vec{l}$$

where v is the velocity of the conductor with respect to the field

Single loop in a constant field:

$$e_{loop} = \phi_{\max}\,\omega\sin(\omega t)$$

ω is the speed of rotation
$\phi_{\max}$ is the maximum flux passing through the coil, i.e. when the loop is perpendicular to the field.

$$\tau_{loop} = ABi\sin\theta$$

τ_{loop} is the developed torque in the loop
A is the coil area
θ is the position of the coil with respect to the field ($\theta = \omega t$)

2. DC Machines

Voltage and torque:

$$E_A = k\phi\omega$$
$$\tau = k\phi I_A$$

with

$$k = \frac{Zp}{2\pi}$$

I_A is the armature current
Z is the total number of conductors
p is the number of poles of the machine

Shunt or separate excitation:

$$I_F = \frac{V_F}{R_F}$$ (V_F=V_T for shunt DC machine)

$$V_T = E_A + R_A I_A$$ (Terminal voltage)

$$\omega = \frac{V_T}{k\phi} - \frac{R_A}{(k\phi)^2}\tau$$ (Torque-speed relationship)

Series excitation:

$\phi = cI_A$, where c is a constant

$$V_T = E_A + I_A(R_A + R_S)$$ (Terminal voltage)

$$\omega = \frac{V_T}{\sqrt{kc}}\frac{1}{\sqrt{\tau}} - \frac{R_A + R_S}{kc}$$ (Torque-speed relationship)

where Rs is the series field resistance.

3. AC Machine Fundamental

Three-phase system:

Y-connection:

$$V_{LL} = \sqrt{3} V_\phi$$

$$I_L = I_\phi$$

Δ-connection:

$$V_{LL} = V_\phi$$

$$I_L = \sqrt{3} I_\phi$$

Power (Real power):

$$P = 3V_\phi I_\phi \cos(\theta) = \sqrt{3} V_{LL} I_L \cos(\theta)$$

Reactive power:

$$Q = 3V_\phi I_\phi \sin(\theta) = \sqrt{3} V_{LL} I_L \sin(\theta)$$

Apparent power:

$$S = 3V_\phi I_\phi = \sqrt{3} V_{LL} I_L$$

Power factor:

$$PF = \cos(\theta)$$

Rotating field:

Electrical-Synchronous angles:

$$\theta_e = \frac{p}{2} \theta_s$$

$$f_e = \frac{p}{2} f_s$$

$$\omega_e = \frac{p}{2} \omega_s$$

where

θ_e, θ_s	are electrical angle and field rotation angle
f_e, f_s	are electrical & synchronous frequencies in Hz
ω_e, ω_s	are electrical & synchronous frequencies in rad/s

$$f_e = \frac{p}{120} n_s$$

where

n_s	is the synchronous speed in rpm
p	is the number of poles

Voltage induced in a coil by a rotating field:

$$E = \sqrt{2} \pi N_C \phi f_e$$

Torque induced by two rotating fields

$$\tau = k \overrightarrow{B_R} \times \overrightarrow{B_S}$$

4. Synchronous Machines

Generators:

$$\overrightarrow{E_A} = \overrightarrow{V_\phi} + R_A \overrightarrow{I_A} + jX_S \overrightarrow{I_A}$$

where

$\overrightarrow{E_A}$	is the armature induced voltage (excitation)
$\overrightarrow{V_\phi}$	is the armature terminal phase voltage
$\overrightarrow{I_A}$	is the armature current
R_A	is the armature resistance
Xs	is the synchronous reactance

Neglecting armature resistance (R_A)

$$P = \frac{3V_\phi E_A}{X_S} \sin \delta$$

$$Q = \frac{3V_\phi E_A}{X_S} \cos \delta - \frac{3V_\phi^2}{X_S}$$

$$\tau = \frac{3V_\phi E_A}{\omega_S X_S} \sin \delta$$

where δ is the torque angle.

Voltage Regulation:

$$VR = \frac{V_{nl} - V_{fl}}{V_{fl}} \times 100\%$$

where

V_{nl}	is the no-load voltage
V_{fl}	is the full-load voltage

Speed Droop:

$$SD = \frac{n_{nl} - n_{fl}}{n_{fl}} \times 100\%$$

where

n_{nl}	is the no-load speed
n_{fl}	is the full-load speed

P-*f* Relationship:

$$P = s_P (f_{nl} - f_{sys})$$

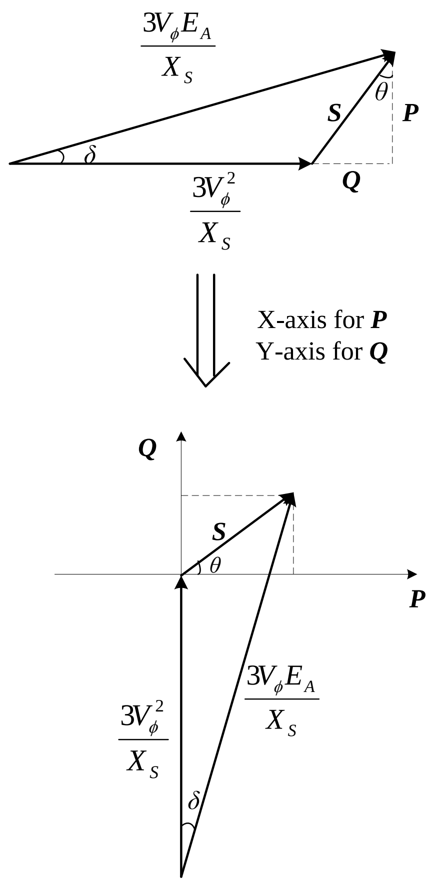
$$s_P = \frac{\Delta P}{\Delta f}$$

where

f_{nl}	is the no-load frequency
f_{sys}	is the operating frequency
s_P	is the slope of P-f curve

Capability diagram:

Scale the generator diagram by $\frac{3V_\phi}{X_S}$:



Motors:

$$\overrightarrow{V_\phi} = \overrightarrow{E_A} + R_A\overrightarrow{I_A} + jX_S\overrightarrow{I_A}$$

5. 3-phase Induction Motors

Slip:

$$s = \frac{n_s - n_m}{n_s} = \frac{\omega_s - \omega_m}{\omega_s}$$

where

n_s, n_m are synchronous and rotor speed in *rpm*

ω_s, ω_m are synchronous and rotor speed in *rad/s*

Slip speed:

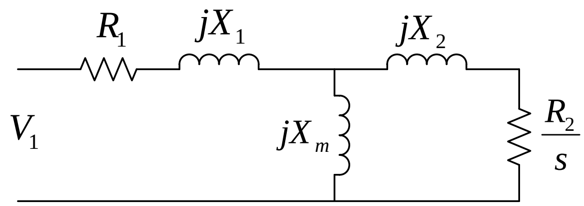
$$n_{sl} = n_s - n_m \text{ in rpm}$$

$$\omega_{sl} = \omega_s - \omega_m \text{ in rad/s}$$

Rotor frequency:

$$f_r = sf_e$$

Induction motor per-phase equivalent model:



Airgap power:

$$P_{AG} = \frac{3I_2^2R_2}{s}$$

Stator copper loss:

$$P_{SCL} = 3I_1^2R_1$$

Rotor copper loss:

$$P_{RCL} = 3I_2^2R_2$$

$$P_{RCL} = sP_{AG}$$

Converted power:

$$P_{conv} = 3I_2^2R_2\frac{1-s}{s}$$

$$P_{conv} = (1-s)P_{AG}$$

Output power:

$$P_{out} = P_{conv} - P_{rotation}$$

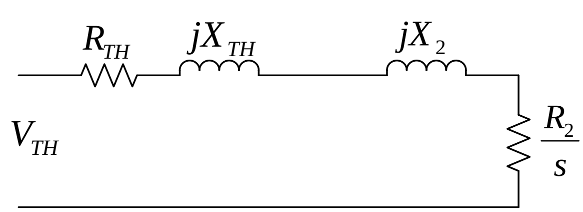
Induced torque:

$$\tau = \frac{P_{conv}}{\omega_m} = \frac{P_{AG}}{\omega_s} = \frac{3I_2^2R_2}{s\omega_s}$$

Output torque (shaft torque):

$$\tau_{out} = \frac{P_{out}}{\omega_m}$$

Thevenin equivalent circuit model:



Thevenin voltage and impedance:

$$\overrightarrow{V_{TH}} = \frac{jX_m}{R_1 + j(X_1 + X_m)}\overrightarrow{V_1}$$

$$V_{TH} = \frac{X_m}{\sqrt{R_1^2 + (X_1 + X_m)^2}}V_1$$

$$Z_{TH} = R_{TH} + jX_{TH} = \frac{jX_m(R_1 + jX_1)}{R_1 + j(X_1 + X_m)}$$

Torque:

$$\tau = \frac{3V_{TH}^2}{(R_{TH} + \frac{R_2}{s})^2 + (X_{TH} + X_2)^2} \frac{R_2}{s} \frac{1}{\omega_s}$$

Maximum torque:

$$s_{\tau\max} = \frac{R_2}{\sqrt{R_{TH}^2 + (X_{TH} + X_2)^2}}$$

$$\tau_{\max} = \frac{3V_{TH}^2}{2\omega_s(R_{TH} + \sqrt{R_{TH}^2 + (X_{TH} + X_2)^2})}$$

Induction motor start codes:

Code Letter	Start kVA/rated Hp
A	0 - 3.15
B	3.15 - 3.55
C	3.55 - 4.0
D	4.0 - 4.5
E	4.5 - 5.0
F	5.0 - 5.6
G	5.6 - 6.3
H	6.3 - 7.1
J	7.1 - 8.0
K	8.0 - 9.0
L	9.0 - 10.0
M	10.0 - 11.0
N	11.0 - 12.5
P	12.5 - 14.0
R	14.0 - 16.0
S	16.0 - 18.0
T	18.0 - 20.0
U	20.0 - 22.4
V	22.4 and up

Induction motor parameter measurement:

Blocked-rotor test:

$$X_1 + X_2 = \frac{f_{rated}}{f_{BR}}(X_1 + X_2)_{BR}$$

where f_{BR} is the blocked rotor test frequency.

Percentage of X_1 and X_2

Rotor Design	X_1	X_2
Wound Rotor	50%	50%
Class A	50%	50%
Class B	40%	60%
Class C	30%	70%
Class D	50%	50%

No load test:

$$P_{nl} = 3I_1^2 R_1 + P_{rotation}$$

$$Q_{nl} = 3I_1^2 (X_1 + X_m)$$

6. 1-phase Induction Motors

Forward impedance:

$$Z_F = R_F + jX_F = \frac{jX_m(\frac{R_2}{s} + jX_2)}{\frac{R_2}{s} + j(X_2 + X_m)}$$

Backward impedance:

$$Z_B = R_B + jX_B = \frac{jX_m(\frac{R_2}{2-s} + jX_2)}{\frac{R_2}{2-s} + j(X_2 + X_m)}$$

Total input impedance:

$$Z_{in} = R_1 + jX_1 + \frac{1}{2}(Z_F + Z_B)$$

Airgap Powers:

$$P_{AG\_F} = \frac{1}{2} I_1^2 R_F$$

$$P_{AG\_B} = \frac{1}{2} I_1^2 R_B$$

$$P_{AG} = P_{AG\_F} + P_{AG\_B}$$

Torque:

$$\tau_F = \frac{P_{AG\_F}}{\omega_s}$$

$$\tau_B = \frac{P_{AG\_B}}{-\omega_s}$$

$$\tau = \tau_F + \tau_B = \frac{P_{AG\_F} - P_{AG\_B}}{\omega_s}$$

Converted power:

$$P_{conv} = \tau\omega_m = \tau(1-s)\omega_s = (P_{AG\_F} - P_{AG\_B})(1-s)$$