

# Stat 651 Project

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2025-04-08

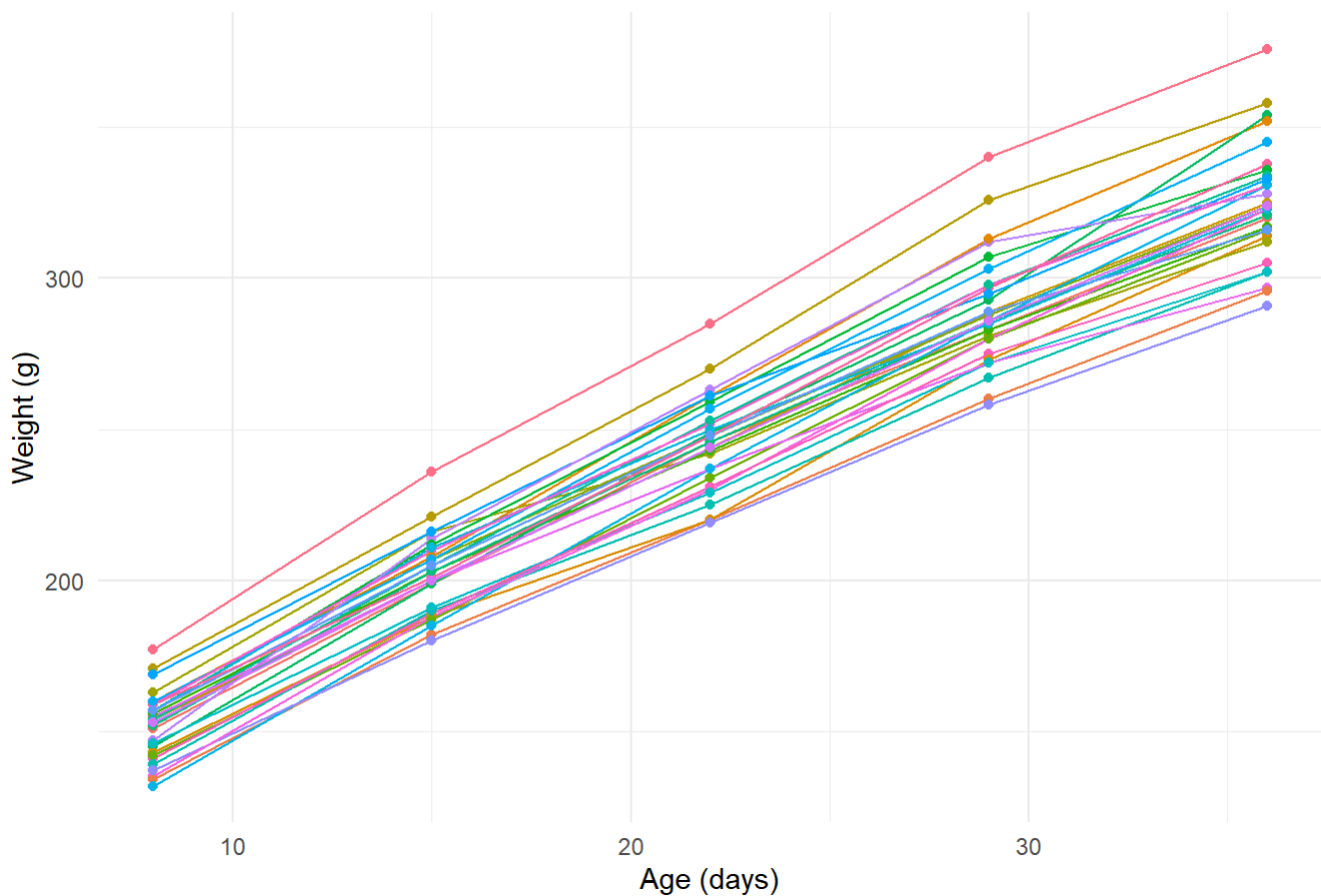
```
# Read in data
ratdata <- read_excel("ratdata.xlsx")

# Pivot longer to get tidy data
rat_long <- ratdata %>%
  pivot_longer(cols = starts_with("rat"), names_to = "rat", values_to = "weight")
```

## EDA

```
# Weights of each rat
ggplot(rat_long, aes(x = age, y = weight, group = rat, color = rat)) +
  geom_line() +
  geom_point() +
  labs(title = "Growth Curves for Each Rat", x = "Age (days)", y = "Weight (g)") +
  theme_minimal() +
  theme(legend.position = "none")
```

Growth Curves for Each Rat

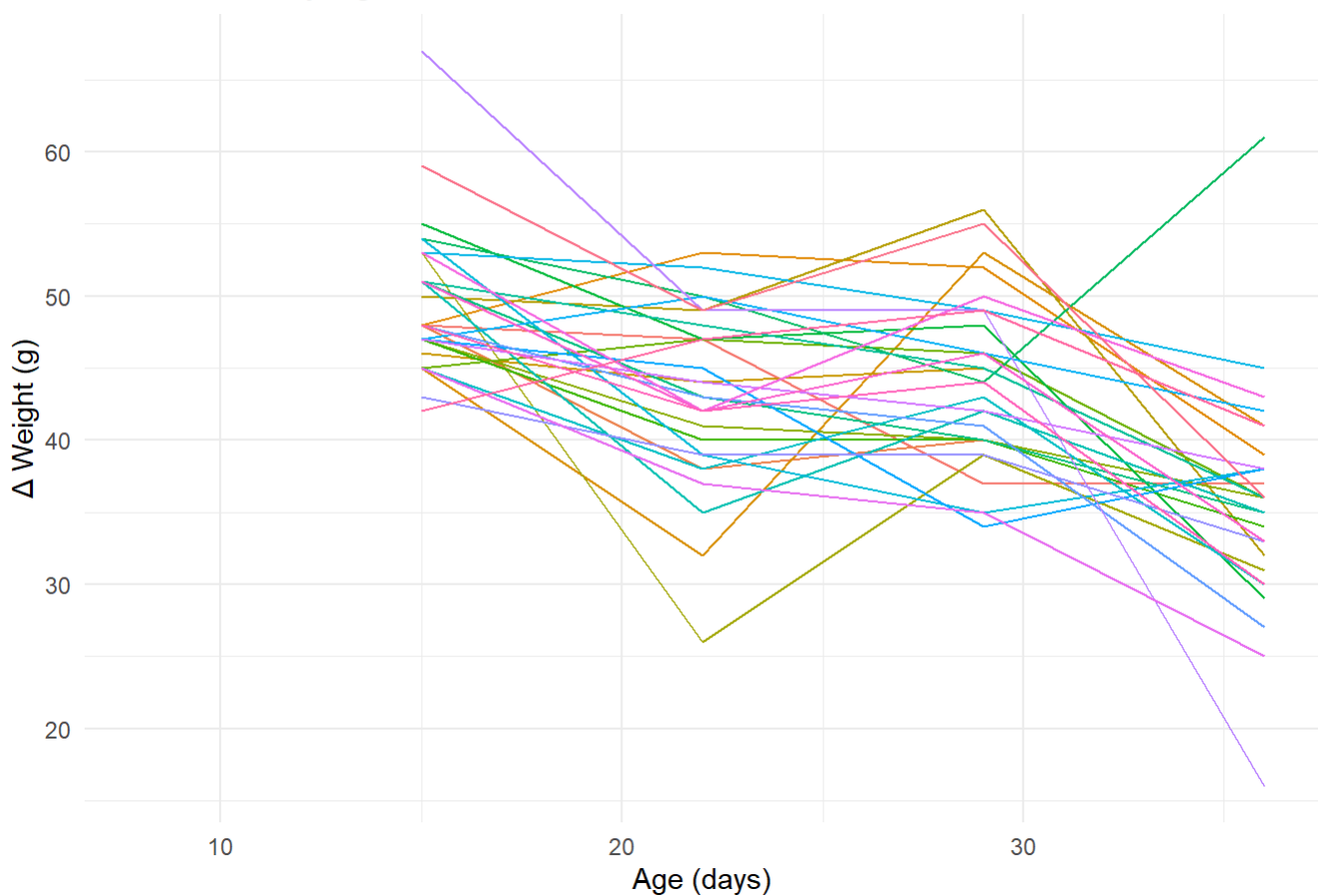


```
# Growth rate for each rat
rat_diff <- ratdata %>%
  dplyr::select(-age) %>%
  mutate(across(everything(), ~ c(NA, diff(.)))) %>%
  mutate(age = ratdata$age)

rat_diff_long <- rat_diff %>%
  pivot_longer(cols = -age, names_to = "rat", values_to = "growth_rate")

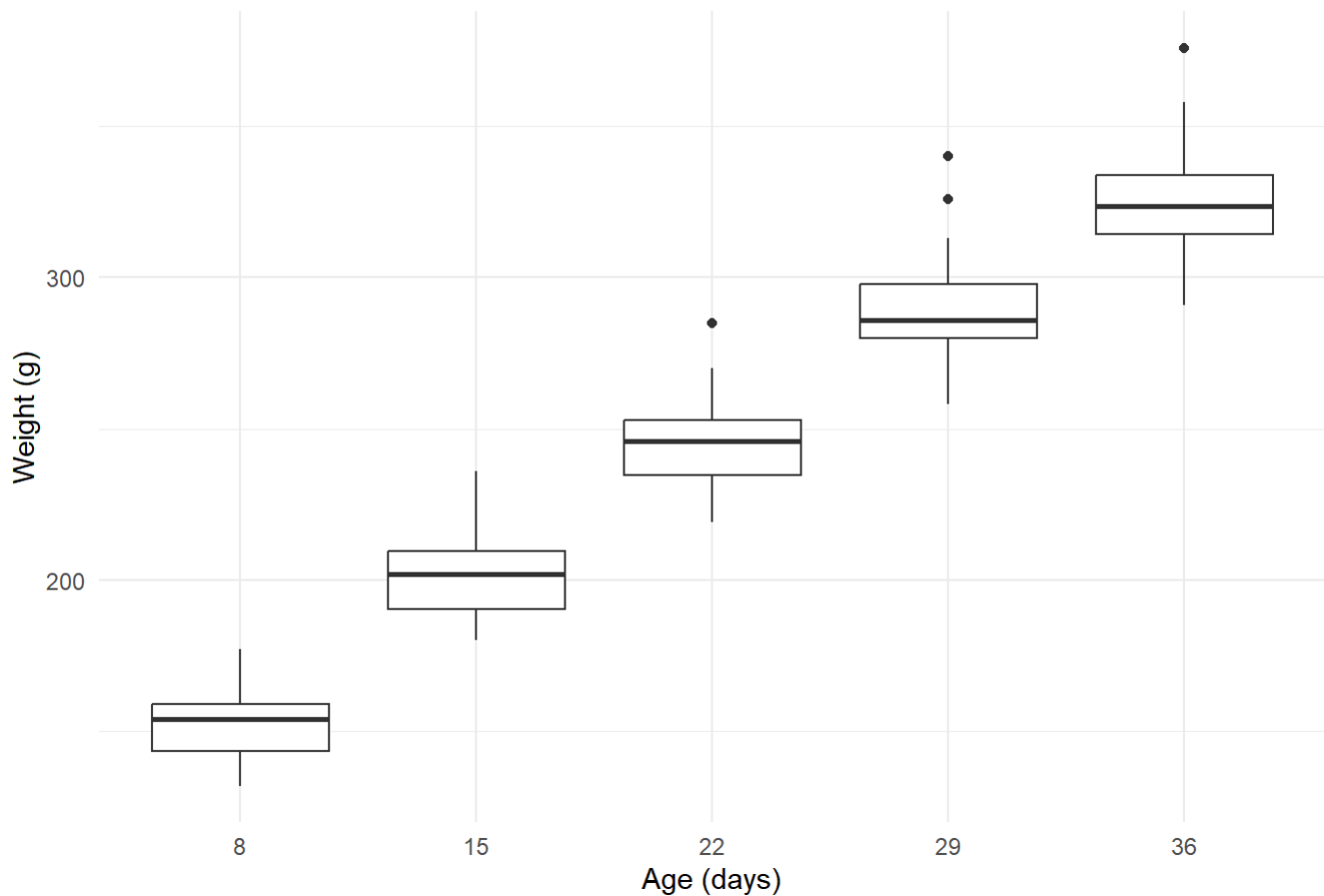
ggplot(rat_diff_long, aes(x = age, y = growth_rate, group = rat, color = rat)) +
  geom_line() +
  labs(title = "Growth Rate by Age", x = "Age (days)", y = "Δ Weight (g)") +
  theme_minimal() +
  theme(legend.position = "none")
```

Growth Rate by Age



```
# Boxplots at each age
ggplot(rat_long, aes(x = factor(age), y = weight)) +
  geom_boxplot() +
  labs(title = "Distribution of Rat Weights at Each Age", x = "Age (days)", y = "Weight (g)")
+
  theme_minimal()
```

## Distribution of Rat Weights at Each Age



The growth curves show that the rats grow in a mostly linear fashion. Obviously there is some variability, but overall I'd say that it's pretty linear. The growth rates are not very consistent (lots of ups and downs), but overall they're all pretty similar. For the most part the growth rate slows down between days 15-22, then speed up between days 22-29, and then decrease again from days 29-36. However, there are a few rate that increase in growth rate during the last time period. There are some exceptions to the rule here, but overall I'd say that the Normal model for  $Y_{ij}$  is fairly reasonable.

```
# Justify the prior
library(MASS)      # For mvnrm
library(ggplot2)   # For prettier plots
library(reshape2)  # For melt
library(gridExtra) # For side-by-side plots
```

```
##
## Attaching package: 'gridExtra'
```

```
## The following object is masked from 'package:dplyr':
##
##      combine
```

```

# Prior parameters
eta <- c(0, 0)
C <- diag(5, 2)
lambda_0 <- 0.1
nu_0 <- 0.1

# Sample from prior
set.seed(123)
n_samples <- 100

# Sample tau ~ Inv-Gamma
# Inv-Gamma(a, b) → 1 / rgamma(n, shape = a, rate = b)
tau <- 1 / rgamma(n_samples, shape = nu_0 / 2, rate = (nu_0 * lambda_0) / 2)

# For each tau, sample alpha_c and beta_c ~ N2(eta, tau * C)
alphas_betas <- t(sapply(tau, function(t) {
  mvrnorm(1, mu = eta, Sigma = t * C)
}))

# Organize samples
df <- data.frame(alpha_c = alphas_betas[,1],
                  beta_c = alphas_betas[,2],
                  tau = tau)

# Marginal density plots
p1 <- ggplot(df, aes(x = alpha_c)) + geom_density(fill = "skyblue") + ggtitle("Prior for alpha_c")
p2 <- ggplot(df, aes(x = beta_c)) + geom_density(fill = "orange") + ggtitle("Prior for beta_c")
p3 <- ggplot(df, aes(x = tau)) + geom_density(fill = "green") + ggtitle("Prior for tau")

# Contour plot of joint (alpha_c, beta_c)
p4 <- ggplot(df, aes(x = alpha_c, y = beta_c)) +
  stat_density_2d(aes(fill = ..level..), geom = "polygon", color = "black") +
  scale_fill_viridis_c() + ggtitle("Joint Prior: alpha_c vs beta_c")

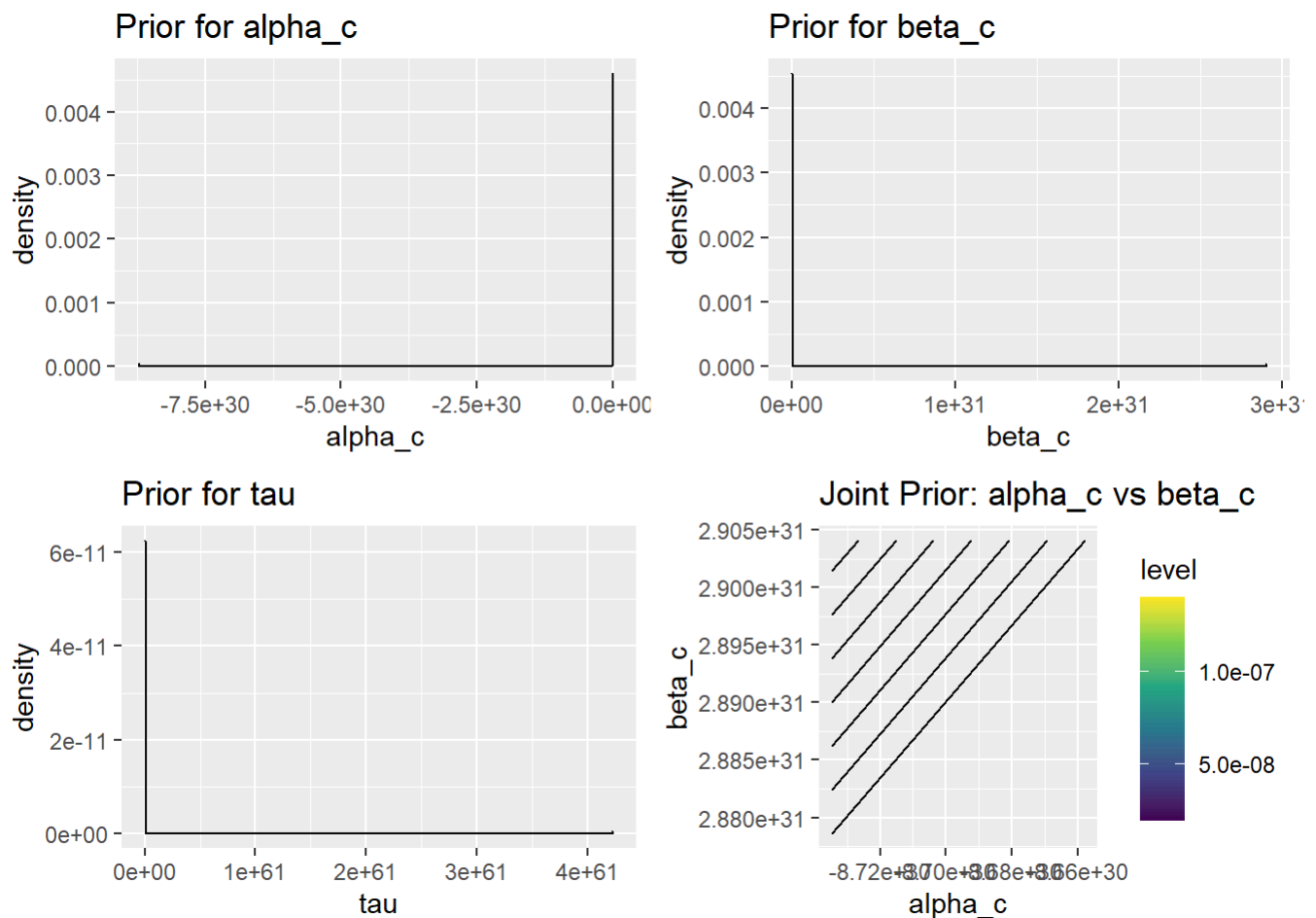
# Plot all together
grid.arrange(p1, p2, p3, p4, ncol = 2)

```

```

## Warning: The dot-dot notation (`..level..`) was deprecated in ggplot2 3.4.0.
## i Please use `after_stat(level)` instead.
## This warning is displayed once every 8 hours.
## Call `lifecycle::last_lifecycle_warnings()` to see where this warning was
## generated.

```



## Question 2

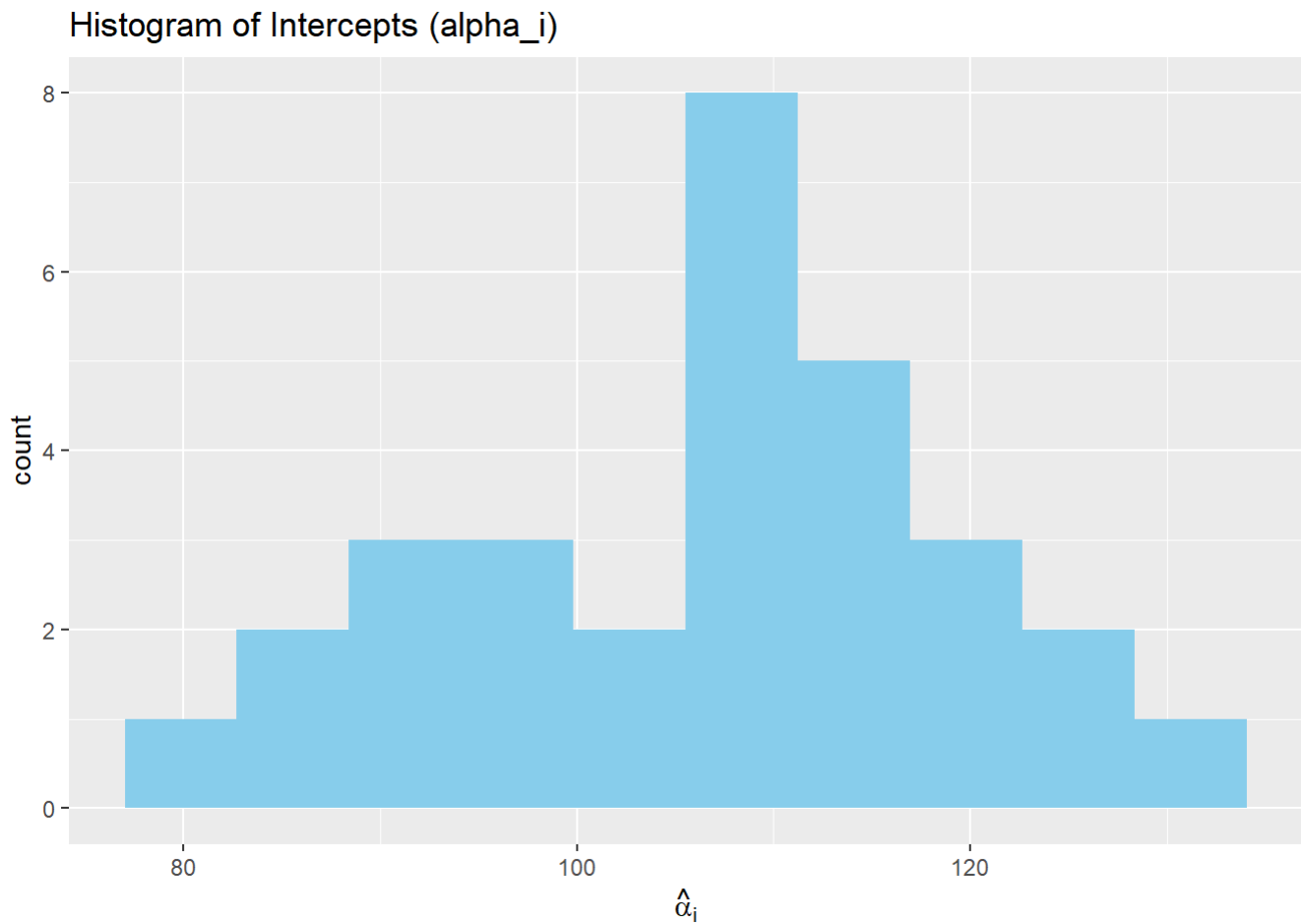
```
# Nest the data by rat
rat_nested <- rat_long %>%
  group_by(rat) %>%
  nest()

# Fit regression to each nested dataset
rat_models <- rat_nested %>%
  mutate(model = map(data, ~lm(weight ~ age, data = .)))

# Extract coefficients from each model
rat_coefs <- rat_models %>%
  mutate(coefs = map(model, broom::tidy)) %>%
  unnest(coefs) %>%
  dplyr::select(rat, term, estimate) %>%
  pivot_wider(names_from = term, values_from = estimate) %>%
  rename(alpha_hat = `(Intercept)`, beta_hat = age)

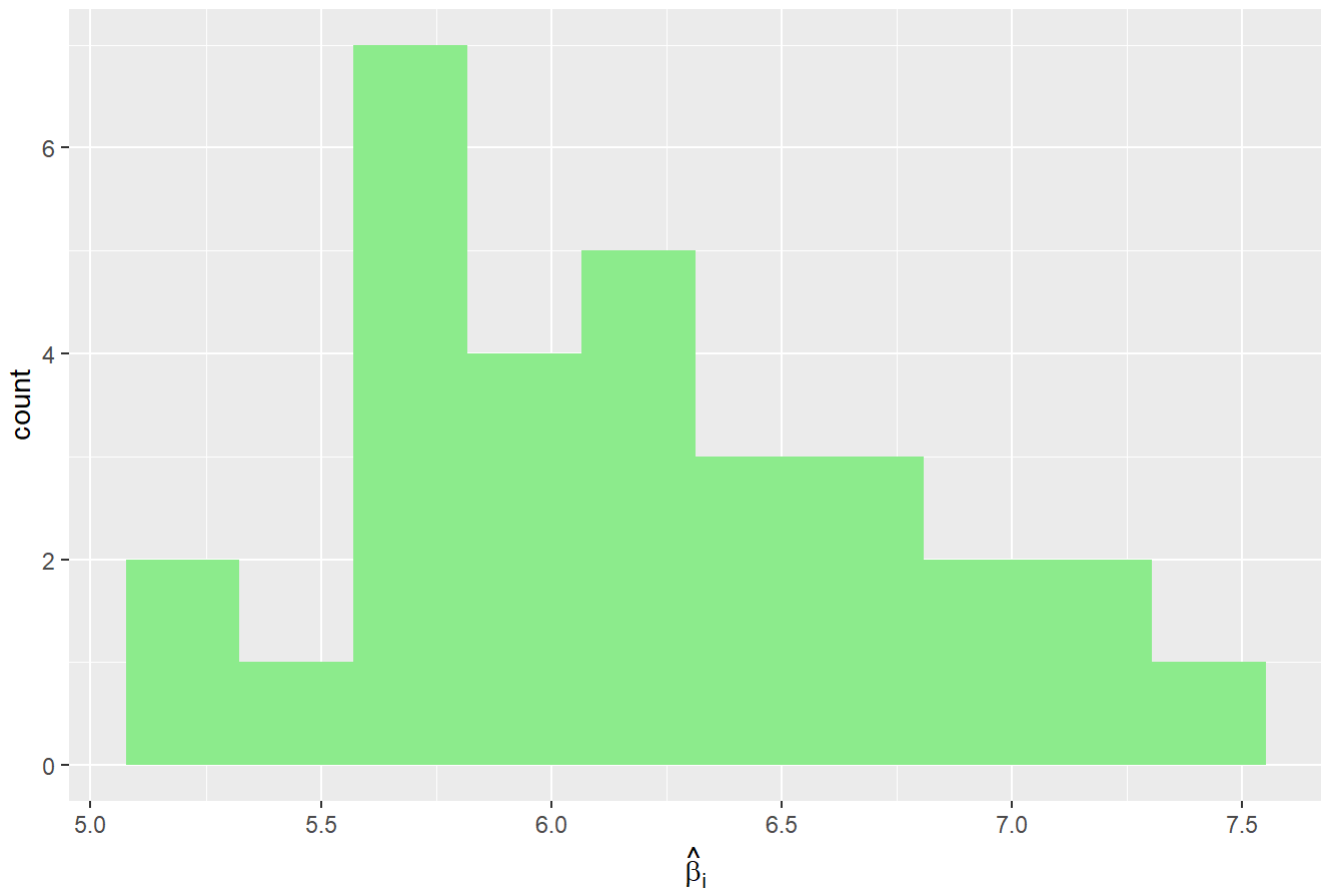
# Check Normality of thetas

# Histogram of intercepts
ggplot(rat_coefs, aes(x = alpha_hat)) +
  geom_histogram(bins = 10, fill = "skyblue") +
  labs(title = "Histogram of Intercepts (alpha_i)", x = expression(hat(alpha)[i]))
```



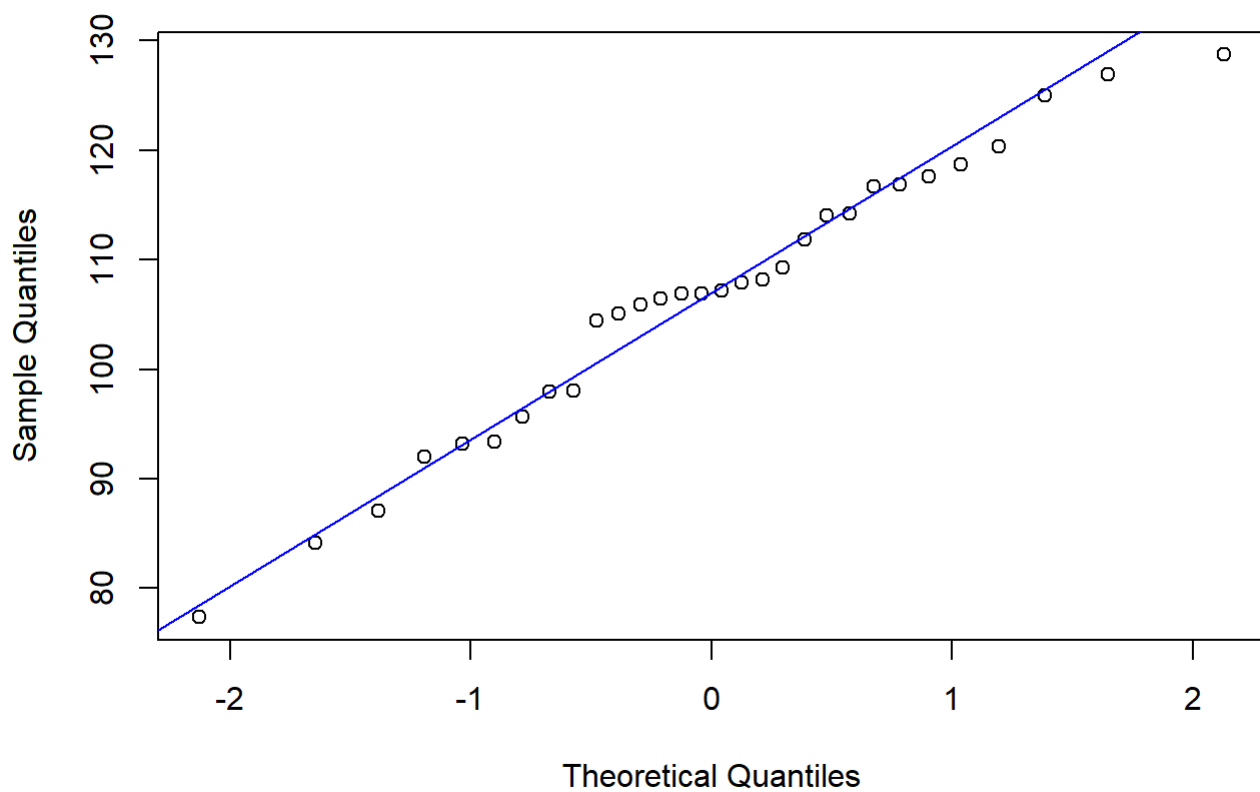
```
# Histogram of slopes
ggplot(rat_coefs, aes(x = beta_hat)) +
  geom_histogram(bins = 10, fill = "lightgreen") +
  labs(title = "Histogram of Slopes (beta_i)", x = expression(hat(beta)[i]))
```

Histogram of Slopes (beta\_i)



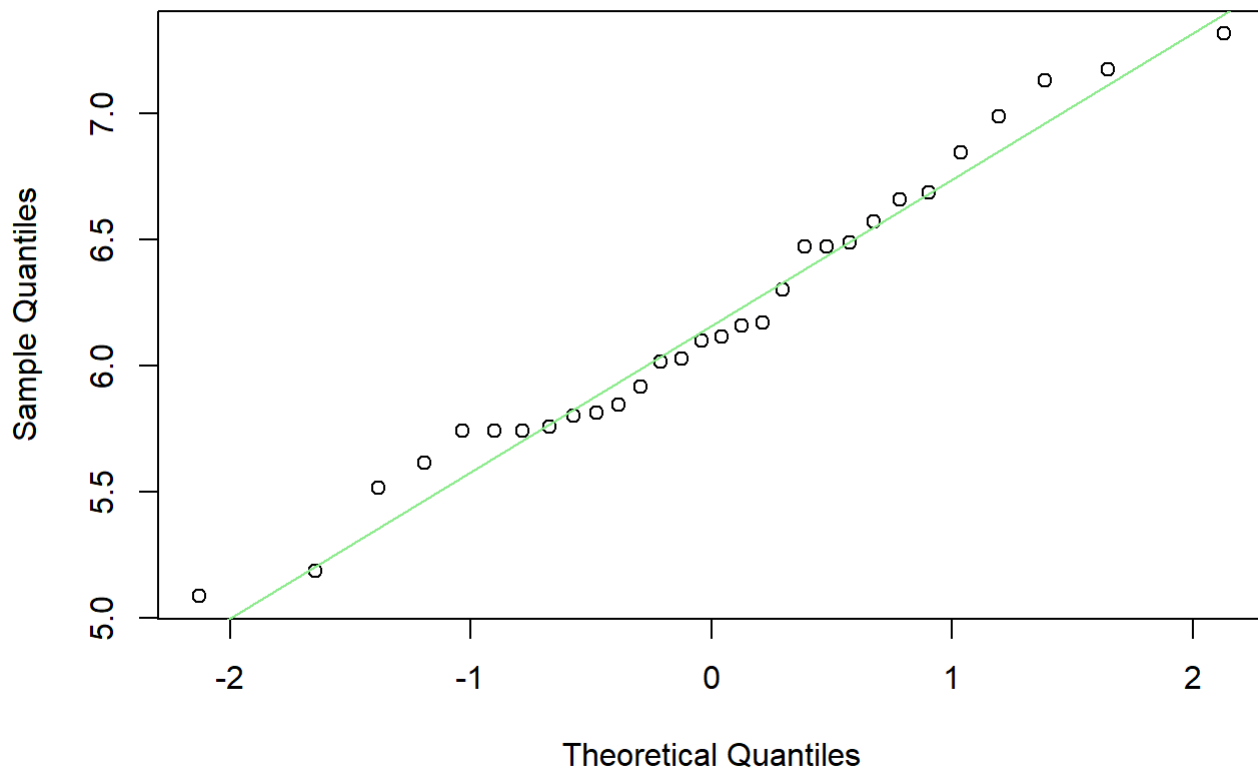
```
# QQ plots  
qqnorm(rat_coefs$alpha_hat); qqline(rat_coefs$alpha_hat, col = "blue")
```

Normal Q-Q Plot



```
qqnorm(rat_coefs$beta_hat); qqline(rat_coefs$beta_hat, col = "lightgreen")
```

### Normal Q-Q Plot



I would say that the normality assumption is reasonable here. The distribution of intercepts is approximately normal. The distribution of slopes is a little right skewed, but I wouldn't say it's too egregious. The Q-Q plots also aren't too terrible.

## Gibbs Sampler



```

# Reshape data
rats <- melt(ratdata, id.vars = c("age"))
colnames(rats) <- c("age", 'id', 'weight')

# Priors
lambda0 <- 0.1
nu0 <- 0.1
eta <- matrix(c(0,0), nrow=2)
Sigma <- matrix(c(10, 0, 0, 10), nrow=2, ncol=2)
Sigma_inv <- solve(Sigma)
C <- matrix(c(5, 0, 0, 5), nrow=2, ncol=2)
C_inv <- solve(C)
V <- solve(30 * Sigma_inv + C_inv)

set.seed(123)

# Prepare data
age <- ratdata$age
n <- 30 # number of rats
t <- 5 # number of time points = 5

# Design matrix
rat_data <- lapply(unique(rats$id), function(j) {
  that_rat <- rats[rats$id == j, ]
  that_rat$weight # Response variable
})
Xi <- cbind(rep(1, 5), c(8, 15, 22, 29, 36)) # Define X_i once
XtX <- t(Xi) %*% Xi
Y_matrix <- do.call(cbind, rat_data) # Combine all Y_i into a matrix (5 x 30)

# Hyperparameters
n_iter <- 100000
burn_in <- 10000
thin <- 10

# Storage
alphas = matrix(nrow=n_iter,ncol=30)
betas = matrix(nrow=n_iter,ncol=30)
mu_cs = matrix(nrow=n_iter,ncol=2)
taus = matrix(nrow=n_iter,ncol=1)
sses = rep(0,n_iter)

# Initialize
alpha = rep(0,30)
beta = rep(0,30)
mu_c = rep(0,2)
tau = 1

# Gibbs sampler
for (i in 1:n_iter) {

  # --- 1. Sample theta_i | Y_i, mu, tau ---
  # Compute D_i_inv for all groups (same for all 30 rats)
  Di_inv <- (1/tau) * XtX + Sigma_inv

```

```

Di <- solve(Di_inv)

# Compute means (2 x 30 matrix)
means <- Di %%% ((1/tau) * (t(Xi) %%% Y_matrix) + Sigma_inv %%% matrix(mu_c, ncol=30, nrow=
2, byrow=FALSE))

# Draw samples for all 30 groups at once (each row is a rat)
new_params <- t(rmvnorm(n = 30, mean = rep(0, 2), sigma = Di)) + means # Ensuring correct
shape

# Extract alpha and beta
alphas[i, ] <- alpha <- new_params[1, ] # First row is alpha
betas[i, ] <- beta <- new_params[2, ] # Second row is beta

# --- 2. Sample mu | theta, tau ---
## Update the group parameters
theta_bar = matrix(c(mean(alpha), mean(beta)), nrow=2)
mu_c = rmvnorm(n = 1,
               mean = V %%% (30 * Sigma_inv %%% theta_bar + C_inv %%% eta),
               sigma = V)
mu_c = as.vector(mu_c)
mu_cs[i,] = mu_c

# --- 3. Sample tau | theta ---
# Compute SSE efficiently
residuals <- Y_matrix - Xi %%% new_params
sse <- sum(residuals^2)
sses[i] <- sse # Store SSE

# Update tau
tau <- rinvgamma(n=1,
                shape = (nu0 + 150)/ 2,
                scale = (1/2) * (nu0 * lambda0 + sse))
taus[i] <- tau
}

# POSTERIOR INFERENCE

# Remove burn-in
alpha_post <- alphas[(burn_in+1):n_iter, ]
beta_post <- betas[(burn_in+1):n_iter, ]
alpha_c_post <- mu_cs[(burn_in+1):n_iter, 1]
beta_c_post <- mu_cs[(burn_in+1):n_iter, 2]
tau_post <- taus[(burn_in+1):n_iter]

# Thin
alpha_post <- alpha_post[seq(1, nrow(alpha_post), by=thin), ]
beta_post <- beta_post[seq(1, nrow(beta_post), by=thin), ]
alpha_c_post <- alpha_c_post[seq(1, length(alpha_c_post), by=thin)]
beta_c_post <- beta_c_post[seq(1, length(beta_c_post), by=thin)]
tau_post <- tau_post[seq(1, length(tau_post), by=thin)]

```

```
# Compute posterior means, variances, and credible intervals
alpha_est <- apply(alpha_post, 2, mean)
alpha_var <- apply(alpha_post, 2, var)
alpha_ci <- apply(alpha_post, 2, function(x) quantile(x, probs = c(0.025, 0.975)))

beta_est <- apply(beta_post, 2, mean)
beta_var <- apply(beta_post, 2, var)
beta_ci <- apply(beta_post, 2, function(x) quantile(x, probs = c(0.025, 0.975)))

alpha_c_est <- mean(alpha_c_post)
alpha_c_var <- var(alpha_c_post)
alpha_c_ci <- quantile(alpha_c_post, probs = c(0.025, 0.975))
beta_c_est <- mean(beta_c_post)
beta_c_var <- var(beta_c_post)
beta_c_ci <- quantile(beta_c_post, probs = c(0.025, 0.975))

tau_est <- mean(tau_post)
tau_var <- var(tau_post)
tau_ci <- quantile(tau_post, probs = c(0.025, 0.975))

# Print posterior estimates
cat("Posterior Mean of alpha:\n", alpha_est, "\n")
```

```
## Posterior Mean of alpha:
## 6.667316 6.630745 6.778173 6.779697 6.533948 6.725573 6.655685 6.732996 6.788809 6.633324
6.703732 6.607456 6.652874 6.718473 6.770628 6.750904 6.674162 6.76905 6.710859 6.70431 6.734
694 6.582309 6.697096 6.768374 6.600971 6.776939 6.737182 6.754197 6.611198 6.702432
```

```
cat("Posterior Variance of alpha:\n", alpha_var, "\n")
```

```
## Posterior Variance of alpha:
## 15.63594 15.45511 15.43135 14.88998 15.12935 15.19445 15.33134 15.48413 15.62299 15.3057
15.10144 15.38704 15.43006 15.0637 15.01497 15.33964 15.12333 15.09833 15.16042 15.39388 15.8
0234 15.50716 15.28125 15.16023 15.3075 15.21455 15.17751 15.00546 15.04692 15.20299
```

```
cat("95% CI for alpha:\n", alpha_ci, "\n\n")
```

```
## 95% CI for alpha:
## -1.048659 14.42171 -1.104099 14.25037 -0.971798 14.52742 -0.8354788 14.32131 -1.144158 1
4.12933 -0.818209 14.40971 -1.197021 14.22993 -0.9412614 14.49281 -0.8589824 14.64125 -0.8530
033 14.47356 -0.8307167 14.34078 -1.026804 14.38797 -0.9369168 14.54719 -0.8896057 14.1826 -
0.8173413 14.39218 -1.002791 14.50161 -0.8390374 14.25726 -0.8782033 14.40379 -0.9388665 14.3
6312 -0.8931014 14.45864 -1.162896 14.45107 -1.159827 14.34182 -1.032787 14.34 -0.8284084 14.
4285 -1.032591 14.1381 -0.8332164 14.57909 -0.9704972 14.30392 -0.5907241 14.33717 -0.9984552
14.2894 -1.067173 14.26645
```

```
cat("Posterior Mean of beta:\n", beta_est, "\n")
```

```
## Posterior Mean of beta:
## 9.783375 10.27388 10.32268 9.367298 9.601935 10.16816 9.363423 10.18162 11.48696 8.984793
10.60972 9.372273 9.908306 10.93497 9.763672 9.935697 9.550815 9.751102 10.37209 9.843294 10.
18381 9.217273 9.303531 9.924102 9.751282 10.39854 10.28911 9.868462 8.910079 9.849183
```

```
cat("Posterior Variance of beta:\n", beta_var, "\n")
```

```
## Posterior Variance of beta:
## 9.783375 10.27388 10.32268 9.367298 9.601935 10.16816 9.363423 10.18162 11.48696 8.984793
10.60972 9.372273 9.908306 10.93497 9.763672 9.935697 9.550815 9.751102 10.37209 9.843294 10.
18381 9.217273 9.303531 9.924102 9.751282 10.39854 10.28911 9.868462 8.910079 9.849183
```

```
cat("95% CI for beta:\n", beta_ci, "\n\n")
```

```
## 95% CI for beta:
## 8.139502 11.40018 8.59136 11.95068 8.657094 11.96094 7.679782 10.97746 7.952186 11.26588
8.528952 11.83996 7.713505 11.05325 8.552631 11.84143 9.815865 13.15352 7.352503 10.64765 8.9
14892 12.28516 7.739273 10.98631 8.268249 11.56366 9.261959 12.58416 8.108349 11.39903 8.3104
79 11.64418 7.925098 11.21899 8.092801 11.37382 8.701472 12.06636 8.173262 11.4954 8.511383 1
1.84849 7.589857 10.89919 7.678441 10.92656 8.290863 11.59992 8.091827 11.40905 8.726808 12.0
4698 8.641312 11.94331 8.239374 11.51634 7.271488 10.5886 8.228946 11.50582
```

```
cat("Posterior Mean of alpha_c:\n", alpha_c_est, "\n")
```

```
## Posterior Mean of alpha_c:
## 6.281169
```

```
cat("Posterior Variance of alpha_c:\n", alpha_c_var, "\n")
```

```
## Posterior Variance of alpha_c:
## 5.262789
```

```
cat("95% CI for alpha_c:\n", alpha_c_ci, "\n\n")
```

```
## 95% CI for alpha_c:
## 1.859327 10.82475
```

```
cat("Posterior Mean of beta_c:\n", beta_c_est, "\n")
```

```
## Posterior Mean of beta_c:
## 9.285139
```

```
cat("Posterior Variance of beta_c:\n", beta_c_var, "\n")
```

```
## Posterior Variance of beta_c:  
## 0.3478579
```

```
cat("95% CI for beta_c:\n", beta_c_ci, "\n\n")
```

```
## 95% CI for beta_c:  
## 8.112458 10.42863
```

```
cat("Posterior Mean of tau:\n", tau_est, "\n")
```

```
## Posterior Mean of tau:  
## 2162.16
```

```
cat("Posterior Variance of tau:\n", tau_var, "\n")
```

```
## Posterior Variance of tau:  
## 89883.6
```

```
cat("95% CI for tau:\n", tau_ci, "\n")
```

```
## 95% CI for tau:  
## 1647.776 2826.678
```

```
# Create a data frame to store the results  
ci_table <- data.frame(  
  Alpha_Mean = alpha_est,           # Posterior mean of alpha  
  Alpha_Lower = alpha_ci[1, ],      # Lower bound of alpha CI  
  Alpha_Upper = alpha_ci[2, ],      # Upper bound of alpha CI  
  Beta_Mean = beta_est,             # Posterior mean of beta  
  Beta_Lower = beta_ci[1, ],        # Lower bound of beta CI  
  Beta_Upper = beta_ci[2, ]         # Upper bound of beta CI  
)  
  
# View the table  
print(ci_table)
```

##	Alpha_Mean	Alpha_Lower	Alpha_Upper	Beta_Mean	Beta_Lower	Beta_Upper
## 1	6.667316	-1.0486585	14.42171	9.783375	8.139502	11.40018
## 2	6.630745	-1.1040991	14.25037	10.273876	8.591360	11.95068
## 3	6.778173	-0.9717980	14.52742	10.322685	8.657094	11.96094
## 4	6.779697	-0.8354788	14.32131	9.367298	7.679782	10.97746
## 5	6.533948	-1.1441581	14.12933	9.601935	7.952186	11.26588
## 6	6.725573	-0.8182090	14.40971	10.168160	8.528952	11.83996
## 7	6.655685	-1.1970214	14.22993	9.363423	7.713505	11.05325
## 8	6.732996	-0.9412614	14.49281	10.181621	8.552631	11.84143
## 9	6.788809	-0.8589824	14.64125	11.486964	9.815865	13.15352
## 10	6.633324	-0.8530033	14.47356	8.984793	7.352503	10.64765
## 11	6.703732	-0.8307167	14.34078	10.609716	8.914892	12.28516
## 12	6.607456	-1.0268038	14.38797	9.372273	7.739273	10.98631
## 13	6.652874	-0.9369168	14.54719	9.908306	8.268249	11.56366
## 14	6.718473	-0.8896057	14.18260	10.934973	9.261959	12.58416
## 15	6.770628	-0.8173413	14.39218	9.763672	8.108349	11.39903
## 16	6.750904	-1.0027908	14.50161	9.935697	8.310479	11.64418
## 17	6.674162	-0.8390374	14.25726	9.550815	7.925098	11.21899
## 18	6.769050	-0.8782033	14.40379	9.751102	8.092801	11.37382
## 19	6.710859	-0.9388665	14.36312	10.372089	8.701472	12.06636
## 20	6.704310	-0.8931014	14.45864	9.843294	8.173262	11.49540
## 21	6.734694	-1.1628959	14.45107	10.183812	8.511383	11.84849
## 22	6.582309	-1.1598271	14.34182	9.217273	7.589857	10.89919
## 23	6.697096	-1.0327869	14.34000	9.303531	7.678441	10.92656
## 24	6.768374	-0.8284084	14.42850	9.924102	8.290863	11.59992
## 25	6.600971	-1.0325908	14.13810	9.751282	8.091827	11.40905
## 26	6.776939	-0.8332164	14.57909	10.398543	8.726808	12.04698
## 27	6.737182	-0.9704972	14.30392	10.289108	8.641312	11.94331
## 28	6.754197	-0.5907241	14.33717	9.868462	8.239374	11.51634
## 29	6.611198	-0.9984552	14.28940	8.910079	7.271488	10.58860
## 30	6.702432	-1.0671729	14.26645	9.849183	8.228946	11.50582

```
# Covariance matrix of alpha_c, beta_c, tau
joint <- cbind(alpha_c_post, beta_c_post, tau_post)
cov(joint)
```

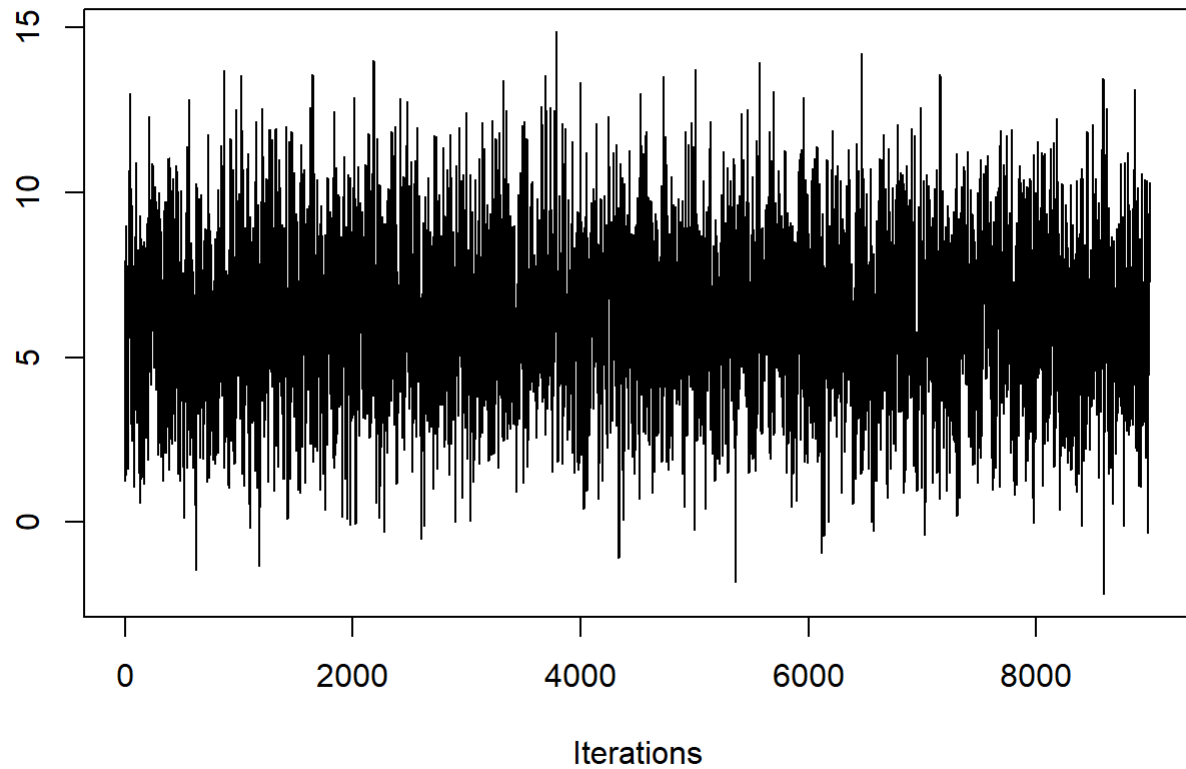
```
##          alpha_c_post beta_c_post    tau_post
## alpha_c_post    5.2627888 -0.1978537 -218.555290
## beta_c_post    -0.1978537  0.3478579   7.908453
## tau_post      -218.5552896  7.9084527 89883.603609
```

### # CONVERGENCE GRAPHICAL DIAGNOSTICS

```
# Convert to mcmc objects
mcmc_alpha <- mcmc(alpha_post)
mcmc_beta <- mcmc(beta_post)
mcmc_alpha_c <- mcmc(alpha_c_post)
mcmc_beta_c <- mcmc(beta_c_post)
mcmc_tau <- mcmc(tau_post)

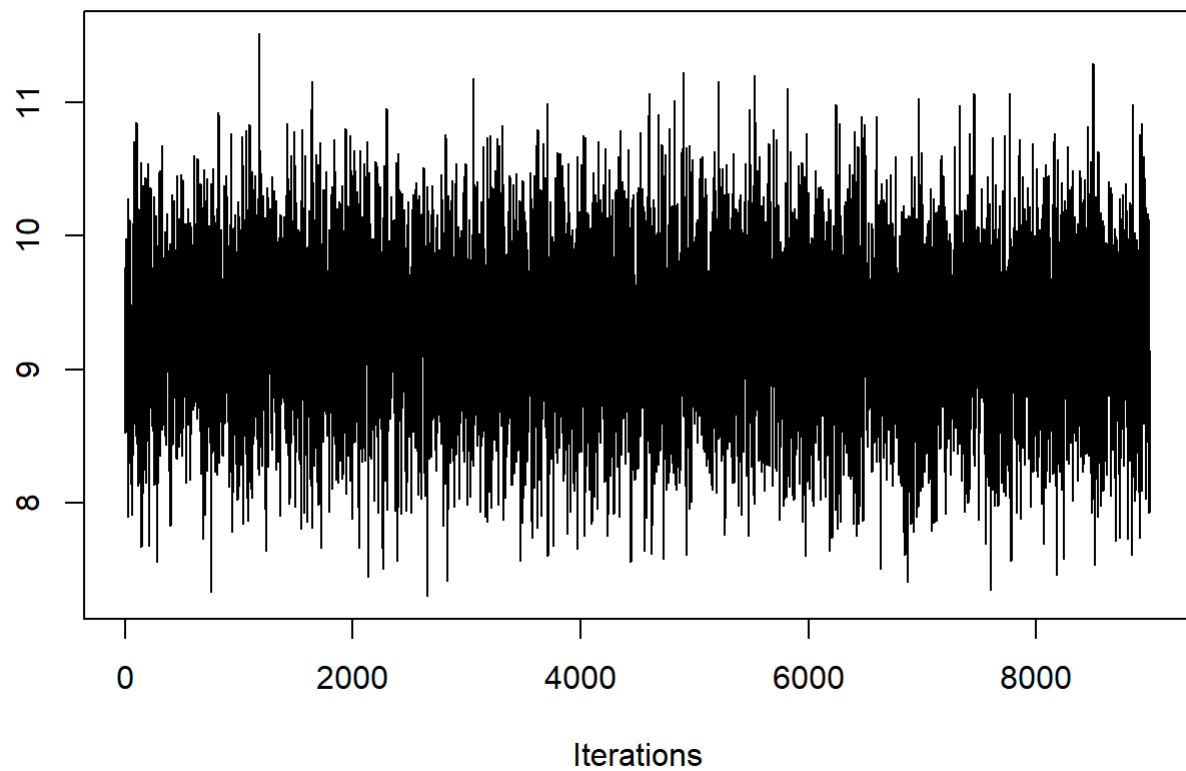
# Trace plots
par(mfrow = c(1,1))
traceplot(mcmc_alpha_c, main = "Traceplot of alpha_c")
```

### Traceplot of alpha\_c



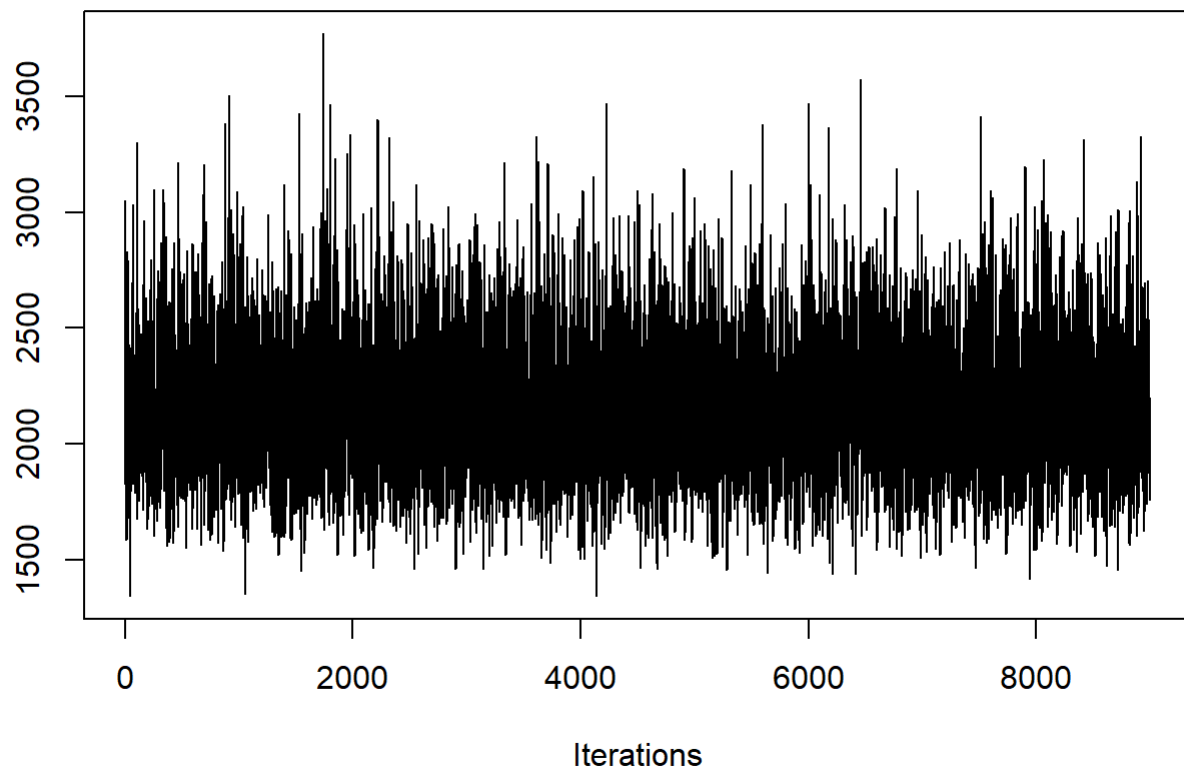
```
traceplot(mcmc_beta_c, main = "Traceplot of beta_c")
```

### Traceplot of beta\_c



```
traceplot(mcmc_tau, main = "Traceplot of tau")
```

### Traceplot of tau



```
# Trace plots for alpha
png("alpha_trace_plots.png", width=1500, height=1000)
par(mfrow=c(6,5))
for (i in 1:30) {
  # Plot trace for alpha (i-th observation)
  plot(1:length(alpha_post[,1]), alpha_post[, i], type = "l",
       main = paste("Trace plot for Alpha", i),
       xlab = "Iteration", ylab = "Alpha Value", col = "black")
}
dev.off()
```

```
## png
## 2
```



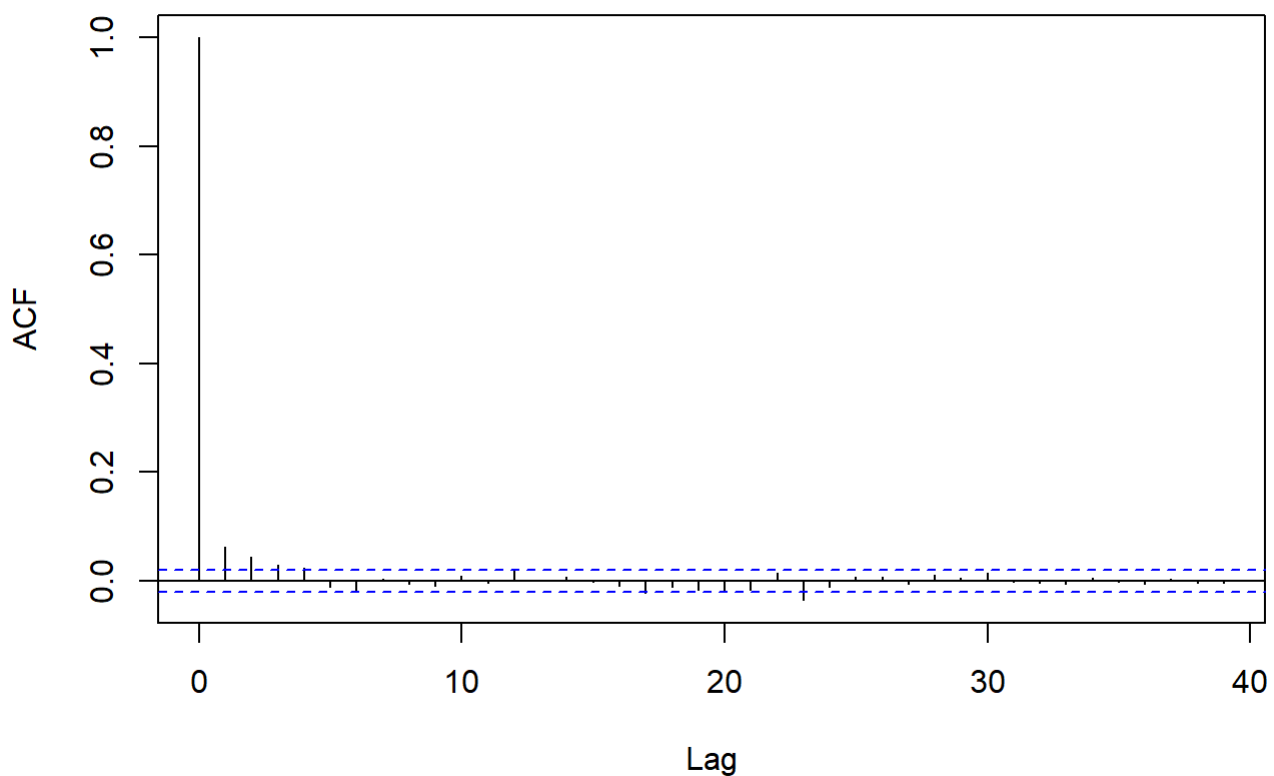
```
# Trace plots for beta
png("beta_trace_plots.png", width=1500, height=1000)
par(mfrow=c(6,5))
for (i in 1:30) {
  # Plot trace for beta (i-th observation)
  plot(1:length(beta_post[,1]), beta_post[, i], type = "l",
       main = paste("Trace plot for Beta", i),
       xlab = "Iteration", ylab = "Beta Value", col = "black")
}

dev.off()
```

```
## png
## 2
```

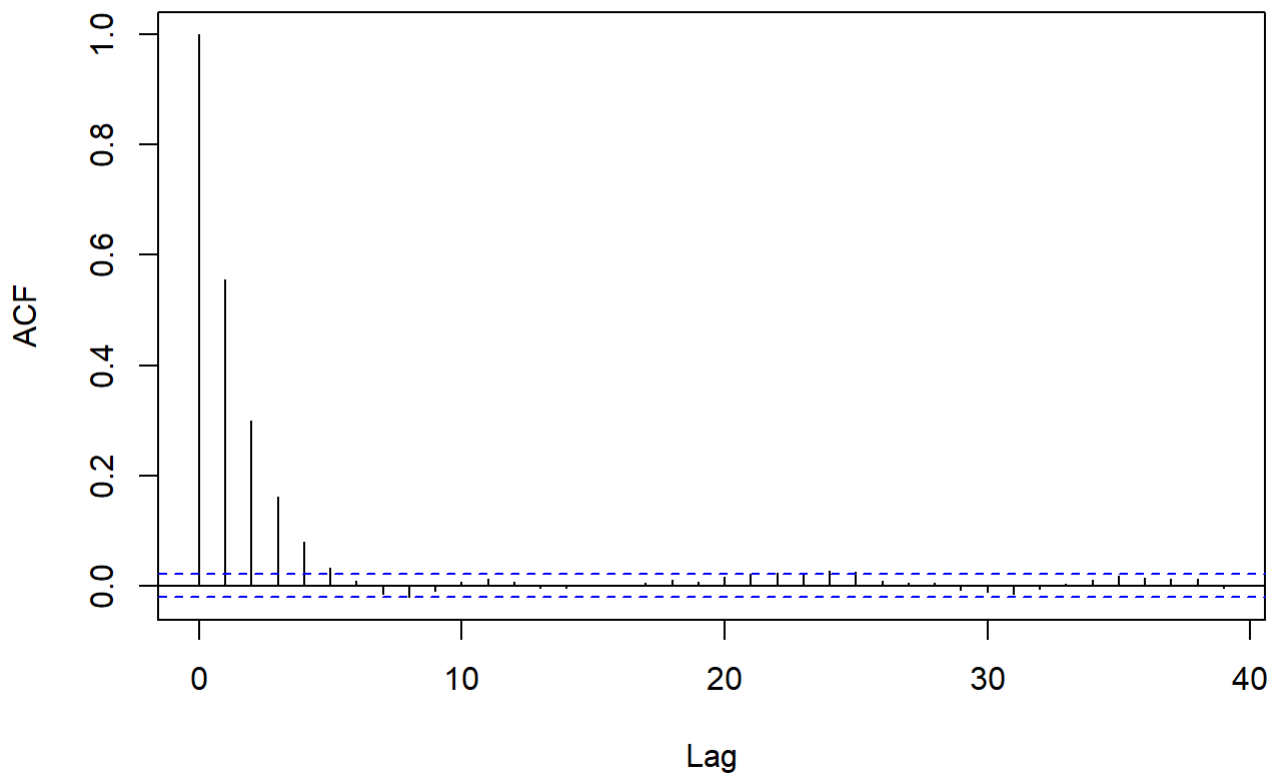
```
# Autocorrelation
par(mfrow=c(1,1))
acf(mcmc_tau, main = "ACF of tau")
```

### ACF of tau



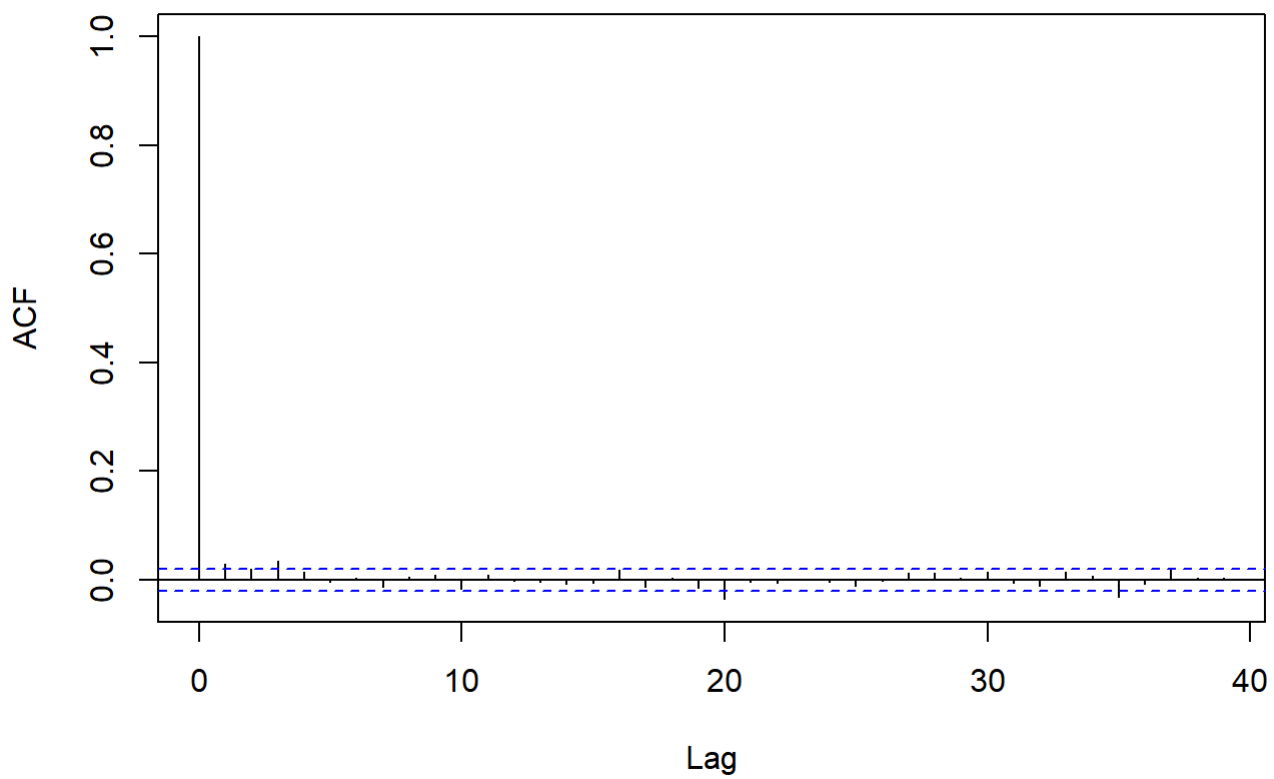
```
acf(mcmc_alpha_c, main = "ACF of alpha_c")
```

### ACF of alpha\_c



```
acf(mcmc_beta_c, main = "ACF of beta_c")
```

### ACF of beta\_c



```
# ACF plots for alpha
png("alpha_acf_plots.png", width=1500, height=1000)
par(mfrow=c(6,5))
for (i in 1:30) {
  # ACF for alpha (i-th observation)
  acf(alpha_post[, i], main = paste("ACF for Alpha", i))
}
dev.off()
```

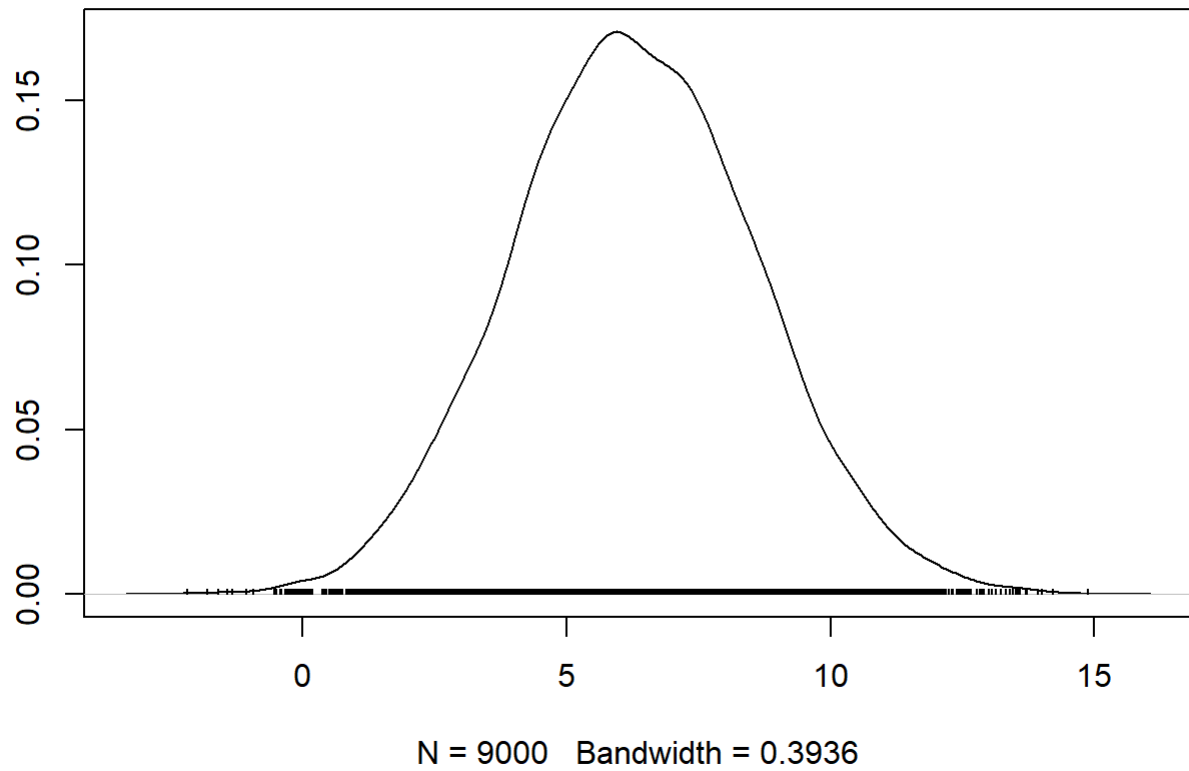
```
## png
## 2
```

```
# ACF plots for beta
png("beta_acf_plots.png", width=1500, height=1000)
par(mfrow=c(6,5))
for (i in 1:30) {
  # ACF for beta (i-th observation)
  acf(beta_post[, i], main = paste("ACF for Beta", i))
}
dev.off()
```

```
## png
## 2
```

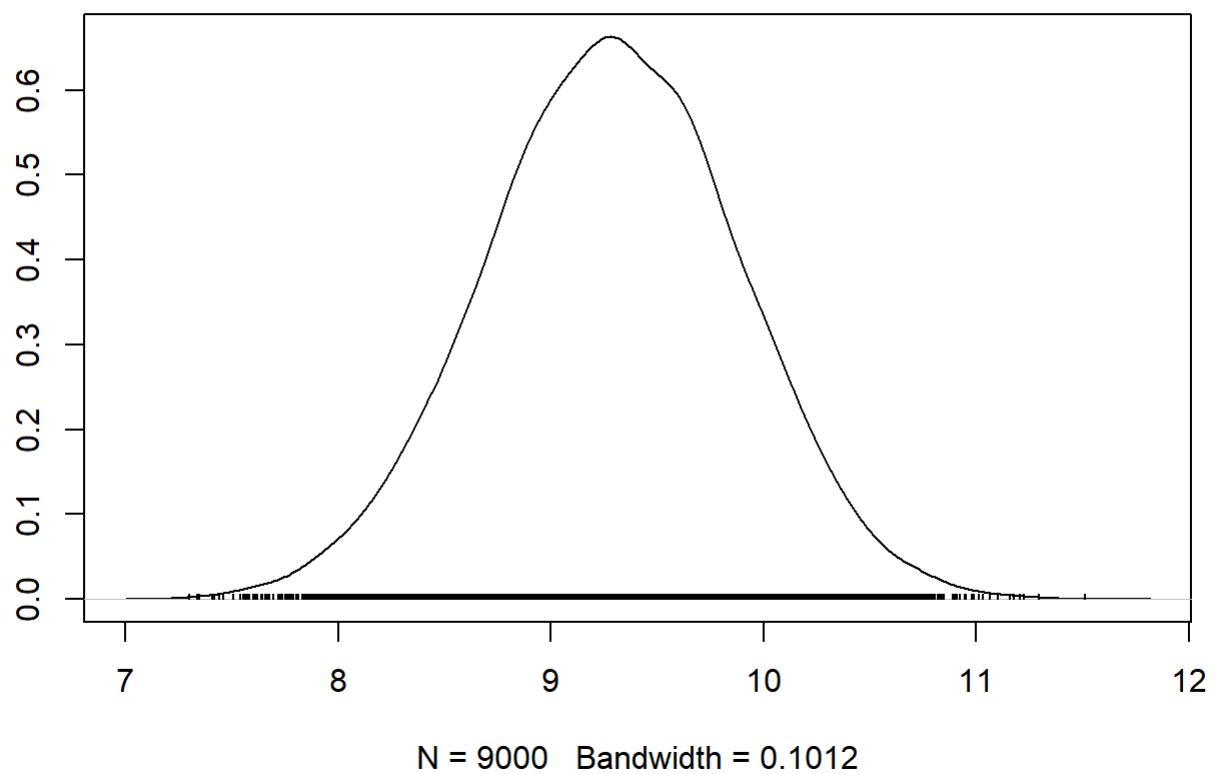
```
# Posterior densities
par(mfrow=c(1,1))
densplot(mcmc_alpha_c, main = "Posterior density of alpha_c")
```

### Posterior density of alpha\_c

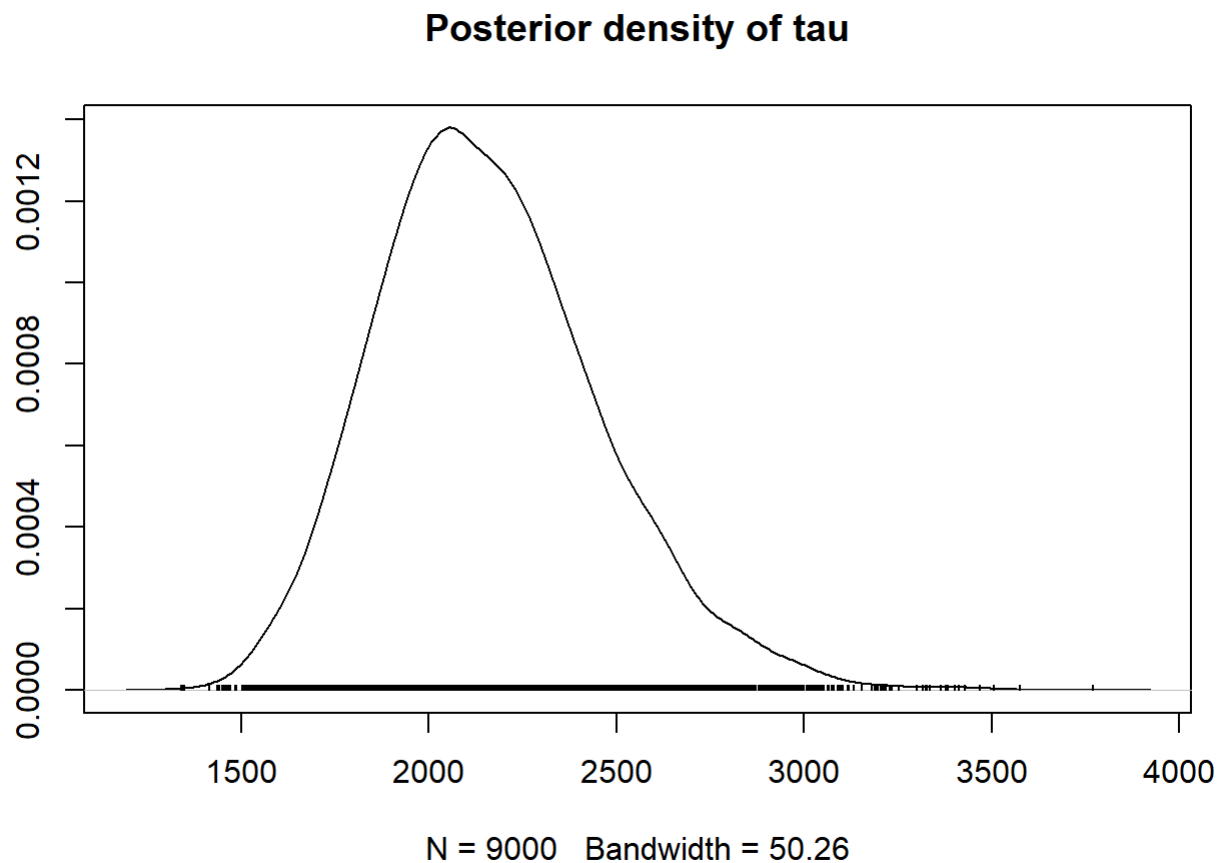


```
densplot(mcmc_beta_c, main = "Posterior density of beta_c")
```

### Posterior density of beta\_c



```
densplot(mcmc_tau, main = "Posterior density of tau")
```



```
# Posterior densities for alpha
png("alpha_density_plots.png", width=1500, height=1000)
par(mfrow=c(6,5))
for (i in 1:30) {
  # Density Plot for alpha (i-th observation)
  densplot(mcmc_alpha[, i], main = paste("Density Plot for Alpha", i))
}
dev.off()
```

```
## png
## 2
```

```
# Posterior densities for beta
png("beta_density_plots.png", width=1500, height=1000)
par(mfrow=c(6,5))
for (i in 1:30) {
  # Density Plot for beta (i-th observation)
  densplot(mcmc_beta[, i], main = paste("Density Plot for Beta", i))
}
dev.off()
```

```
## png
## 2
```

```
# Geweke Tests for each parameter

geweke_results_alpha <- apply(alpha_post, 2, function(chain) {
  geweke.diag(mcmc(chain))$z
})

geweke_results_beta <- apply(beta_post, 2, function(chain) {
  geweke.diag(mcmc(chain))$z
})

geweke_results_alpha_c <- geweke.diag(mcmc_alpha_c)$z

geweke_results_beta_c <- geweke.diag(mcmc_beta_c)$z

geweke_results_tau <- geweke.diag(mcmc_tau)$z

print(list(geweke_results_alpha, geweke_results_beta, geweke_results_alpha_c, geweke_results_beta_c, geweke_results_tau))
```

```
## [[1]]
## [1] -0.3810342  0.3757453 -0.6466500 -1.7067673 -0.9068367 -1.4256248
## [7] -1.1590214 -1.6364037 -0.6551472 -0.9101827 -1.8100018 -0.8062032
## [13] -1.1829874 -1.5179295 -0.8851523 -1.1514061 -0.3335064 -1.2697546
## [19] -0.7802281 -1.8412930 -1.5494567 -0.5081327 -0.9544819 -0.5983633
## [25] -0.4354705 -1.9139924 -0.6176586 -0.5264184 -1.3628785  0.1671091
##
## [[2]]
## [1]  0.716810922  0.403106702  1.726454116  0.761766330  1.879849369
## [6]  0.615711328  0.256272308 -0.006404438  1.605324468  0.018395054
## [11] -0.349733177  0.614962349  0.882148372 -0.127280873 -1.933425679
## [16]  0.437498473  0.272254332  1.165727064  1.977643040  0.668673325
## [21] -2.157728573  0.423928164 -0.665050906  1.067509748  1.789676462
## [26] -0.972874585  0.342931200  1.519293720  0.370981263 -1.009252332
##
## [[3]]
##      var1
## -1.257055
##
## [[4]]
##      var1
##  2.056386
##
## [[5]]
##      var1
## -0.3676366
```

None of them are far away enough from 0 to worry me too much, so I'm happy to conclude that all these have done a good job of converging!