## Grant Molnar

# Curriculum Vitae

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#### Education

PhD, Mathematics, Dartmouth College, Hanover, NH Fall 2018 - Spring 2023

Advisor: John Voight

Thesis: Counting elliptic curves with a cyclic m-isogeny over  $\mathbb Q$ 

MA, Mathematics, Dartmouth College, Hanover, NH Fall 2018 - Winter 2020

Advisor: John Voight

MS, Mathematics, Brigham Young University, Provo, UT Winter 2017 - Summer 2018

Advisors: Michael Griffin and Paul Jenkins Thesis: *The arithmetic of modular grids* 

**BS, Mathematics**, *Brigham Young University*, Provo, UT Fall 2013 - Fall 2016

### Professional Experience

#### Academic Researcher at BYU and Dartmouth

Fall 2013 - Spring 2023

At BYU, I proved results about modular forms and algorithms on graphs. At Dartmouth, I instigated successful research into summing divergent series, the Prisoner's Dilemma, and the Ramanujan-Robin criterion. My thesis develops the arithmetic statistics of elliptic curves with cyclic m-isogeny. See Research Publications and Preprints below.

#### Math Instructor at BYU and Dartmouth

Winter 2017 - Spring 2023

I taught 30-student calculus courses in three different styles: lecturing, virtually with a "flipped classroom", and with handouts and group exercises. I TAed for lower-division courses, and by special request for upper-division courses as well. See Teaching Experience below.

### Computer Skills

#### Fluent in...

 $\mathsf{Bash} \bullet \mathsf{Git} \bullet \mathsf{Go} \bullet \mathsf{E}\mathsf{T}_{\mathsf{F}}\mathsf{X} \bullet \mathsf{Python} \bullet \mathsf{R} \bullet \mathsf{SageMath} \bullet \mathsf{YAML}$ 

#### Experience with...

C++ • HTML • Magma • Mathematica • MatLab • MongoDB • PARI/GP • Rust

#### Research Publications

Counting elliptic curves over the rationals with a 7-isogeny, with John Voight

Research in Number Theory **9**, 75 (2023). arXiv: 2212.11354

We count by height the number of elliptic curves over the rationals, both up to isomorphism over the rationals and over an algebraic closure thereof, that admit a cyclic isogeny of degree 7.

# Reactive means in the Iterated Prisoner's Dilemma, with Caroline Hammond and Feng Fu Applied Mathematics and Computation 458 128201 (2023). arXiv: 2302.13909

The Iterated Prisoner's Dilemma (IPD) is a well studied framework for understanding direct reciprocity and cooperation in pairwise encounters. However, measuring the morality of various IPD strategies is still largely lacking. Here, we partially address this issue by proposing a suit of plausible morality metrics to quantify four aspects of justice. We focus our closed-form calculation on the class of reactive strategies because of their mathematical tractability and expressive power. We define reactive means as a tool for studying how actors in the IPD and Iterated Snowdrift Game (ISG) behave under typical circumstances. We compute reactive means for four functions intended to capture human intuitions about "goodness" and "fair play". Two of these functions are strongly anticorrelated with success in the IPD and ISG, and the other two are weakly anticorrelated with success. Our results will aid in evaluating and comparing powerful IPD strategies based on machine learning algorithms, using simple and intuitive morality metrics.

# The arithmetic of modular grids, with Michael Griffin and Paul Jenkins Mathematika 68, 1080-1119 (2022). arXiv: 2012.14403

A modular grid is a pair of sequences  $(f_m)_m$  and  $(g_n)_n$  of weakly holomorphic modular forms such that for almost all m and n, the coefficient of  $q^n$  in  $f_m$  is the negative of the coefficient of  $q^m$  in  $g_n$ . Zagier proved this coefficient duality in weights 1/2 and 3/2 in the Kohnen plus space, and such grids have appeared for Poincaré series, for modular forms of integral weight, and in many other situations. We give a general proof of coefficient duality for canonical row-reduced bases of spaces of weakly holomorphic modular forms of integral or half-integral weight for every group  $\Gamma \subseteq \operatorname{SL}_2(\mathbb{R})$  commensurable with  $\operatorname{SL}_2(\mathbb{Z})$ . We construct bivariate generating functions that encode these modular forms, and study linear operations on the resulting modular grids.

# **Odd, spoof perfect factorizations**, with the BYU Computational Number Theory Group Journal of Number Theory **234**, 31-47 (2022). Quanta article. arXiv: 2006.10697

We investigate the integer solutions of Diophantine equations related to perfect numbers. These solutions generalize the example, found by Descartes in 1638, of an odd, "spoof" perfect factorization  $3^2 \cdot 7^2 \cdot 11^2 \cdot 22021^1$ . More recently, Voight found the spoof perfect factorization  $3^4 \cdot 7^2 \cdot 11^2 \cdot 19^2 \cdot (-127)^1$ . No other examples appear in the literature. We compute all nontrivial, odd, primitive spoof perfect factorizations with fewer than seven bases – there are twenty-one in total. We show that the structure of odd, spoof perfect factorizations is extremely rich, and there are multiple infinite families of them. This implies that certain approaches to the odd perfect number problem that use only the multiplicative nature of the sum-of-divisors function are unworkable. On the other hand, we prove that there are only finitely many nontrivial, odd, primitive spoof perfect factorizations with a fixed number of bases.

# Zagier duality for level p weakly holomorphic modular forms, with Paul Jenkins

The Ramanujan Journal 50, 93-109 (2019). arXiv: 1709.10023

We prove Zagier duality between the Fourier coefficients of canonical bases for spaces of weakly holomorphic modular forms of prime level p with  $11 \le p \le 37$  with poles only at the cusp at  $\infty$ , and special cases of duality for an infinite class of prime levels. We derive generating functions for the bases for genus 1 levels.

### Graphs with the strong Havel-Hakimi property, with Michael Barrus

Graphs and Combinatorics 32, 1689-1697 (2016). arXiv: 1505.00085

The Havel–Hakimi algorithm iteratively reduces the degree sequence of a graph to a list of zeroes. As shown by Favaron, Mahéo, and Saclé, the number of zeroes produced, known as the residue, is a lower bound on the independence number of the graph. We say that a graph has the strong Havel–Hakimi property if in each of its induced subgraphs, deleting any vertex of maximum degree reduces the degree sequence in the same way that the Havel–Hakimi algorithm does. We characterize graphs having this property (which include all threshold and matrogenic graphs) in terms of minimal forbidden induced subgraphs. We further show that for these graphs the residue equals the independence number, and a natural greedy algorithm always produces a maximum independent set.

### Research Preprints

A database of basic numerical invariants of Hilbert modular surfaces, with Eran Assaf, Angelica Babei, Ben Breen, Edgar Costa, Juanita Duque-Rosero, Aleksander Horawa, Jean Kieffer, Avinash Kulkarni, Sam Schiavone, and John Voight

Preprint. arXiv: 2301.10302

We describe algorithms for computing geometric invariants for Hilbert modular surfaces, and we report on their implementation.

#### Multiplicative summations into algebraically closed fields, with Robert Dawson

Preprint. arXiv: 2111.09938

In this paper, extending our earlier program, we derive maximal canonical extensions for multiplicative summations into algebraically closed fields. We show that there is a well-defined analogue to minimal polynomials for a series algebraic over a ring of series, the "scalar polynomial". When that ring is the domain of a summation  $\mathfrak{S}$ , we derive the related concepts of the  $\mathfrak{S}$ -minimal polynomial for a series, which is mapped by  $\mathfrak{S}$  to a scalar polynomial. When the scalar polynomial for a series has the form  $(t-a)^n$ , a is the unique value to which the series can be mapped by an extension of the original summation.

# Telescopic, multiplicative, and rational extensions of summations, with Robert Dawson Preprint. arXiv: 2105.04592

A summation is a shift-invariant R-module homomorphism from a submodule of R to R or another ring. Dawson formalized a method for extending a summation to a larger domain by telescoping. In this paper, we revisit telescoping, we study multiplicative closures of summations (such as the usual summation on convergent series) that are not themselves multiplicatively closed, and we study rational extensions as a generalization of telescoping.

## **Expository Notes**

#### Fast-growing series are transcendental, with Robert Dawson

Unpublished. arXiv: 2102.12995

Let R be a subring of  $\mathbb{C}[[z]]$ , and let  $X \in \mathbb{C}[[z]]$ . The Newton-Puiseux Theorem implies that if the coefficients of X grow sufficiently rapidly relative to the coefficients of the series in R, then X is transcendental over R. We provide an alternative proof of this result by establishing a relationship between the coefficients of A(X) and A'(X), where A(T) is a polynomial over  $\mathbb{C}[[z]]$ .

## Manuscripts in Preparation

A family of analogues to the Ramanujan-Robin criterion, with Steve Fan and Mits Kobayashi Odd, spoof quasiperfect factorizations, with Jonathon Hales Visualizing Basel's problem, with Mits Kobayashi

### Selected Git Projects

#### **Spoof Quasiperfect Factorizations**

Fall 2021

 $\verb|https://github.com/grantmolnar/Spoof-Quasiperfect-Factorizations|\\$ 

## Awards and Fellowships

Dartmouth Graduate Fellowship

Gridley Fund for Graduate Mathematics

NSF Graduate Research Fellowship Honorable Mention

BYU Academic Scholarship

BYU Math Department Scholarship

Fall 2018 - Summer 2019

April 2018 - Summer 2019

Spring 2015 - Winter 2016

Fall 2016

BYU Math Department Scholarship	Fall 2015
BYU Academic Scholarship	Fall 2013 - Winter 2014
Teaching Experience	
Dartmouth College, Hanover, NH (Instructor)	
Math 8 (Calculus of One and Several Variables)	Fall 2021
Math 1 (Algebra and Calculus)	Fall 2020
Dartmouth College, Hanover, NH (Teacher's Assistant)	
Math 100 / Computer Science 49/149 (Random Walk)	Spring 2022
Math 100 / Computer Science 49/149 (Decision Theory)	Spring 2021
Math 23 (Differential Equations)	Winter 2020
Math 3 (Calculus)	Fall 2019
Math 22 (Linear Algebra)	Spring 2019
Math 22 (Linear Algebra)	Fall 2018
Brigham Young University, Provo, UT (Instructor)	
Math 112 (Calculus I)	Summer 2017
Brigham Young University, Provo, UT (Teacher's Assistant)	
Math 112 (Calculus I)	Fall 2017
Math 113 (Calculus II)	Winter 2017
	Willter 2017
Brigham Young University, Provo, UT (Grader)	
Math 570 (Matrix Analysis)	Winter 2017
Math 112 (Calculus I)	Fall 2015
Thesis Defenses	
Counting elliptic curves with a cyclic $m$ -isogeny over $\mathbb Q$	April 24, 2023
Dartmouth College PhD Thesis Defense	·
The arithmetic of modular grids	June 22, 2018
BYU Master's Thesis Defense	
Invited Talks	
Counting elliptic curves with a cyclic $m$ -isogeny	February 9, 2023
Job Talk at Metron	
Counting elliptic curves with a 7-isogeny	January 19, 2023
BYU Colloquium	
Counting elliptic curves with a 7-isogeny	January 11, 2023
Simons Collaboration Annual Meeting	
Counting 7-isogenies	January 6, 2023
2023 Joint Mathematics Meeting	
A family of analogues to the Ramanujan-Robin Criterion BYU Number Theory Seminar	October 27, 2022

#### Other Conference Talks Intersecting varieties with transcendental graphs March 8, 2023 Arizona Winter School 2023: Unlikely Intersections The LCM product and Grönwall's theorem October 3, 2021 2021 Maine-Québec Number Theory Conference (virtual) Formal summation of divergent series September 26, 2020 2020 Conférence de Théorie des Nombres Québec-Maine (virtual) The arithmetic of modular grids October 5, 2019 2019 Maine-Québec Number Theory Conference The arithmetic of modular grids March 8, 2019 33<sup>rd</sup> Automorphic Forms Workshop The arithmetic of modular grids July 17, 2018 Building Bridges: 4th EU/US Summer School + Workshop on Automorphic Forms Zagier duality in level p modular spaces March 21, 2018 32<sup>nd</sup> Automorphic Forms Workshop Zagier duality in level p modular spaces March 3, 2018 2018 BYU Student Research Conference Zagier duality in level p modular spaces May 24, 2017 Modular Forms are Everywhere Conference Weakly holomorphic modular forms of level 11 March 7, 2017 31st Automorphic Forms Workshop Weakly holomorphic modular forms of level 11 March 4, 2017 2017 BYU Student Research Conference Congruence relations in modular forms of prime levels greater than 7 March 21, 2015 2015 BYU Student Research Conference Residues and independence numbers of graphs March 15, 2014 2014 BYU Student Research Conference Other Seminar Talks The Fox H-function March 28, 2023 Dartmouth Graduate Student Seminar Geometric deep learning February 14, 2023 Dartmouth Graduate Student Seminar **Nearing nearrings** September 14, 2022 Dartmouth Graduate Student Seminar Universal algebra and coalgebra March 30, 2022 Dartmouth Graduate Student Seminar February 16, 2022 Cogalois theory Dartmouth Graduate Student Seminar Cyclic resolution of singularities January 25, 2022 Reading Seminar on Hilbert Modular Surfaces What if the Riemann hypothesis is false? January 19, 2022

Dartmouth Graduate Student Seminar

Inverse semigroups: groups without identity	November 9, 2021
Dartmouth Graduate Student Seminar	November 8, 2021
Counting 7-isogenies  Dartmouth Algebra and Number Theory Seminar	November 0, 2021
A sober look at pointless topology	September 21, 2021
Dartmouth Graduate Student Seminar	September 21, 2021
Fast-growing series are transcendental	April 21, 2021
Dartmouth Graduate Student Seminar (virtual)	·
Coalgebras and Hopf algebras	April 15, 2021
Reading Seminar on Affine Group Schemes (virtual)	
Absurd equalities and Runge's method: the degenerate case	March 30, 2021
Dartmouth Algebra and Number Theory Seminar (virtual)	
p-adic Hodge theory	March 2, 2021
Reading Seminar on Classical and Quadratic Chabauty (virtual)	
Odd, spoof quasiperfect factorizations	February 9, 2021
Dartmouth Algebra and Number Theory Seminar (virtual)	
A primer in social choice theory	January 20, 2021
Dartmouth Graduate Student Seminar (virtual)	
Examples of Kedlaya's algorithm	November 19, 2020
Reading Seminar on Classical and Quadratic Chabauty (virtual)	
The LCM product and Grönwall's theorem	November 17, 2020
Dartmouth Algebra and Number Theory Seminar (virtual)	
Reactive means and the prisoner's dilemma	October 7, 2020
Dartmouth Graduate Student Seminar (virtual)	
Formal summation of divergent series: an algebraic approach	April 28, 2020
Dartmouth Algebra and Number Theory Seminar (virtual)	
On the infinitude of the natural numbers	February 12, 2020
Dartmouth Graduate Student Seminar	
Variations of Hodge structures	November 26, 2019
Reading Seminar on Shimura Varieties	N
Hodge structures	November 14, 2019
Reading Seminar on Shimura Varieties	Navarahar 12, 2010
Savage's expected utility and making good decisions  Dartmouth Graduate Student Seminar	November 13, 2019
Geometric and generalized calculus	October 8, 2019
Dartmouth Graduate Student Seminar	October 0, 2013
Summing divergent series	July 31, 2019
Dartmouth Graduate Student Seminar	•
Real analysis: a nonstandard approach	April 17, 2019
Dartmouth Graduate Student Seminar	
Why save the universe? Set theory with a universal set	January 16, 2019
Dartmouth Graduate Student Seminar	Ctl 00 0010
The arithmetic of modular grids	September 26, 2018
Dartmouth Graduate Student Seminar	

Dioids and idempotent geometry II	March 13, 2018
BYU Algebra Seminar  Dioids and idempotent geometry I  BYU Algebra Seminar	February 27, 2018
<b>Zagier duality in level</b> $p$ <b>modular spaces</b> BYU Number Theory Seminar	February 22, 2018
Average values of arithmetic functions BYU Number Theory Seminar	October 12, 2017
Weakly holomorphic modular forms of level 11 BYU Number Theory Seminar	February 9, 2017
Generating functions for canonical bases of certain level 11 weakly holomorphic modular forms BYU Number Theory Seminar	January 26, 2017
High rank elliptic curves with prescribed torsion BYU Number Theory Seminar	December 1, 2016
Heuristics for elliptic curves of high rank BYU Number Theory Seminar	October 20, 2016
A lemma regarding the Feit-Thompson conjecture BYU Number Theory Seminar	November 12, 2015
Algebraic number theory and the Feit-Thompson conjecture BYU Number Theory Seminar	October 1, 2015
Congruence relations in modular forms of prime levels greater than $7$ BYU Number Theory Seminar	March 19, 2015
Hensel's lemma BYU Number Theory Seminar	February 12, 2015
Newton polynomials BYU Number Theory Seminar	March 27, 2014
Leadership and Community Service	
Referee Journal of Number Theory Mathematische Zeitschrift	2018 - Present 2022 - Present
Reviewer zbMATH Open	2021 - Present
K-12 Outreach Utah Math Olympiad Committee Member Activity Leader at Dartmouth Science Day (Games & Surreal Numbers) Activity Leader at Lebanon Ward Pi Day	2015 - Present April 8, 2023
(Trachtenberg Arithmetic & Random Walk) Session Leader at Dartmouth Sonia Kovalevsky Day	March 25, 2023
(Trachtenberg Arithmetic) Activity Leader at Lebanon Ward Pi Day (Random Walk)	May 21, 2022 April 8, 2022

Exploring Mathematics Camp Leader (Graph Theory)	July 27-31, 2020
Exploring Mathematics Camp Leader (Cryptography)	July 13-17, 2020
Activity Leader at Lebanon Ward Pi Day (Random Walk)	March 7, 2020
Volunteer at Dartmouth Sonia Kovalevsky Day	May 11, 2019
Activity Leader at Dartmouth Science Day (Nim)	May 4, 2019
Math Circles Guest Speaker (Tropical Algebra)	February 24, 2018
Math Circles Guest Speaker (Continued Fractions)	October 21, 2017
Proofreader for Utah State Math Contest	February 2017
Math Circles Counselor	2013 - 2015
Dartmouth Graduate Student Council	
Ad Hoc Healthcare Committee Founder and Co-Chair	2022 - 2023
<ul> <li>Budget Committee Member</li> </ul>	Summer 2022
Finance Officer	2021 - 2022
<ul> <li>Budget Committee Member</li> </ul>	Summer 2021
Representative for Math Department	2020 - 2021
<ul> <li>Service Committee Member</li> </ul>	2020 - 2021
<ul> <li>Budget Committee Member</li> </ul>	Summer 2020
Dartmouth Directed Reading Program	
Mentor (Fuzzy Logic)	Winter 2023
Mentor (Decision Theory)	Spring 2022
Mentor (Fractional Calculus)	Winter 2022
Other	
LMFDB Contributor	2019 - 2023
Dartmouth Algebra and Number Theory Seminar Organizer	2019 - 2022
BYU Putnam Team Captain	2014 - 2016
Other	
Achievements and Honors	
$3^{ m rd}$ Place in Virginia Tech Regional Math Competition	October 2016
$2^{ m nd}$ Place in Search for High Rank Elliptic Curve	
at 2016 Connecticut Summer School in Number Theory	August 2016
Gold Palm Eagle Scout	December 2012
Black Belt (Taekwondo)	August 2012
Special Training	
Dartmouth Mathematics Teaching Seminar	Spring 2020
BYU Mathematics Teaching Seminar	Fall 2018
Academic Interests	
Analytic Number Theory • Arithmetic Geometry • Game Theory • Logic	
Personal Interests	
Reading ● Writing ● Philosophy ● Hiking	

Conferences and Workshops Attended

2024 Cimona Callabanation on Anithmatic Connection	
2024 Simons Collaboration on Arithmetic Geometry,	January 11 12 2024
Number Theory, and Computation Annual Meeting Virtual (Simons Foundation's Gerald D. Fischbach Auditorium, New `	January 11-12, 2024
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	August 31 - September 1, 2022
Virtual (Harvard University, Cambridge)	NA 0.10 0000
Hilbert Modular Forms Infrastructure Week 4	May 8-12, 2023
Dartmouth College, Hanover	
Arizona Winter School 2023: Unlikely Intersections	March 4-8, 2023
University of Arizona, Tucson	
2023 Simons Collaboration on Arithmetic Geometry,	
Number Theory, and Computation Annual Meeting	January 11-12, 2023
Simons Foundation's Gerald D. Fischbach Auditorium, New York City	
2023 Joint Mathematics Meetings	January 4-7, 2023
Hynes Convention Center, Boston	D 1 10 10 000
Hilbert Modular Forms Infrastructure Week 3	December 12-16, 2022
Dartmouth College, Hanover	
Modular Curves Workshop 2	November 5-9, 2022
MIT, Boston	A
Big Data Conference 2022	August 26, 2022
Virtual (Harvard University, Cambridge)	
Explicit Methods for Modularity	April 11-15, 2022
Virtual (Simons Collaboration)	
Arizona Winter School 2022: Automorphic Forms Beyond $\mathrm{GL}_2$	March 5-9, 2022
Virtual (University of Arizona, Tucson)	
Hilbert Modular Forms Infrastructure Week 2	February 21-25, 2022
Dartmouth College, Hanover	
2021 Maine-Québec Number Theory Conference	October 2-3, 2021
Virtual (University of Maine, Orono)	
Spring 2021 Algebraic Geometry Northeastern Series	May 4-5, 2021
Virtual (Brown University, Providence)	
$21^{ m st}$ Algebra, Geometry and Combinatorics Day	April 10, 2021
Virtual (University of Notre Dame, Notre Dame)	
2021 Joint Mathematics Meetings	January 6-9, 2021
Virtual (American Math Society, Mathematical Association of American	ca)
Fall 2020 Algebraic Geometry Northeastern Series	October 23-25, 2020
Virtual (Stony Brook University, Stony Brook)	
2020 Conférence de Théorie des Nombres Québec-Maine	September 26-27, 2020
Virtual (Université Laval, Québec)	
14 <sup>th</sup> Algorithmic Number Theory Symposium	June 29 - July 4, 2020
Virtual (University of Auckland, Auckland)	•
UNCG Summer School in Computational Number Theory and Al	lgebra:
Ergodic Theory with Applications to Continued Fractions	May 18-22, 2020
Virtual (University of North Carolina, Greensboro)	·

2019 Maine-Québec Number Theory Conference	October 5-6, 2019
University of Maine, Orono	
Fall 2019 Algebraic Geometry Northeastern Series	September 20-22, 2019
Boston College, Boston	
33rd Automorphic Forms Workshop	March 6-10, 2019
Duquesne University, Pittsburgh	
Building Bridges: $4^{th}$ EU/US Summer School $+$	
Workshop on Automorphic Forms and Related Topics	July 9-20, 2018
Alfréd Rényi Institute of Mathematics, Budapest	
32 <sup>nd</sup> Automorphic Forms Workshop	March 19-22, 2018
Tufts University, Medford	
Modular Forms are Everywhere	May 15-26, 2017
Max Planck Institute for Mathematics, Bonn	
$31^{\mathrm{st}}$ Automorphic Forms Workshop	March 6-9, 2017
East Tennessee State University, Johnson City	
2017 BYU Student Research Conference	March 4, 2017
Brigham Young University, Provo	
2016 Connecticut Summer School in Number Theory	August 8-14, 2016
University of Connecticut, Storrs	
2015 BYU Student Research Conference	March 21, 2015
Brigham Young University, Provo	
2014 BYU Student Research Conference	March 15, 2014
Brigham Young University, Provo	