Availability

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1 Reliability

Any system, operated long enough, will experience failures. We let the reliability, R as a function of the time t that the system has been operated since it started to be:

$$R(t) = e^{-\lambda t} \tag{1}$$

This equation suggests that as a particular system is running for an increasing period of time, the likelihood of it not experiencing a failure falls, Figure 1, according to the parameter λ. We call λ the *failure rate*.

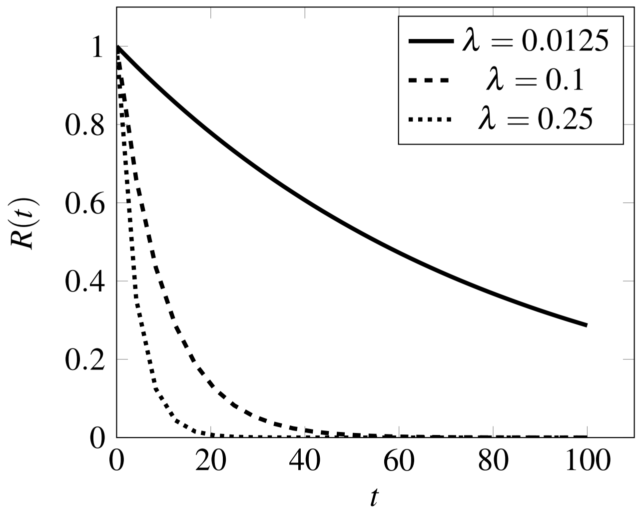

Figure 1: Reliability as a function of time

1.1 Failure rate

Assuming that a system on average will have N_f failures, observed over a total time of T_p, we define the failure rate to be:

$$\lambda = \frac{N_f}{T_p} \tag{2}$$

Meaning that λ has units of inverse time.

Example 1 (Failure rate)**.** A system's manager recorded 1 failure in a 4 year period. What is the system's failure rate per year?

$$\lambda = \frac{N_f}{T_p} \tag{3}$$
$$= \frac{1}{4\,\text{year}} \tag{4}$$
$$= 0.25\,\text{year}^{-1} \tag{5}$$

2 Bathtub curve

In practice, the failure rate is elevated at the beginning and end of a system's life, following the so-called bathtub curve, Figure 2. The failure rate of any system normally excludes these particular periods and can be assumed to be constant for a particular system.

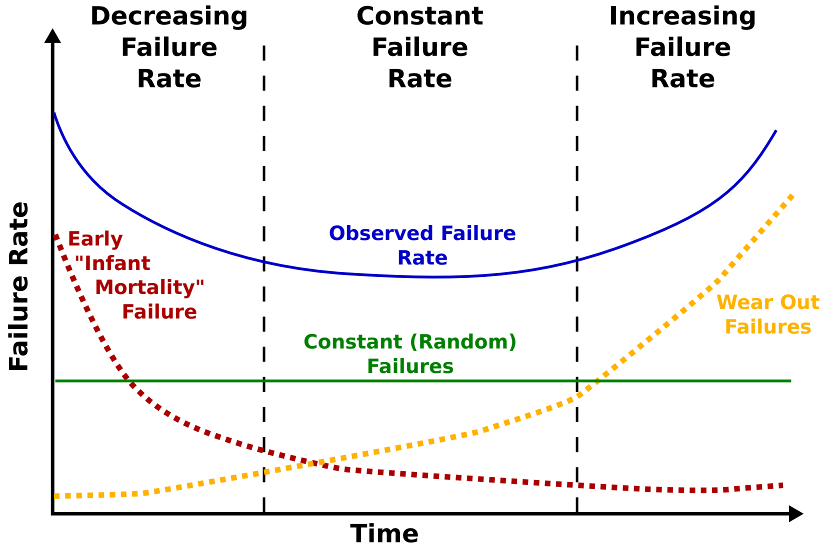

Figure 2: Bathtub curve

3 Mean time between failures

The Mean Time Between Failures (MTBF) is a reciprocal of the failure rate, and has units of time:

$$\text{MTBF} = \frac{1}{\lambda} \tag{6}$$

It is generally accepted that the mean time between failures relates only to the middle portion of the so-called "bathtub curve" of reliability, Figure 2.

Example 2 (Mean time between failures)**.** A system on average fails on two occasions in five years. Calculate the mean time between failures.

$$\begin{aligned}
\text{MTBF} &= \frac{1}{\lambda} && (7)\\
&= \frac{1}{\frac{N_f}{T_p}} && (8)\\
&= \frac{T_p}{N_f} && (9)\\
&= \frac{5}{2} && (10)\\
&= 2.5\,\text{year} && (11)
\end{aligned}$$

4 Mean time to repair

Assuming that a failure has occured, it normally requires a repair time, during which the system is unavailable. Averaged, we say that a particular type of failure has a Mean Time To Repair (MTTR).

5 Inherent availability

The inherent availability of a system tells us for what percentage of time it is likely to be available. It is based on two ideas:

1. The system is available between failures, which should occur at intervals of the MTBF.
2. When a failure occurs, it will be unavailable for time taken to repair, ie the MTTR.

So, the availability is essentially determined by how often a repair is needed and how long it takes. Knowing the MTBF and MTTR, we can estimate the inherent availability, A_i of a system:

$$A_i = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{12}$$

Example 3 (Inherent availability)**.** A system has an MTBF of 24 days and an MTTR of 12 hours. Calculate the inherent availability. Assuming we take a base unit of days, so that 12 hours = 0.5 days.

$$\begin{aligned}
A_i &= \frac{24}{24 + 0.5} && (13)\\
&= 98\,\% && (14)
\end{aligned}$$

6 Operational availability

The opearational availability of a system extends the inherent availability to incorporate scheduled maintenance downtime.

7 N

We say that to provide a particular function, we need a particular number of units N.

8 Common patterns

8.1 N+1

If we need N units, we add an additional unit to cover failures. The additional unit is denoted $+1$. This gives us the designation $N+1$.

8.2 2N

If we need N units, we duplicate each individual unit giving us the designation $2N$.

Note that in practice sometimes the $2N$ and $N+1$ configurations lead to the same number of units, but usually one designation will be preferred over another to convey meaning.