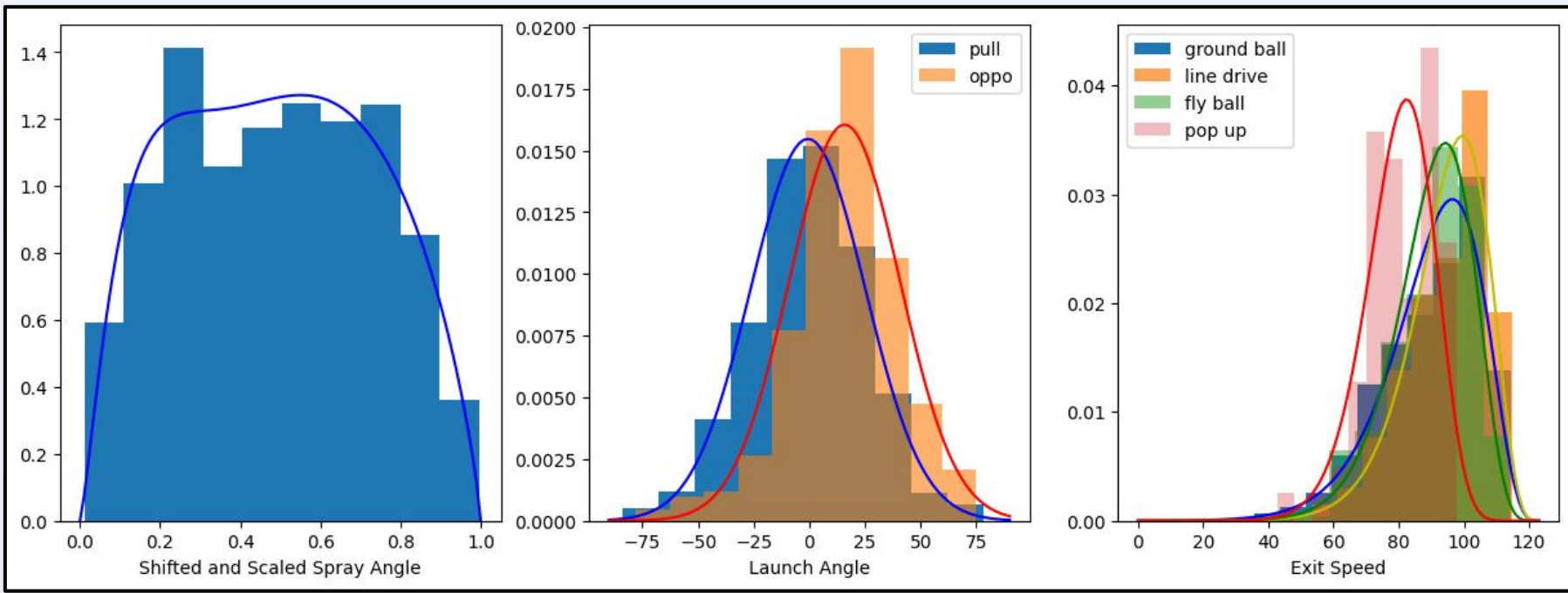


Introduction

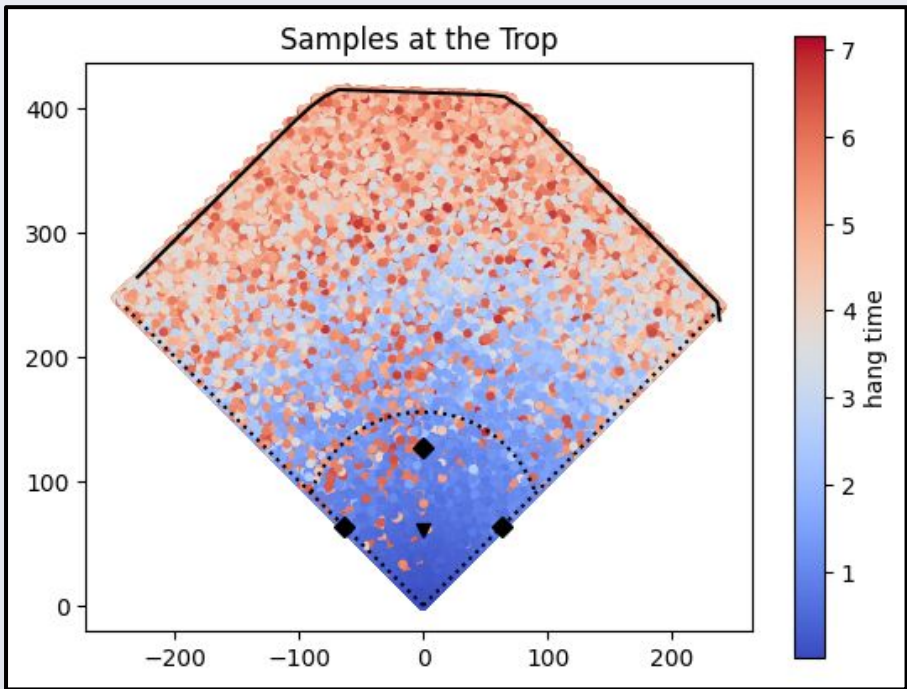
In order to minimize the number of runs scored against them, the defensive players on a baseball team must be optimally positioned such that they can field balls hit against them and record outs. This positioning can be determined from player specific batted ball distributions.



Probability distributions for spray angle, launch angle, and exit speed for Nathaniel Lowe

We combined these distributions with physics equations to simulate the full trajectory of a ball. The figure on the right shows fly balls sampled from these distributions and simulated at Tropicana Field.

We used logistic regression models to determine the probability of an out, given the trajectory of a ball and the positioning of the fielders. We trained separate models for fly balls and ground balls.

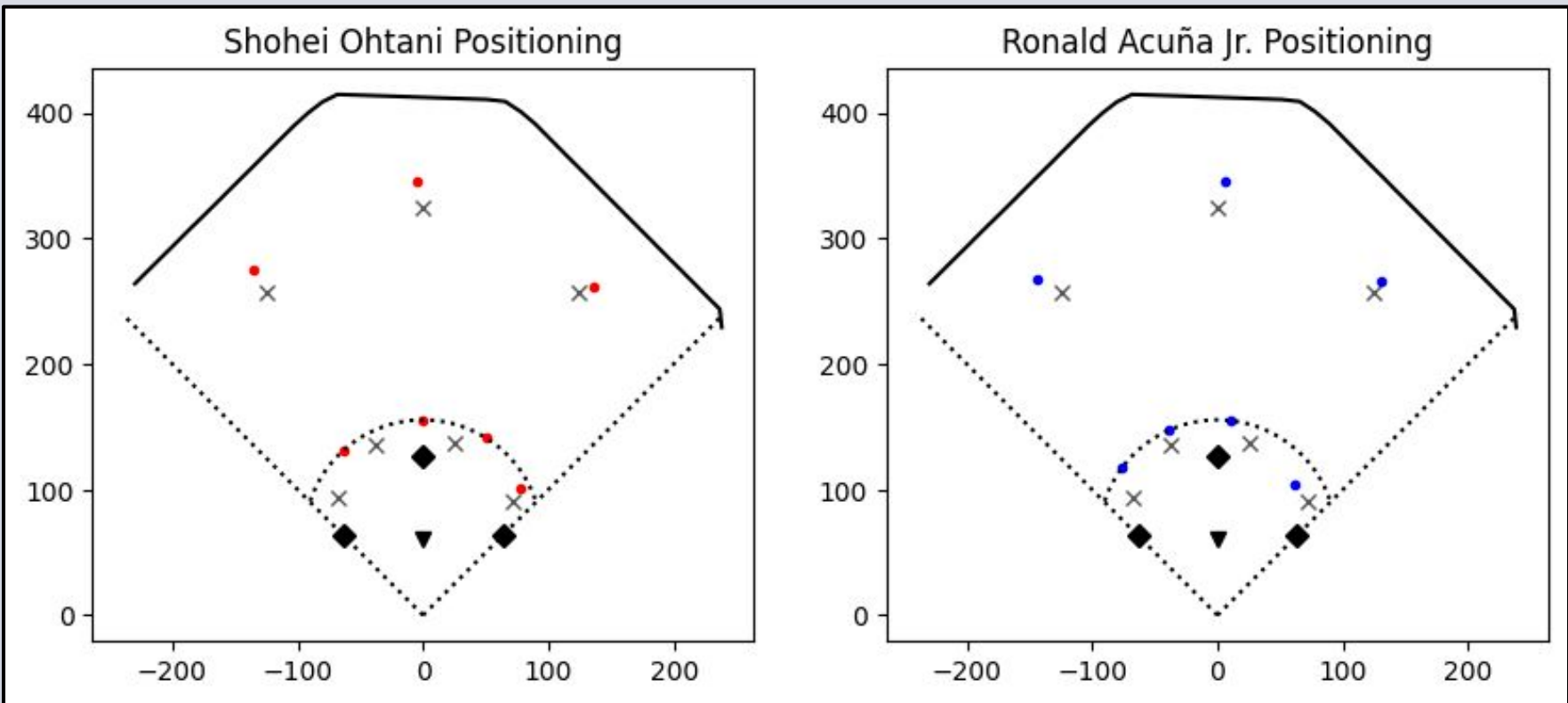


Because outs and runs are closely related, we were able to convert out probability to expected run value.

Given a specific batted ball distribution, any positioning strategy can now be given an expected run value. This allows us to optimize the positioning strategy such that it minimizes runs scored.

Optimal Positioning Against Specific Batters

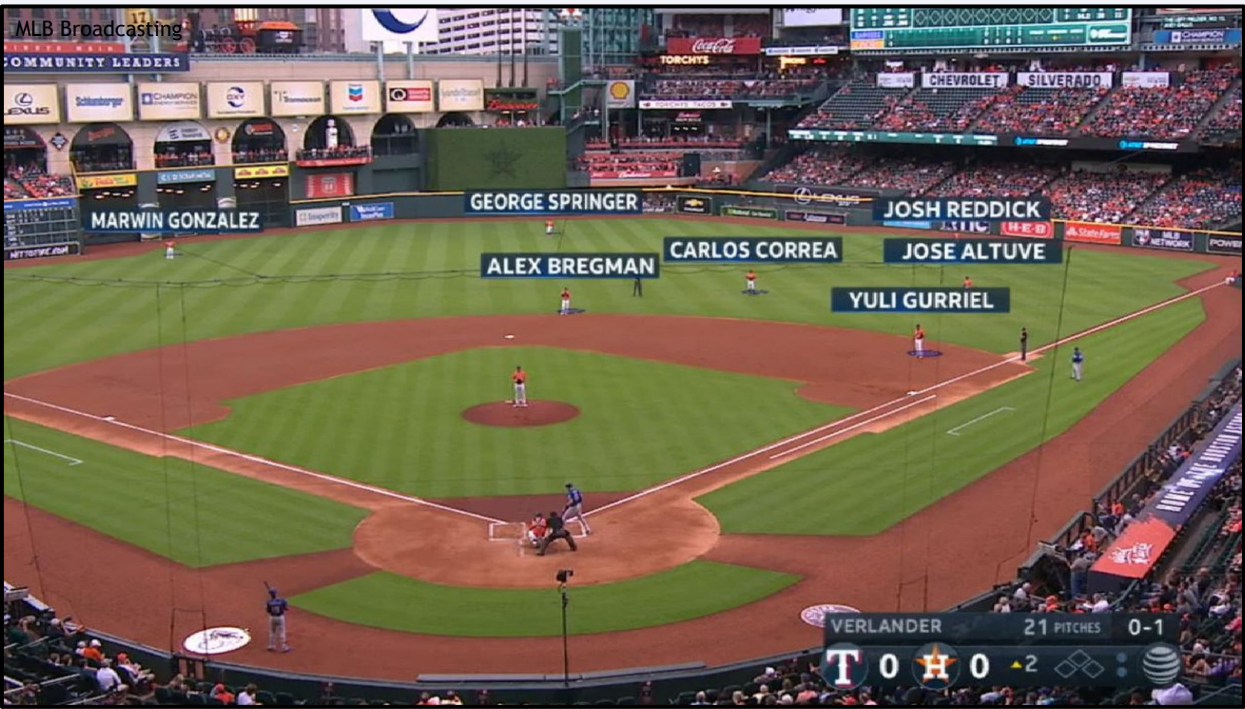
After beginning with starting positions of the seven fielders behind the pitcher and catcher, we used gradient descent to determine a new set of positions that minimizes runs against a particular batter (or batted ball distribution).



Optimal positioning against Shohei Ohtani and Ronald Acuña Jr. in order to minimize runs scored. The X's are the starting positions and the red and blue dots are the optimal positions.

The Need for Equilibrium

Minimizing runs against a single batted ball distribution is only optimal if the batter has no control over the distribution they hit from. This is not the case; some batters have exhibited the ability to change their distribution when shifted against, and any batter can choose to bunt. We formulated an equilibrium such that the defense minimizes runs under the assumption that the batter will respond by maximizing runs.

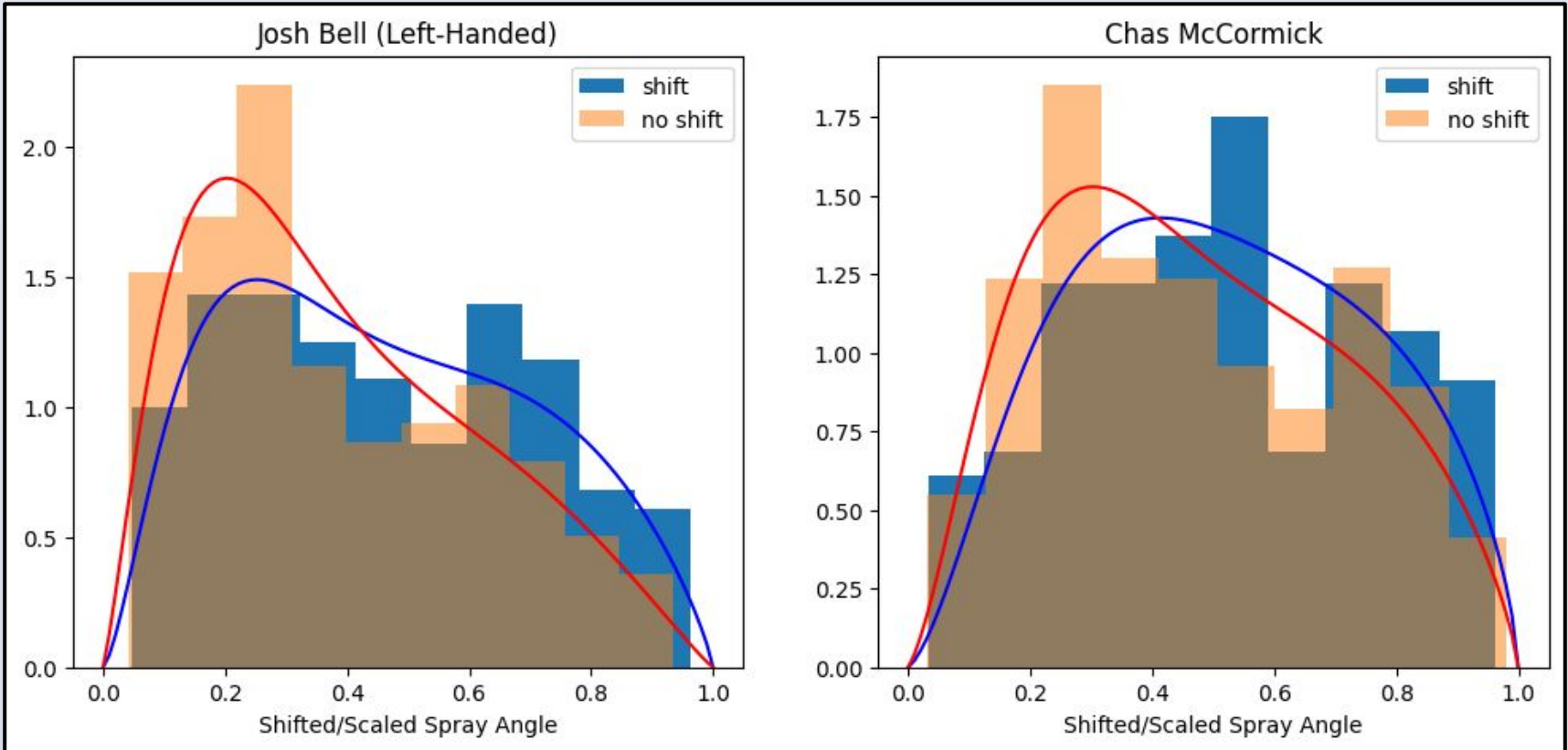


$$p_s^* \in \operatorname{argmin}_{p_s \in P_s} \max_{d \in D} \mathbb{E}_{b \sim d} [(1 - f(b, p_s)) \cdot f_{\text{woba}}(b)] = \operatorname{argmin}_{p_s \in P_s} \max_{d \in D} \frac{1}{N} \sum_{j=1}^N (1 - f(b_j, p_s)) \cdot f_{\text{woba}}(b_j)$$

$$d^* \in \operatorname{argmax}_{d \in D} \min_{p_s \in P_s} \mathbb{E}_{b \sim d} [(1 - f(b, p_s)) \cdot f_{\text{woba}}(b)] = \operatorname{argmax}_{d \in D} \min_{p_s \in P_s} \frac{1}{N} \sum_{j=1}^N (1 - f(b_j, p_s)) \cdot f_{\text{woba}}(b_j)$$

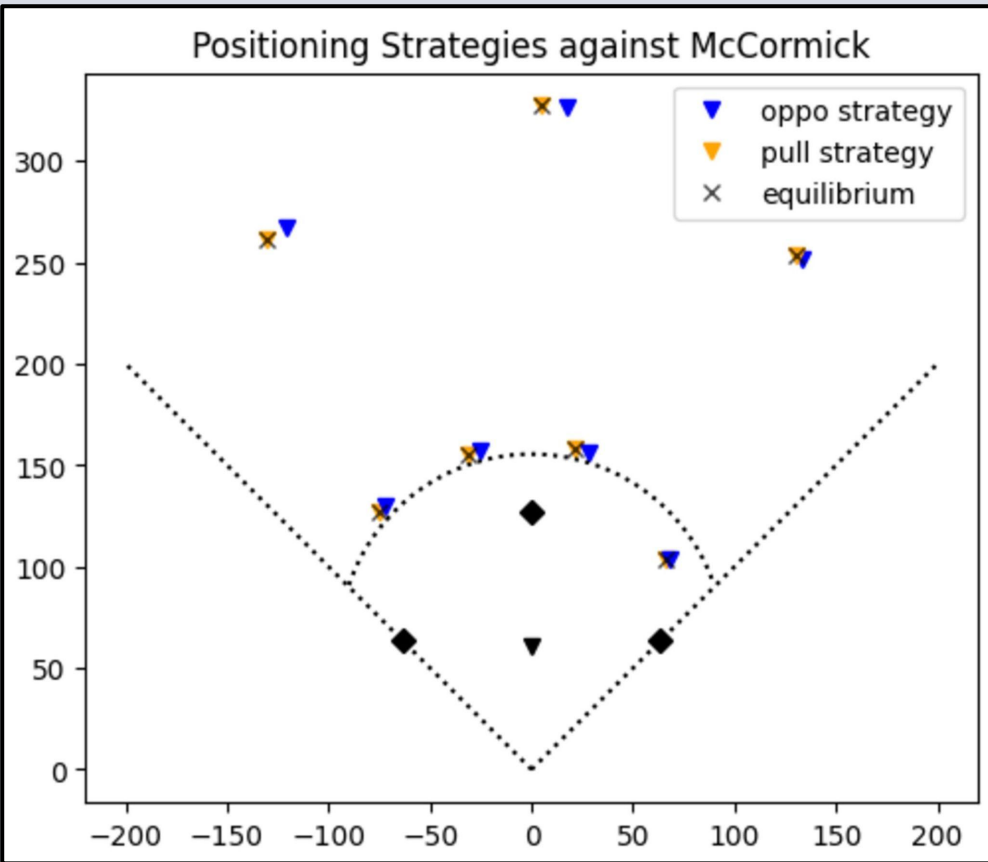
Defending Against Batters Who Can “Beat the Shift”

Batters are more focused on making solid contact than they are on aiming their hit, so changing their batted ball distribution is extremely difficult. However, some batters such as Josh Bell and Chas McCormick have demonstrated the ability to choose between pull-heavy and more opposite-heavy distributions, depending on the positioning of the defense.



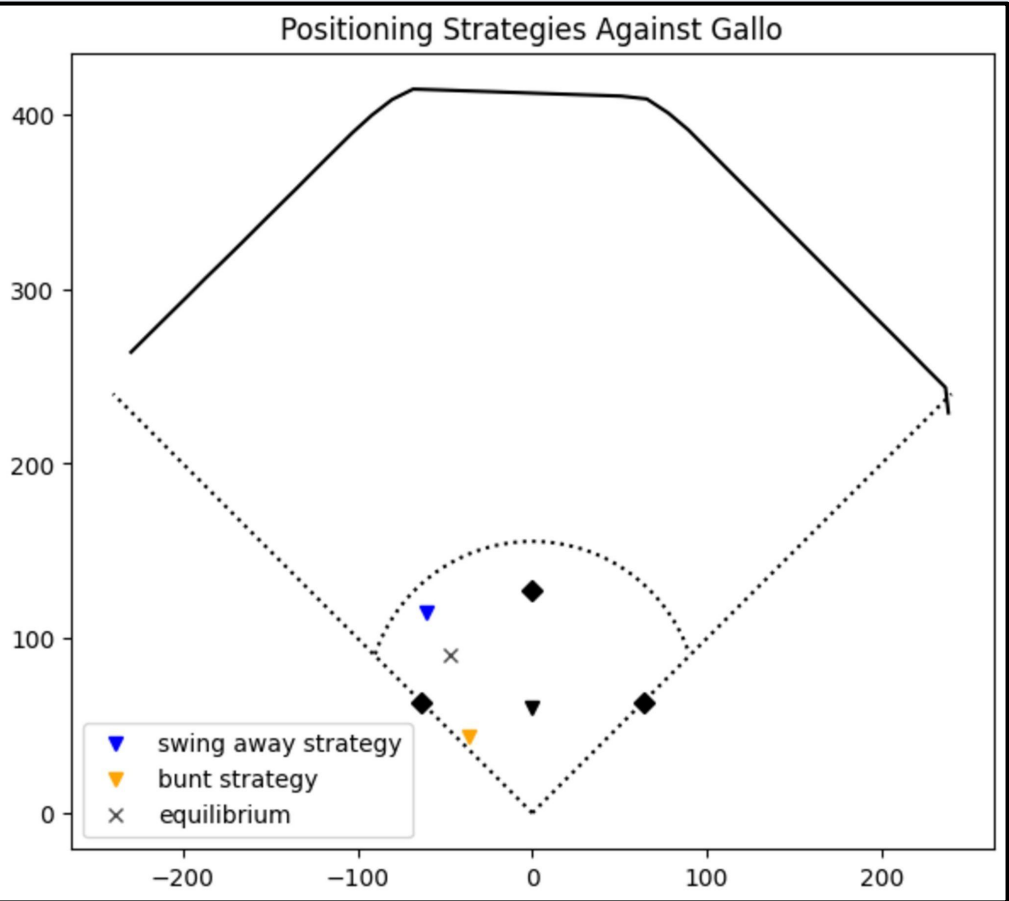
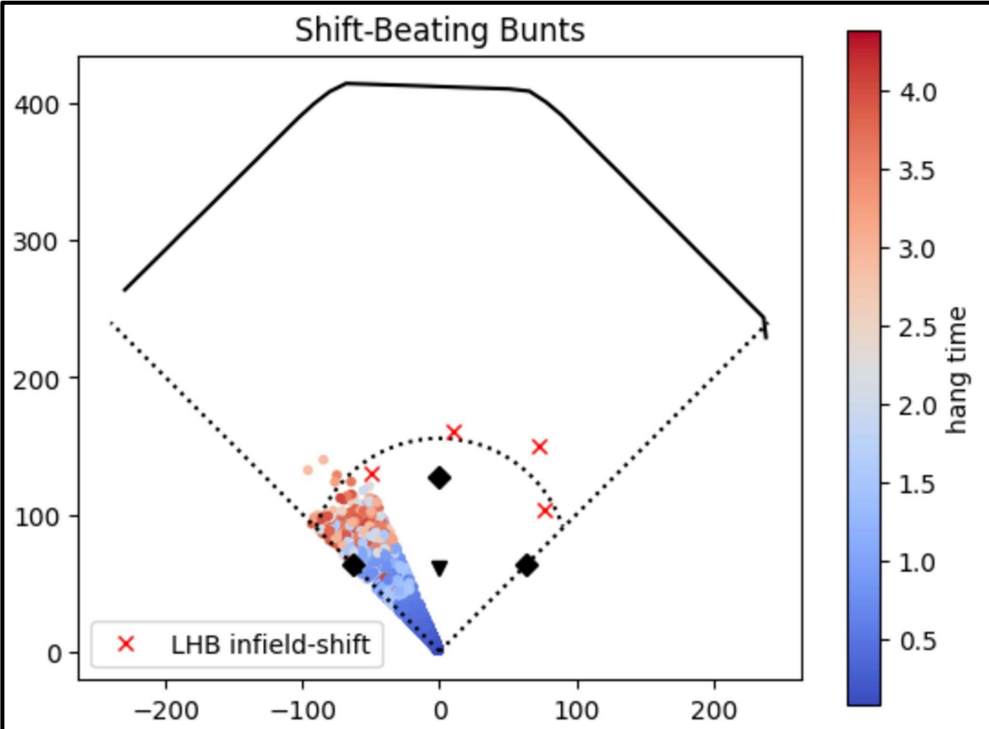
When defenses shift towards the pull side of the field, Bell and McCormick hit the ball towards the opposite side of the field more frequently than usual

The figure on the right shows the optimal positioning strategies against McCormick’s pull distribution and opposite distribution, along with the equilibrium strategy. The equilibrium strategy minimizes hits under the assumption that McCormick chooses the distribution that maximizes hits. What is interesting about the equilibrium strategy is that it is located on top of the pull strategy. This means that, despite McCormick’s ability to hit the ball to the opposite side, he does not generate hits by doing this. This may be because he struggles to get solid contact when hitting to the opposite side. This may not be the case with other batters.



Protecting Against the Bunt

Most batters are not able to reliably change their batted ball distributions like Bell and McCormick. However, any left-handed batter should be able to beat an extreme shift with a bunt. Joey Gallo is an example of a batter who can do this. Because he often pulls the ball as a left-handed batter, defenses frequently shift their infield towards the first-base side of the field. This leaves the third baseline wide open for a well-placed bunt.



For the defense to optimally defend against potential bunts, positioning strategies must take into account that any batter they face can choose between their typical batted-ball distribution and a bunt. Defenses must then assume that the batter will always choose the distribution that will score the most runs given the current positioning strategy.

We once again used gradient descent to find the equilibrium positioning. In this positioning, a batter who always chooses the correct distribution will have their hits created minimized.

As shown above, the third baseman should play deep and towards second base if he knows Gallo will swing away. On the other hand, he should be almost halfway to home plate if Gallo bunts. Because he does not know what Gallo will do, he should stand at the equilibrium position, which is robust against all of Gallo’s strategies. We used batting average instead of runs for Gallo’s equilibrium.

	Anti-Bunt Defense	Shifted Defense	Equilibrium Defense
Bunt	0.036	0.574	0.222
Swing Away	0.290	0.223	0.226

The above table shows that the equilibrium positioning minimizes Gallo’s batting average assuming that Gallo always makes the optimal decision. This is a big assumption to make because batters and coaches are often reluctant to bunt even when it is the right decision. This means defenses can shift away from the equilibrium without needing to worry about the bunt. Batters could take advantage of this by bunting more and forcing defenses back into the equilibrium positioning. If defenses were forced out of the shift like this, batters would get more hits, leading to more runs scored and a more entertaining game.

