Formulas 2018

Waveforms Calculations and Power Quality

(a) 1-phase ac supply harmonics and power quality

Power Factor: $PF = \frac{P}{S} = \frac{I_1}{I_s}\cos\phi_1 = CDF \times DPF$

Real Power Flow: $P = V_s I_{s1r} = V_s I_1 \cos\phi_1$

Apparent Power: $S = V_s I_s$

$I_s = \sqrt{I_1^2 + I_2^2 + I_3^2 + I_4^2 + I_5^2 + ...}$, $I_s = \sqrt{I_1^2 + I_H^2}$, $I_H = \sqrt{I_2^2 + I_3^2 + I_4^2 + ...}$

Current Distortion Factor: $CDF = \frac{I_1}{I_s}$

Displacement Power Factor: $DPF = FPF = \cos\phi_1 = \frac{I_{1r}}{I_1}$

ac supply current: $THD_R = 100\frac{I_H}{I_S}\ \%$, $THD_F = 100\frac{I_H}{I_1}\ \%$

$$THD_R = \frac{THD_F}{\sqrt{1+THD_F^2}},\ THD_F = \frac{THD_R}{\sqrt{1-THD_R^2}}$$

$$CDF = \frac{1}{\sqrt{1+THD_F^2}},\ CDF = \sqrt{1-THD_R^2}$$

(b) Basic Waveform Calculation

Harmonics of a square-wave of voltage magnitude V_o:

$$V_{n,pk} = V_o\frac{4}{n\pi}, n = 1,3,5,7,9,11,13 \text{ etc...}, V_{n,rms} = V_o\frac{2\sqrt{2}}{n\pi}$$

Power drawn from a dc source V_{dc}: $P = V_{dc} \times I_{dc}$

Power drawn from an ac source V_1: $P = V_1 \times I_1 \times \cos\phi_1$

Power dissipated in a resistor V_1: $P_R = I_{rms}^2 \times R$

RMS of a voltage with 2 components V1(t), V2(t):

$$V_{rms} = \sqrt{V_{1,rms}^2 + V_{2,rms}^2} = \sqrt{V_1^2 + V_2^2}$$

In general: $I_{rms} = \sqrt{I_{dc}^2 + I_{1,rms}^2 + I_{2,rms}^2 + I_{3,rms}^2 + ...}$

$$I_{rms} = \sqrt{I_{dc}^2 + I_1^2 + I_2^2 + I_3^2 + ...} = \sqrt{I_{dc}^2 + I_1^2 + I_H^2}$$

$$I_H = \sqrt{I_2^2 + I_3^2 + I_4^2 + ...},\ I_{ac} = \sqrt{I_1^2 + I_2^2 + I_3^2 + ...} = \sqrt{I_{rms}^2 - I_{dc}^2}$$

A current ramping between I_p and I_m has an rms/dc/ac:

$$I_{rms} = \sqrt{\frac{1}{3}\left(I_p^2 + I_p I_m + I_m^2\right)},\ I_{dc} = \frac{I_p + I_m}{2},\ I_{ac} = \sqrt{I_{rms}^2 - I_{dc}^2}$$

A current lasting for δ of a cycle has an rms/dc/ac of:

$$I_{rms} = \sqrt{\frac{\delta}{3}\left(I_p^2 + I_p I_m + I_m^2\right)},\ I_{dc} = \delta\left(\frac{I_p + I_m}{2}\right),\ I_{ac} = \sqrt{I_{rms}^2 - I_{dc}^2}$$

1-phase diode rectifier: R and R-E load

(a) Including the effect of θ_c and α

Full-wave R/E load (R-load)

$$\alpha = a\sin\left(\frac{E + 2V_{Don}}{\sqrt{2}V_s}\right), \theta_c = \pi - 2\alpha, \theta_c^o = (180 - 2\alpha)^\circ$$

$$I_{o,rms} = \frac{1}{R + 2r_D}\sqrt{\frac{V_s^2}{\pi}\sin 2\alpha - \frac{4\sqrt{2}}{\pi}V_s\left(E + 2V_{Don}\right)\cos\alpha + \left[V_s^2 + \left(E + 2V_{Don}\right)^2\right]\left(1 - \frac{2\alpha}{\pi}\right)}$$

$$I_{o,dc} = \frac{1}{R + 2r_D}\left[\frac{2\sqrt{2}}{\pi}V_s\cos\alpha - \left[E + 2V_{Don}\right]\left(1 - \frac{2\alpha}{\pi}\right)\right]$$

(b)

$$I_{o,rms} = \frac{V_{o,rms}}{R}, I_{o,dc} = \frac{V_{o,dc}}{R}$$

(i) **Half-Wave R-Load:** $V_{o,dc} \approx \frac{\sqrt{2}}{\pi}V_s - 0.5V_{Don}$, $V_{o,rms} \approx \frac{V_s - V_{Don}}{\sqrt{2}}$

(ii) **Full-Wave R-Load**: $V_{o,dc} \approx \frac{2\sqrt{2}}{\pi}V_s - 2V_{Don} = 0.9V_s - 2V_{Don}$,

$V_{o,rms} \approx V_s - 2V_{Don}$

1-Phase diode rectifier continuous conduction

For simplicity, the equations given neglect V_{Don}

$$Z = \sqrt{R^2 + (\omega L)^2},\ \tan\phi = \frac{\omega L}{R}$$

(a) Full-Wave R/L Load continuous conduction

$$V_{O,dc} = \frac{2\sqrt{2}}{\pi}V_S = 0.9V_S, V_{O,rms} = V_S,$$

$$V_{On} = \frac{4}{\left(n^2 - 1\right)\pi}V_S\quad n = 2,4,6,8....\quad I_{On} = \frac{V_{On}}{\sqrt{R^2 + (\omega L)^2}}\quad n = 2,4,6,8....,$$

$$I_{O,ac} \approx \frac{V_{O2}}{\sqrt{R^2 + (2\omega L)^2}}$$

(b) Full-Wave LC filter, R load, continuous conduction

$$V_{O,dc} = \frac{2\sqrt{2}}{\pi}V_S = 0.9V_S, V_{O,rms} = V_S\quad V_{On} = \frac{4}{\left(n^2 - 1\right)\pi}V_S\quad n = 2,4,6,8....,$$

$$I_{On} = \frac{V_{On}}{n\omega L}\quad n = 2,4,6,8....\quad V_{O2} = \frac{4}{3\pi}V_S \approx 0.424V_S,\ I_{O,ac} \approx I_{O2} = \frac{V_{O2}}{2\omega L}$$

(c) Full-Wave R./L Load, cont. cond., ripple free

FPF = cos Φ_1 = cos α

$$P_{in} = \frac{2\sqrt{2}}{\pi}V_S I_{DC}, I_S = I_{DC}, S = V_S I_{DC}$$

$$PF = \frac{P_{in}}{S} = \frac{2\sqrt{2}}{\pi},\ CDF = \frac{2\sqrt{2}}{\pi},\ DPF = 1$$

$$I_{S1} = \frac{2\sqrt{2}}{\pi} I_{DC},\ I_H = I_{DC}\sqrt{1-\frac{8}{\pi^2}}$$

THDF = 0.48, THDR = 0.44

(d) Full-Wave Capacitor Smoothed

$$V_R = \frac{I_{DC}}{2f_S C}\left(1-\frac{\theta_c}{\pi}\right),\ V_{R,rms} = \frac{V_R}{2\sqrt{3}},\quad \text{If } V_R = r.V_{DC}\text{: } V_{DC} = \frac{V_{pk}}{1+\frac{r}{2}}$$

$$\theta_c \approx 2\times a\cos\left(1-\frac{V_R}{\sqrt{2}V_S}\right) \text{ or } V_R \approx \sqrt{2}V_S\left(1-\cos\frac{\theta_c}{2}\right)$$

$$V_{DC} = V_{pk} - \frac{V_R}{2},\ V_m = \sqrt{2}V_S,\ V_{pk} = \sqrt{2}V_S - 2V_{Don}$$

$$I_m = 1.57\left(\frac{180}{\theta_c^o}\right) I_{DC},\ I_{O,dc} = \frac{2\theta_c}{\pi^2} I_m,\ I_{O,dc} = I_{DC}, I_{DC} = \text{av.load current}$$

$$I_{O,rms} = I_S = I_{DC}\sqrt{1.234\left(\frac{180}{\theta_c^o}\right)},\ I_C = I_S = I_{DC}\sqrt{1.234\left(\frac{180}{\theta_c^o}\right)-1}$$

3-phase diode rectifier

(a) Performance when the output current is ripple-free

$$S = \sqrt{2}V_{LL}I_{DC},\ PF = \frac{3}{\pi},\ V_{o,dc} = 1.35V_{LL} - 2V_{Don}$$

$$I_n = \frac{\sqrt{6}}{n} I_{DC},\ I_H = 0.24 I_{DC},\ I_S = \sqrt{\frac{2}{3}} I_{DC}$$

$$THD_F = 31\%,\ THD_R = 31\%$$

(b) Continuous Conduction Operation

$$V_{o,dc} = \frac{3\sqrt{2}}{\pi}V_{LL} - 2V_{Don},\ V_{o,rms} = V_{LL}\sqrt{1+\frac{3\sqrt{3}}{2\pi}} - 2V_{Don}$$

$$V_{on} = \frac{6}{(n^2-1)\pi}V_{LL},\ n = 6, 12, 18, \ldots,\ V_{o6} = \frac{6}{35\pi}V_{LL} \approx \frac{5.5}{100}V_{LL}$$

(c) half-wave

$$V_{o,dc} = \frac{3}{\sqrt{2}\pi}V_{LL} - V_{Don}$$

(d) definition of base values for using per-unit

$$V_{base} = V_{LL},\ f_{base} = 60\,\text{Hz},\ P_{base} = \text{rectifier input power}$$

$$I_{base} = \frac{P_{base}}{\sqrt{3}V_{LL}},\ X_{base} = R_{base} = \frac{V_{LL}}{\sqrt{3}I_{base}} = \frac{V_{LL}^2}{P_{base}}$$

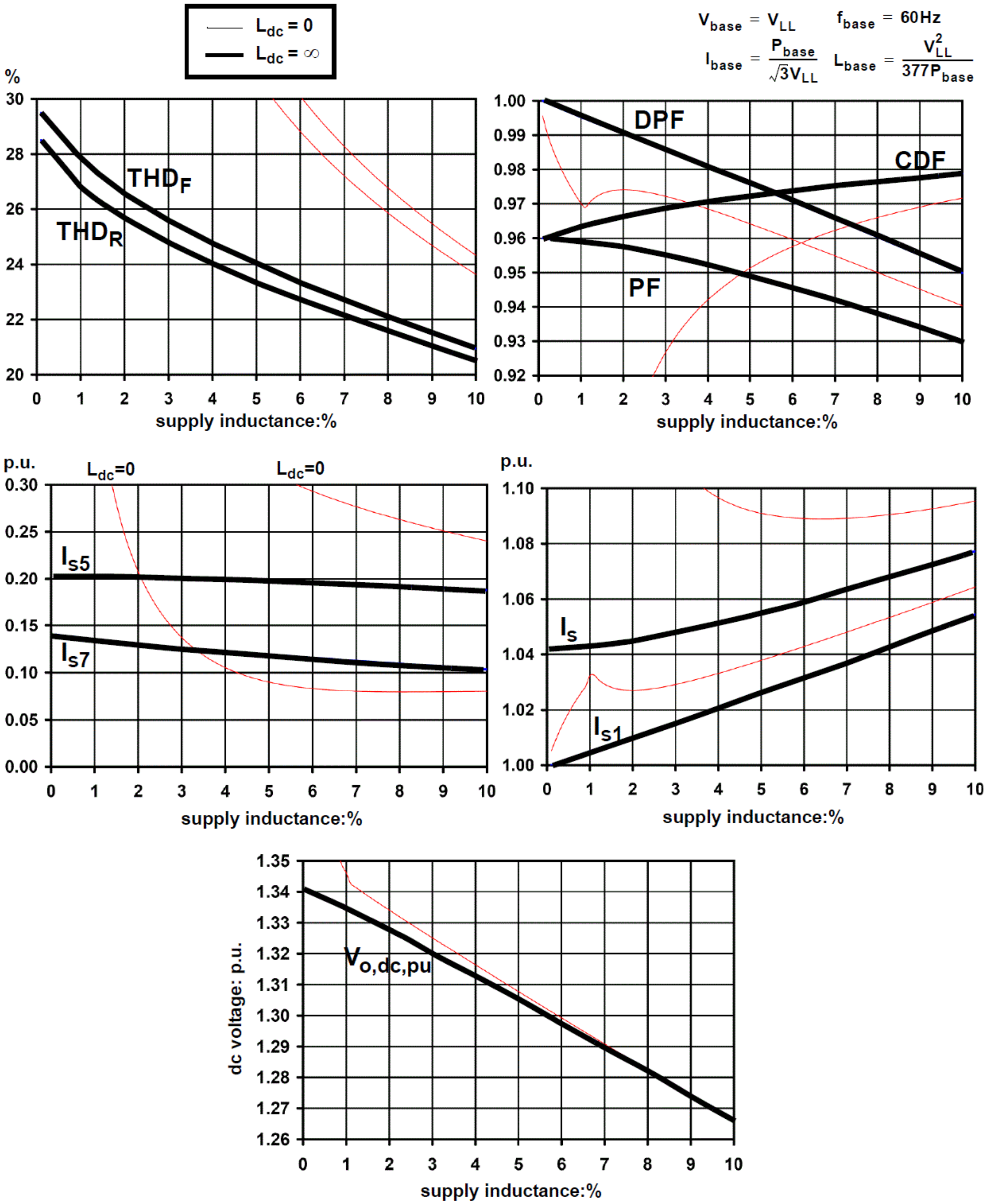
$$L_{base} = \frac{V_{LL}^2}{2\pi f_s P_{base}},\ L_{pu} = \frac{L}{L_{base}}$$

(e) Commutation Overlap

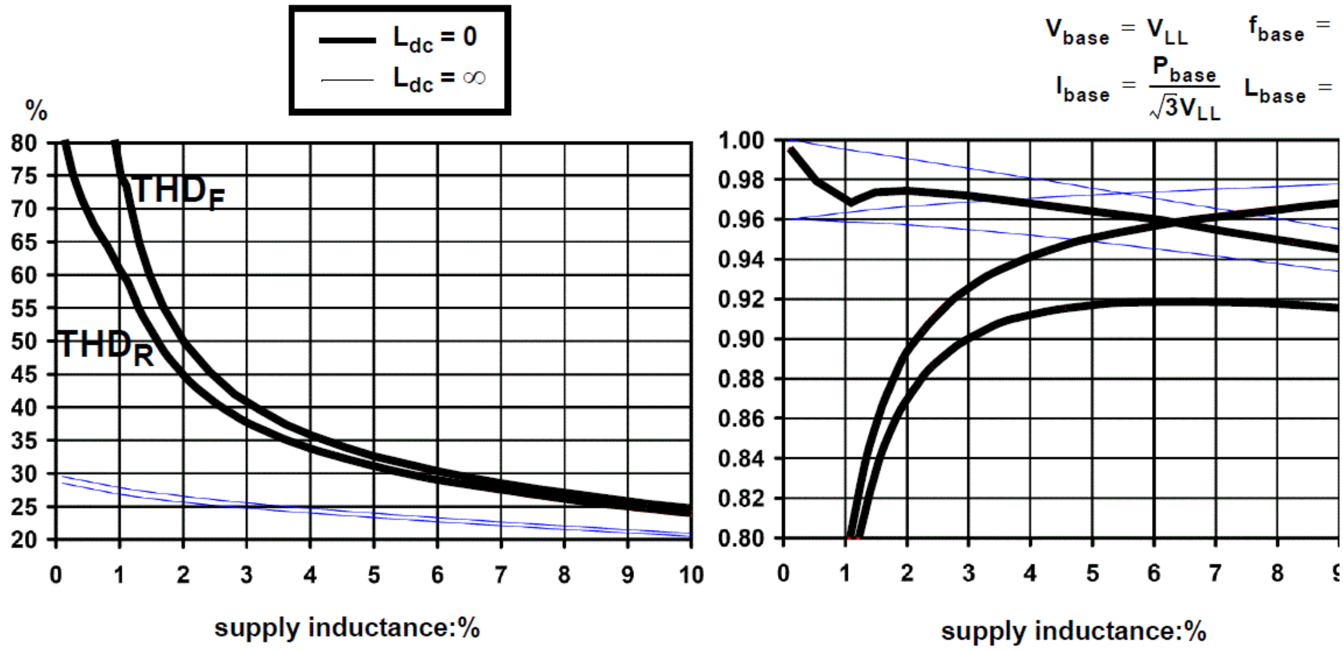
$$\mu = a\cos\left(1-\frac{\sqrt{2}I_{DC}X_s}{V_{LL:}}\right),\ V_{O,dc} = 1.35V_{LL} - 0.955 I_{DC}X_S$$

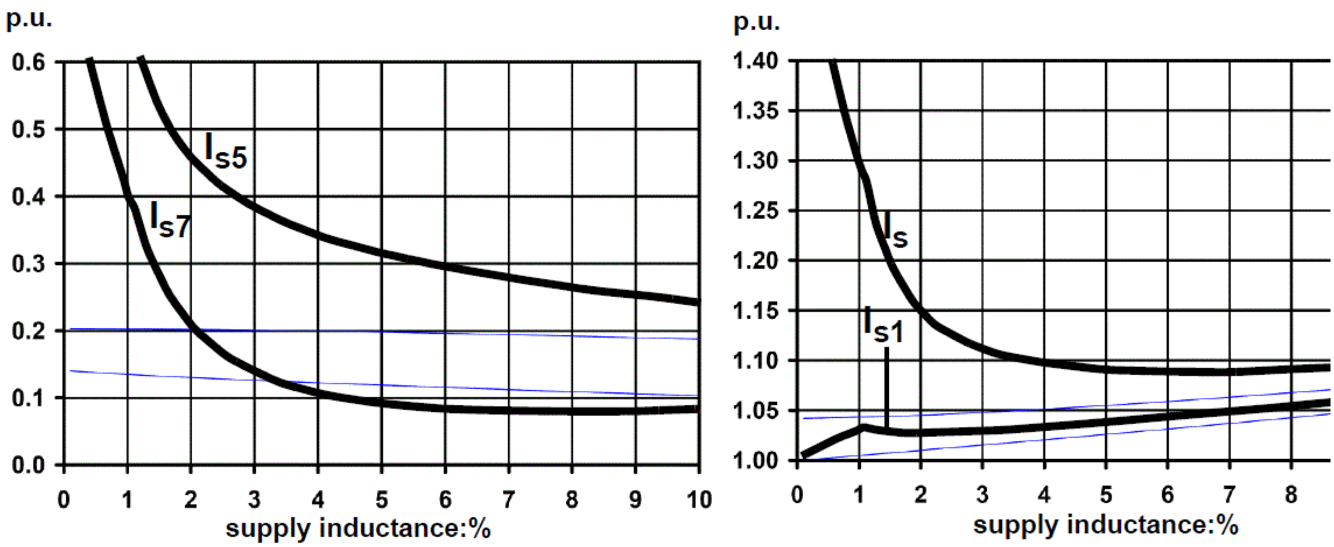
$$DPF = \frac{1+\cos\mu}{2},\ PF \approx 0.955\times DPF$$

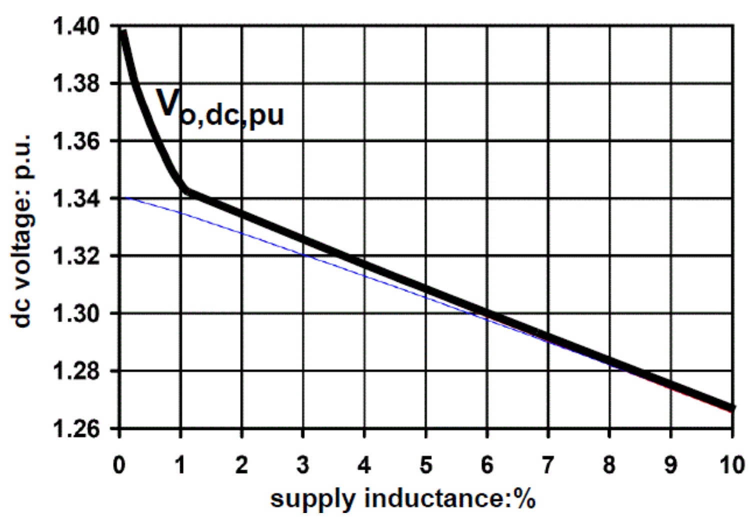
Performance of the 3-phase diode rectifier: $L_{dc} = \infty$



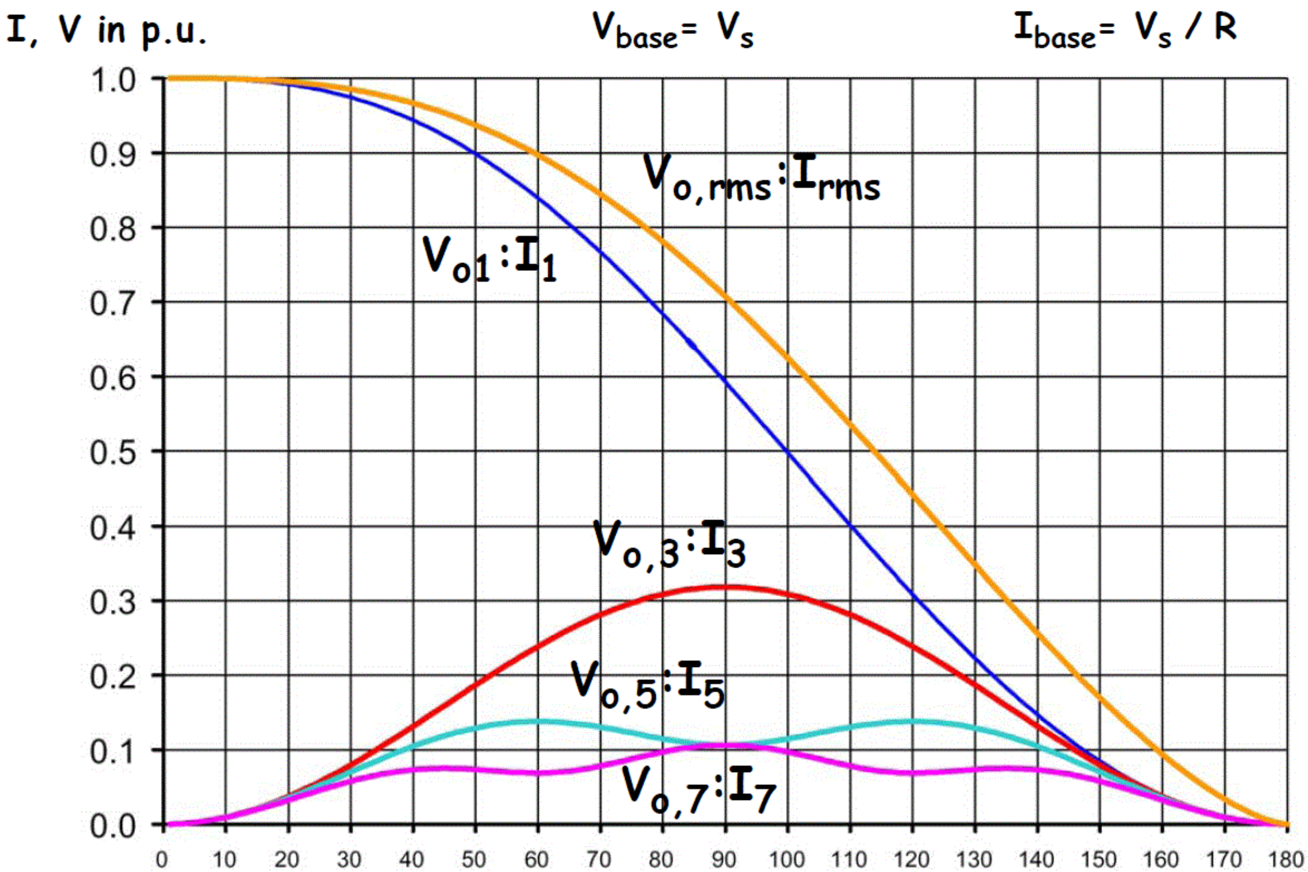
Performance of the 3-phase diode rectifier: $L_{dc} = 0$







Performance of the 1-phase ac regulator with an R-Load



1-phase full-wave regulator

R-load: $V_{o,rms} = V_S\sqrt{\dfrac{\pi-\alpha+\dfrac{\sin 2\alpha}{2}}{\pi}}$

$$I_{nr} = \frac{I_b}{\pi}\left(\frac{\sin(n+1)\alpha}{n+1}+\frac{\sin(n-1)\alpha}{n-1}\right)\quad n = 3, 5, 7, \ldots$$

$$I_{ni} = \frac{I_b}{\pi}\left(\frac{1-\cos(n-1)\alpha}{n-1}-\frac{1-\cos(n+1)\alpha}{n+1}\right)\quad n = 3, 5, 7, \ldots$$

$$I_{1r} = \frac{I_b}{\pi}\left(\pi-\alpha+\frac{\sin 2\alpha}{2}\right),\ I_{1i} = \frac{I_b}{\pi}\left(\frac{\cos 2\alpha-1}{2}\right),\ I_b = \frac{V_S}{R}$$

R-L Load: solve for β where: $\sin(\beta-\phi)-\sin(\alpha-\phi)e^{\frac{(\alpha-\beta)}{\tan\phi}} = 0$

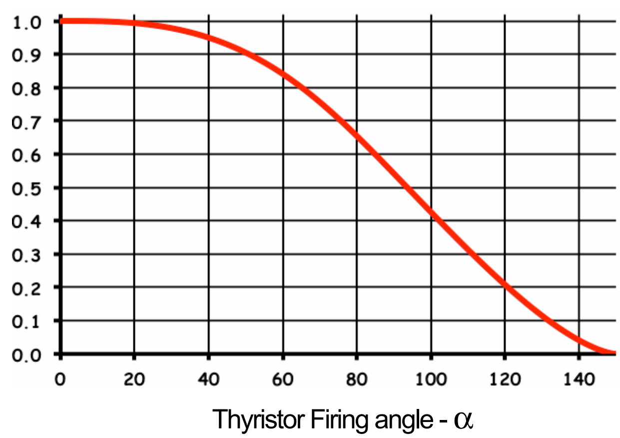
$$\phi = a\tan\left(\frac{\omega L}{R}\right)$$

3-phase ac regulators: Y connected R-load

$$V_{O,rms}^{pu} = \sqrt{1-\frac{3\alpha}{2\pi}+\frac{3}{4\pi}\sin 2\alpha}\,,0\le\alpha\le 60°$$

$$V_{O,rms}^{pu} = \sqrt{\frac{1}{2}+\frac{9}{8\pi}\sin 2\alpha+\frac{3\sqrt{3}}{8\pi}\cos 2\alpha}\,,\ 60°\le\alpha\le 90°$$

$$V_{O,rms}^{pu} = \sqrt{\frac{5}{4}-\frac{3\alpha}{2\pi}+\frac{3}{8\pi}\sin 2\alpha+\frac{3\sqrt{3}}{8\pi}\cos 2\alpha}\,,\ 90°\le\alpha\le 150°$$

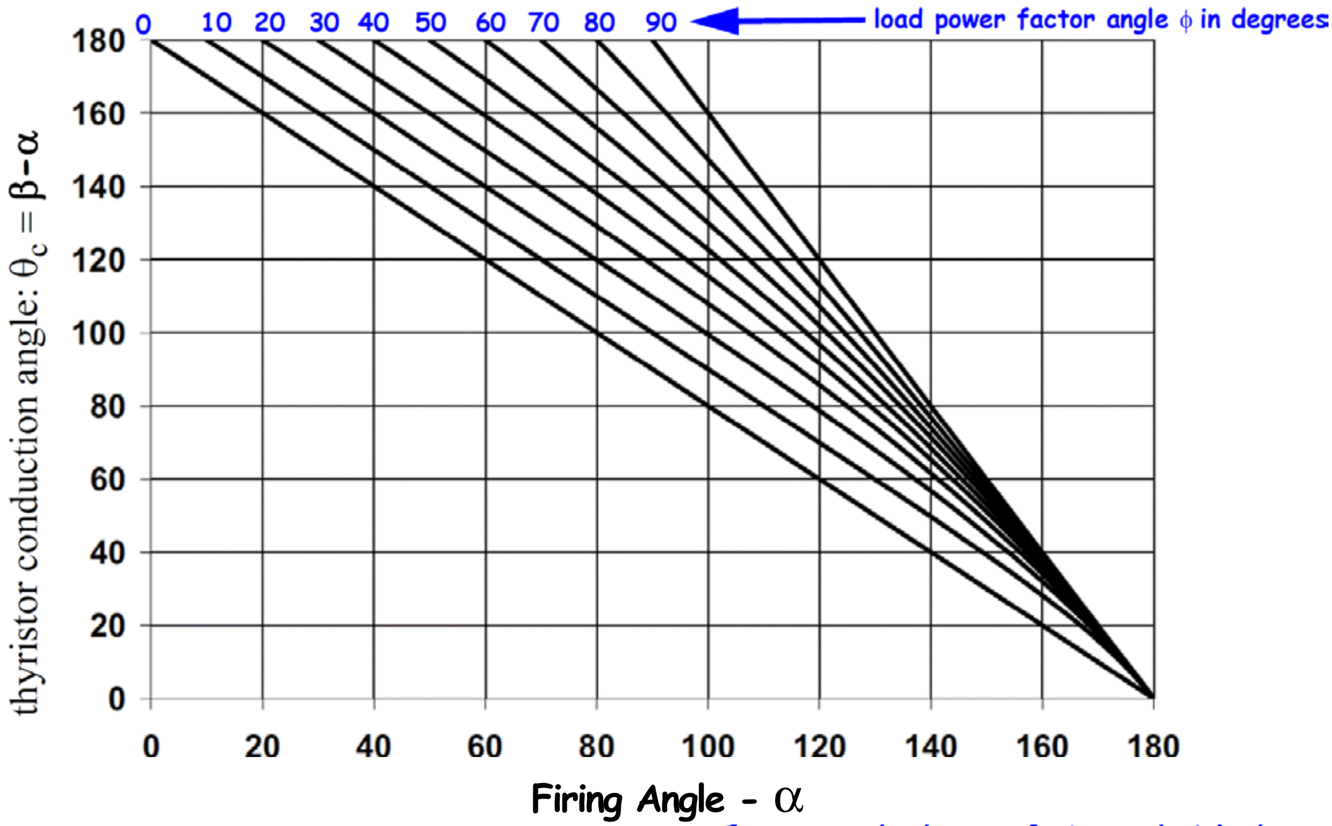


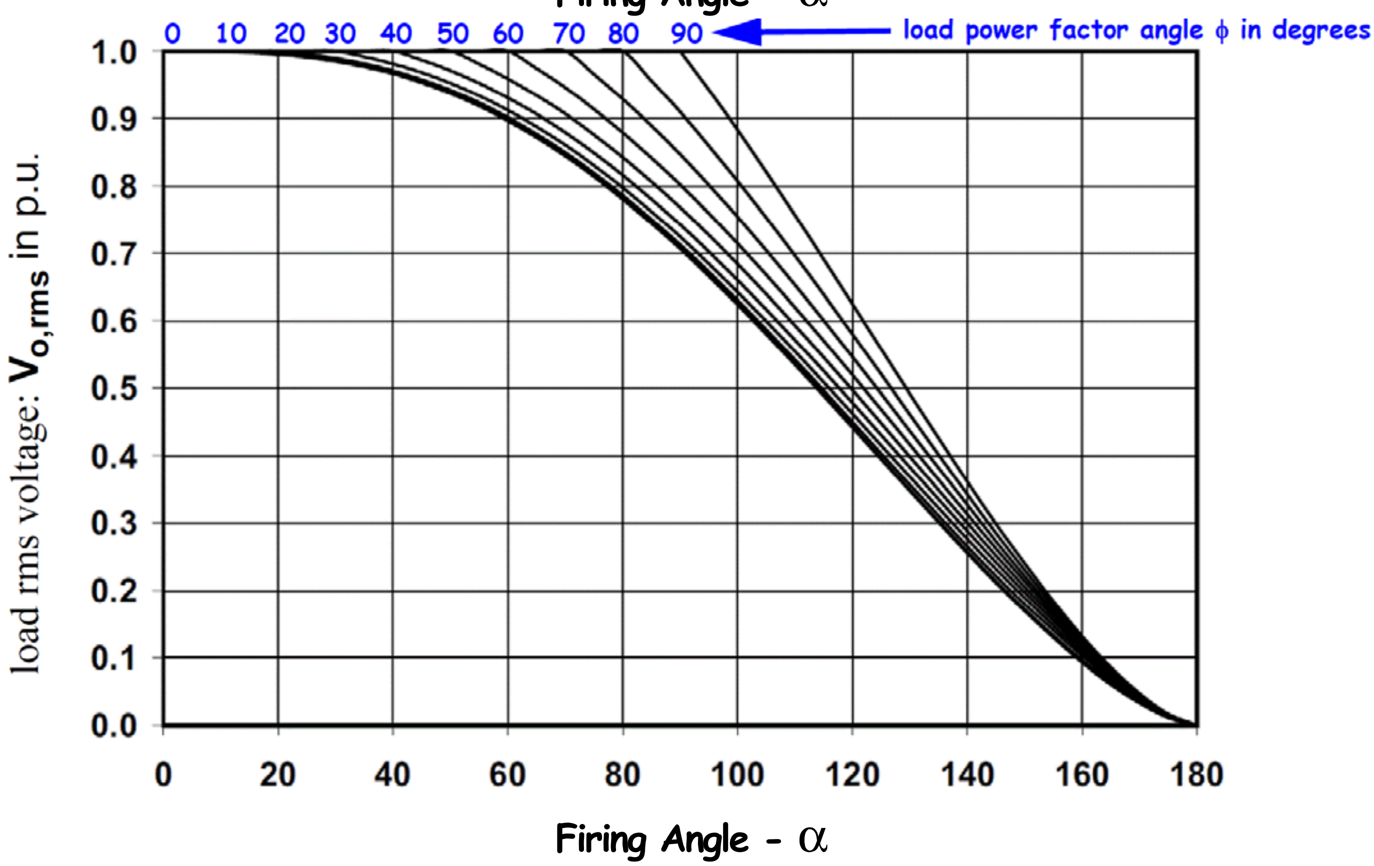
Heatsinks

$$t_j - t_a = P_D\times\left[R_{jc}+R_{ca}//\left(R_{ch}+R_{ha}\right)\right]$$

t_j = junction temperature: °C, t_c = case temperature: °C
t_h = heatsink temperature: °C, t_a = ambient temperature:
R_{jc} = junction to case thermal resistance °C/W
R_{ca} = case to ambient thermal resistance °C/W
R_{ch} = case to heatsink thermal resistance °C/W
R_{ha} = heatsink to ambient thermal resistance °C/W
P_D = power semiconductor power loss - W

Performance of 1-phase AC regulator with an R-L Load





Power Semiconductor Losses

$I_p = I_o + \frac{\Delta I_o}{2}, I_m = I_o - \frac{\Delta I_o}{2}$

switching losses: $I_p = I_o + \frac{\Delta I_o}{2}, I_m = I_o - \frac{\Delta I_o}{2}$

$P_1 = \frac{E \times I_m}{2} \times t_1 \times f_c \text{ watts}, \; P_2 = \frac{E \times I_p}{2} \times t_2 \times f_c \text{ watts}$

$P_{sw} = \frac{E \times I_O}{2} f_c (t_1 + t_2) \text{ watts, if } I_p = I_m = I_O$

conduction losses

$P_{on} = V_{on} \times I_{device,dc}, \; I_{device,dc} = \delta \frac{I_p + I_m}{2}$

$P_{on} = I^2_{device,rms} \times R_{on}, \; I_{device,rms} = \sqrt{\frac{\delta}{3}\left(I_p^2 + I_p I_m + I_m^2\right)}$

DC Choppers

Motoring: $V_{O,dc} = E_a + I_a R_a$

Regenerating: $V_{O,dc} = E_a - I_a R_a$

$E_a = K \times \Phi_f \times \omega_r$ if the field flux Φ_f varies

$E_a = K_1 \times \omega_r$ if the field flux Φ_f is constant

Motor output torque: $T_e = K_1 \times I_a$

Motor output power: $P_e = E_a \times I_a = T_e \times \omega_r$

Unipolar PWM: $V_{O,dc} = \delta V_{dc}, \; \Delta I_a = \frac{V_{dc}}{f_c L_a} \times \delta(1-\delta)$

Bipolar PWM: $V_{O,dc} = (2\delta - 1)V_{dc}, \; \Delta I_a = \frac{V_{dc}}{f_c L_a} \times 2\delta(1-\delta)$

Discontinuous conduction with unipolar pwm

critical conduction has: $\Delta I_a = 2 I_{O,dc} \Rightarrow \Delta I_a = \delta(1-\delta)\frac{V_{dc}}{f_c L_a}$

condition for always continuous conduction; $2\frac{I_a f_c L_a}{V_{dc}} \geq 0.25$

range of δ for disc. cond., solve for δ in: $\delta^2 - \delta + 2\frac{I_a f_c L_a}{V_{dc}} = 0$

load current: $I_{O,ac} = \frac{\Delta I_a}{2\sqrt{3}}$

switch currents:

$I_{rms} = \sqrt{\frac{\delta}{3}\left(I_p^2 + I_p I_m + I_m^2\right)}, \; I_{dc} = \frac{I_p + I_m}{2}, \; I_{ac} = \sqrt{I_{rms}^2 - I_{dc}^2}$

Assume: I_a = Imotor,dc = $I_{O,dc}$

$I_p = I_a + \frac{\Delta I_a}{2}, \; I_m = I_a - \frac{\Delta I_a}{2}$

(a) motor

$P_{Ra} = I^2_{motor,rms} R_a, \; P_{Ea} = P_e = E_a \times I_a$

$E_a = V_{O,dc} - I_a R_a = \delta V_{dc} - I_a R_a, \; T_e = \frac{P_e}{\omega_r}, \; P_{in} = P_{Ra} + P_{Ea}$

(b) mosfet: $P_{mos} = I^2_{switch,rms} R_{DSon}$ BJT: $P_{BJT} = I_{switch,dc} \times V_{CE,on}$

(c) diode: $P_D = I^2_{diode,rms} r_D + I_{diode,dc} V_{Don}$

1-phase H-bridge PWM Inverter

m_a = maximum amplitude modulation depth = $v_{s,pk}/v_{c,pk}$
m_a = maximum amplitude modulation depth= $v_{s,pk}$ / $v_{c,pk}$
$V_{s,pk}$ = peak of the sinusoidal reference signal
$V_{c,pk}$ = peak of the triangular carrier signal
E = bridge dc-link voltage V_O = bridge output voltage
$V_{O,1}$ = fundamental of V_O I_O = bridge output current
ΔI_O= pk-pk ripple current of I_O L= bridge output inductance
m_f = frequency modulation ratio= f_c/f_1
f_c = freq. of the carrier f_1 = freq. of the ref. signal

(a) Unipolar pwm switching: 0,±E

fundamental output voltage: $V_{O1} = \frac{m_a}{\sqrt{2}}E, \; V_{AN,1} = \frac{m_a}{2\sqrt{2}}E$

output pk-pk current ripple with an L filter: $\Delta I_O = \frac{E}{8 f_c L}$

(b) Bipolar pwm switching: ± E

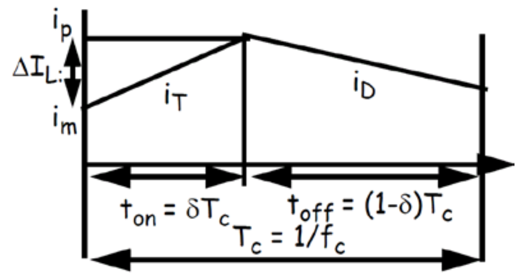
fundamental output voltage: $V_{O1} = \frac{m_a}{\sqrt{2}}E, \; V_{AN,1} = \frac{m_a}{2\sqrt{2}}E$

pk-pk current ripple with an L filter $\Delta I_O = \frac{V_{dc}}{2 f_c L}$

DC DC Converters: Continuous Conduction: $I_O > I_{OB}$

$I_O \geq I_{OB}$ $\qquad f_c = \dfrac{1}{T_c}$

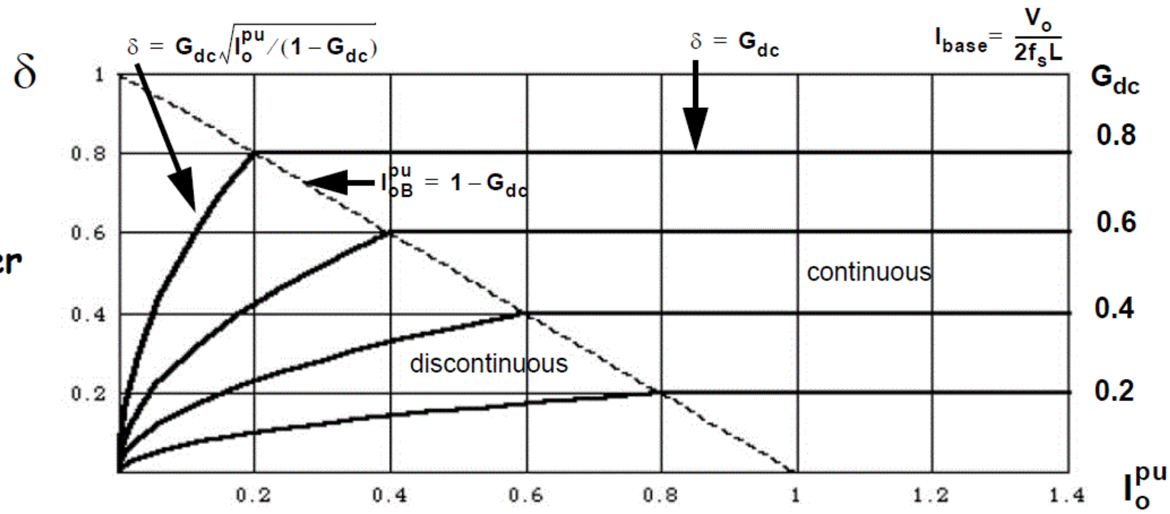
$G_{dc} = \dfrac{V_o}{V_i}$ $\qquad \delta = \dfrac{t_{ON}}{T_c}$



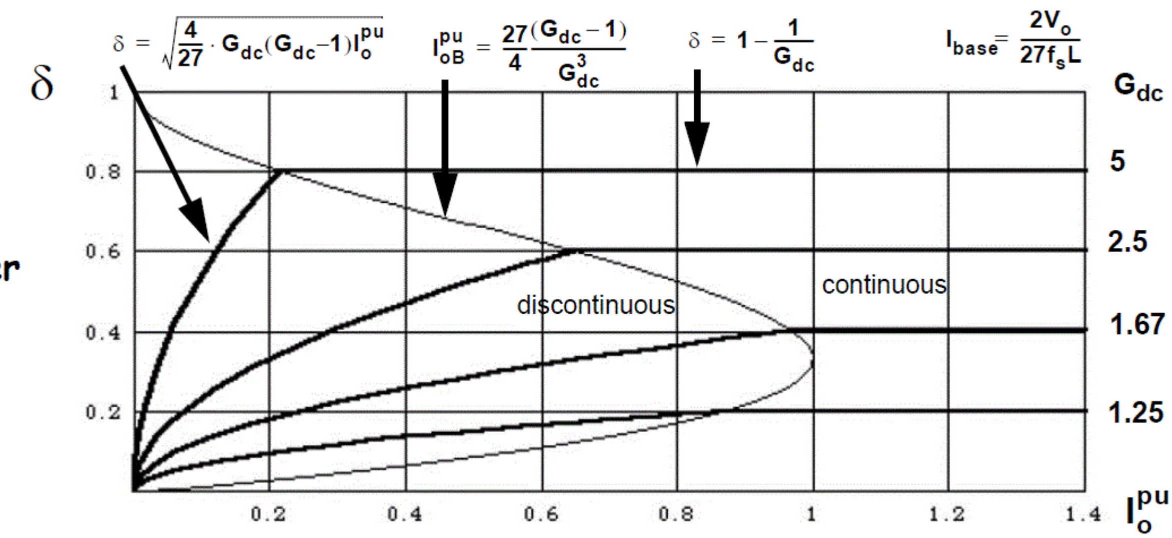
	Buck	Boost	Buck-Boost
I_{base}	$\dfrac{V_O}{2f_cL}$	$\dfrac{2}{27}\times\dfrac{V_O}{f_cL}$	$\dfrac{V_O}{2f_cL}$
I_{OB}^{pu}	$1-G_{dc}$	$\dfrac{27}{4}\dfrac{\left(G_{dc}-1\right)}{G_{dc}^3}$	$\dfrac{1}{\left(G_{dc}+1\right)^2}$
G_{dc}	δ	$\dfrac{1}{1-\delta}$	$\dfrac{\delta}{1-\delta}$
δ	G_{dc}	$1-\dfrac{1}{G_{dc}}$	$\dfrac{G_{dc}}{G_{dc}+1}$
ΔI_L	$\dfrac{\left(V_i-V_o\right)}{f_cL}\delta$	$\dfrac{V_i}{f_cL}\delta$	$\dfrac{V_i}{f_cL}\delta$
$I_p \quad I_m$	$I_p = I_O + \dfrac{\Delta I_L}{2}$ $I_m = I_O - \dfrac{\Delta I_L}{2}$	$I_p = I_i + \dfrac{\Delta I_L}{2}$ $I_m = I_i - \dfrac{\Delta I_L}{2}$	$I_p = I_i + I_O + \dfrac{\Delta I_L}{2}$ $I_m = I_i + I_O - \dfrac{\Delta I_L}{2}$

DC-DC Converters: Performance Curves

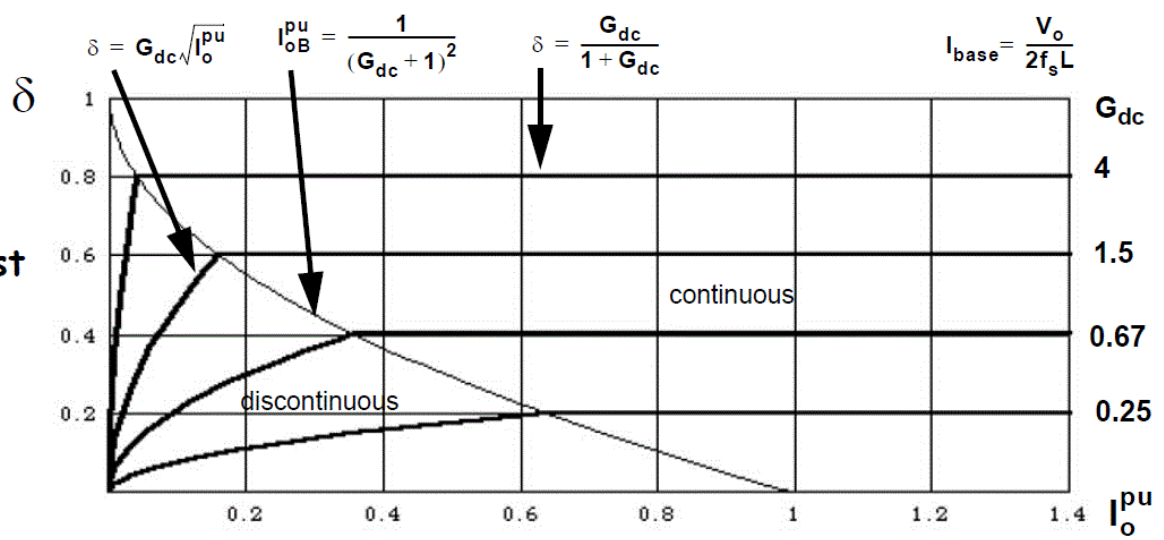
Buck Converter



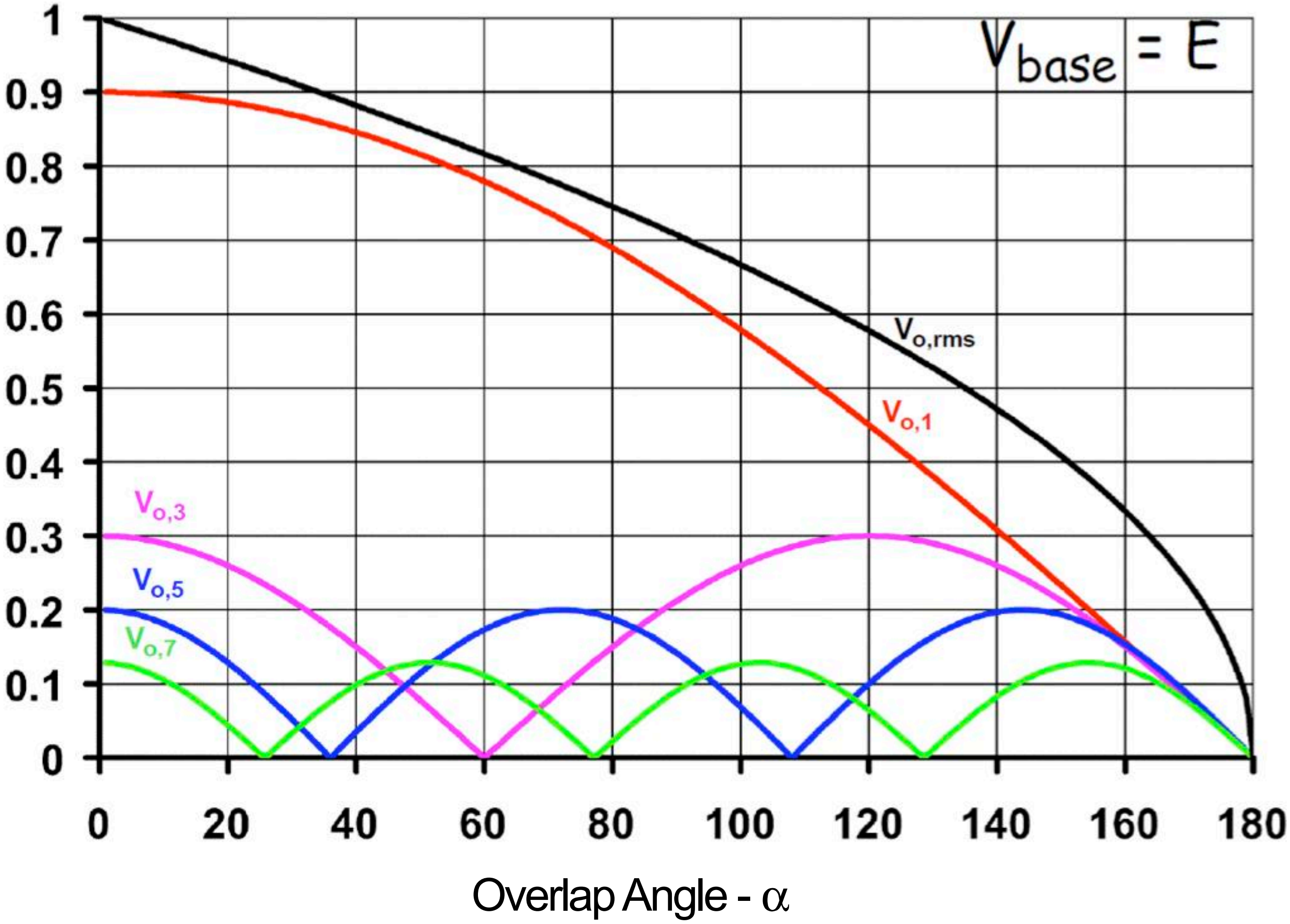
Boost Converter



Buck-Boost Converter



1-Phase H-Bridge Performance Curves with Square-Wave Switching



E = dc-link voltage

V_O = bridge output voltage $V_{O,n}$ = n^{th} harmonic

α = overlap angle in deg./rad. $V_{O,1}$ = fund. harmonic of V_O

$V_{o,rms}$ = rms of V_O n = n^{th} harmonic

$$V_{O,n} = \frac{0.9}{n} E \cos\left(n\frac{\alpha}{2}\right),\ n = 3, 5, 7, \ldots$$

$$V_{O,1} = 0.9E\cos\left(\frac{\alpha}{2}\right)$$

harmonics of a square-wave voltage magnitude V_o

assume perfect square wave, $\alpha = 0°$, V_o = E

$$V_n = \frac{4}{\pi} V_O,\ n = 1, 3, 5, 7, 9, 11, 13, \text{ etc..}$$

$$\Delta I_{O,max} = \frac{E}{2f_s L}\left(1 - \frac{8}{\pi^2}\right) = 0.189\left(\frac{E}{2f_s L}\right)$$