Power

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September 13, 2021

1 Electrical basics

Table 1 summarises key electrical quantities.

Quantity	Symbol	Unit	
Voltage	V	Volt	V
Current	I	Ampere	A
Resistance	R	Ohm	Ω
Power	P	Watt	W

Table 1: Key electrical quanities

Note that voltage is a difference between *two* points. Often measured in any system with respect to a common earth / ground.

1.1 Ohm's law

Voltage, current and resistance are related by Ohm's law, which can be written in terms of V, I or R. Re-arrange to calculate required quantity.

$$V = R \cdot I \tag{1}$$

$$\Rightarrow I = \frac{V}{R} \tag{2}$$

$$\Rightarrow R = \frac{V}{I} \tag{3}$$

Example 1 (Ohm's law)**.** A 7 Ω resistance carries a current of 2 A. Determine the voltage across the component.

$$V = 7 \times 2 \tag{4}$$

$$= 14\,\mathrm{V} \tag{5}$$

Example 2 (Ohm's law with rearrangement)**.** When an electrical component is connected across a battery nominally supplying 12 V a current of 3 A flows. Calculate the resistance of the component.

$$V = R \cdot I \tag{6}$$
$$\Rightarrow R = \frac{V}{I} \tag{7}$$
$$= \frac{12}{3} \tag{8}$$
$$= 4\,\Omega \tag{9}$$

1.2 Power

Power quantifies how much energy is converted from one form to another per unit time. Measured in Joule per second $\mathrm{J\,s^{-1}}$, more commonly the Watt W. Just as with Ohm's law, the power relation, Equation 10, can be rearranged to give V or I:

$$P = V \cdot I \tag{10}$$
$$\Rightarrow V = \frac{P}{I} \tag{11}$$
$$\Rightarrow I = \frac{P}{V} \tag{12}$$

Example 3 (Power calculation)**.** A graphics card is supplied from the 12 V power supply in a computer. The current flowing is measured at 5 A. Determine the power consumed by the graphics card.

$$P = 12 \times 5 \tag{13}$$
$$= 60\,\mathrm{W} \tag{14}$$

Example 4 (Power calculation with rearrangement)**.** A computer power supply delivers 6 W to a hard disk drive on the 12 V line. Determine the current flowing in the cable.

$$I = \frac{P}{V} = \frac{6}{12} \tag{15}$$
$$= 0.5\,\mathrm{A} \tag{16}$$

2 Mains electricity

In Ireland, mains electricity is supplied at a *nominal* 230 V 50 Hz. Mains wiring in Ireland generally involves three conductors:

Live (or hot, or phase) carries a 230 V RMS AC voltage.

Neutral provides the return path for current on the live conductor, and under normal conditions will be the negative of that.

Earth is connected to earth and bonded to metal casings.

2.1 Circuit protection

Fuses: a piece of thin wire encased in a holder that is deliberatly designed to melt if the current exceeds the fuse rating.

Circuit Breakers (MCB): electromechanical devices that will trip when the current exceeds the circuit breaker's rating.

Residual current device (RCD): protect from electric shock by detecting any leakage of current to earth by comparing live and neutral currents. Trips if these differ by more than a set amount ΔI, normally 30 mA. Other names: GFI, ELCB.

Residual Current Breaker Overload (RCBO): combined MCB and RCD functionality in one device.

2.2 Alternating current (AC)

Mains electricity is supplied in most parts of the world as alternating current (AC). This means that the instantaneous voltage $v(t)$ varies sinusoidally with respect to time.

$$v(t) = V_{\max}\sin(2\pi f t) \tag{17}$$

A single cycle of a generic AC waveform is shown in Figure 1

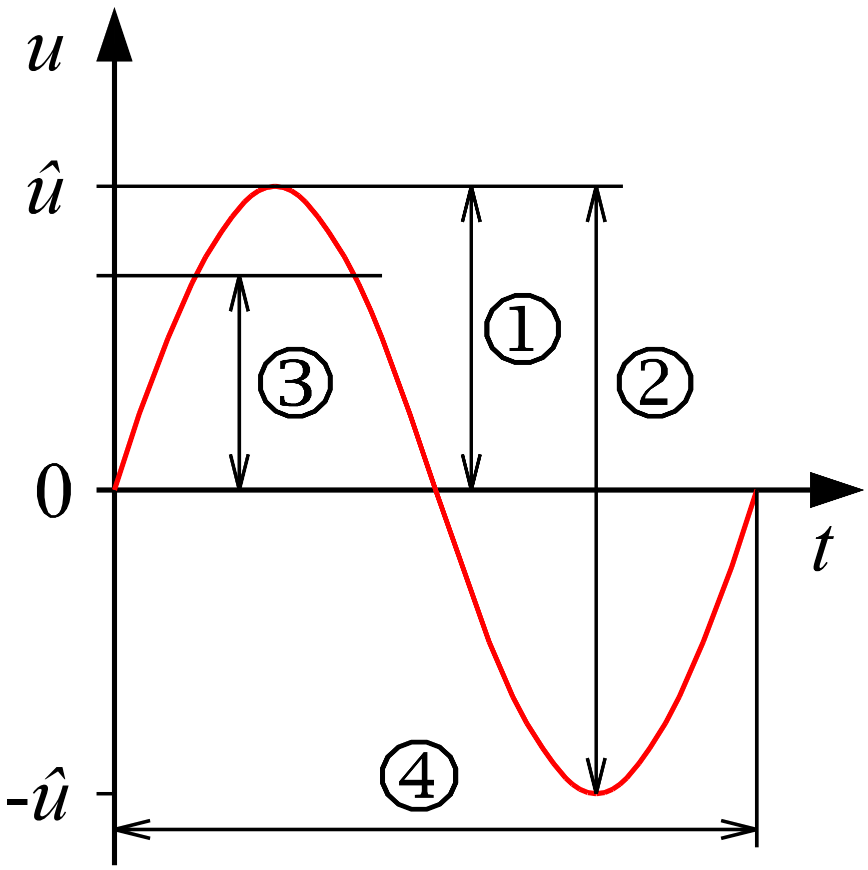

Figure 1: AC waveform properties

The **maximum voltage** $V_{\max}$ of a sinusoid is the amplitude of the sine wave in both directions. The most positive value is $V_{\max}$ whilst the most negative value is $-V_{\max}$. We can thus define the **peak-to-peak** amplitude as the difference between these two values:

$$V_{\text{PK-PK}} = V_{\max} - (-V_{\max}) \tag{18}$$
$$= 2V_{\max} \tag{19}$$

Example 5 (Peak-to-peak to amplitude)**.** Calculate the amplitude of an AC waveform with a 650 V peak-to-peak amplitude.

$$V_{\text{max}} = \frac{V_{\text{PK-PK}}}{2} \tag{20}$$
$$= \frac{650}{2} \tag{21}$$
$$= 325\,\text{V} \tag{22}$$

2.2.1 RMS Voltage

The voltage in western Europe is a nominal 230 V RMS. This is a root mean square value, which is equivalent to the heating power that the same DC voltage would deliver.

$$V_{\text{max}} = \sqrt{2}V_{\text{RMS}} \tag{23}$$
$$\Rightarrow V_{\text{RMS}} = \frac{V_{\text{max}}}{\sqrt{2}} \tag{24}$$

Example 6 (RMS to peak voltage)**.** Calculate the amplitude of a 230 V RMS AC supply.

$$V_{\text{max}} = \sqrt{2}V_{\text{RMS}} \tag{25}$$
$$= \sqrt{2} \times 230 \tag{26}$$
$$= 325\,\text{V} \tag{27}$$

Example 7 (Peak to RMS voltage)**.** Calculate the RMS voltage of an AC supply with an amplitude of 100 V.

$$V_{\text{RMS}} = \frac{V_{\text{max}}}{\sqrt{2}} \tag{28}$$
$$= \frac{100}{\sqrt{2}} \tag{29}$$
$$= 70.7\,\text{V} \tag{30}$$

2.2.2 Frequency / Period

A single cycle lasts for a period of time, T. The period is directly related to the frequency:

$$T = \frac{1}{f} \tag{31}$$
$$\Rightarrow f = \frac{1}{T} \tag{32}$$

Example 8 (Frequency to period)**.** Calculate the period of a signal that repeats at 20 Hz.

$$T = \frac{1}{20} \tag{33}$$
$$= 0.05\,\text{s} \tag{34}$$

Example 9 (Period to frequency)**.** Determine the frequency of a signal with a period of 40 ms.

$$f = \frac{1}{40 \times 10^{-3}} \tag{35}$$
$$= 25\,\text{Hz} \tag{36}$$

3 Computer power supplies

Power Supply Units (PSUs) are used to convert the mains-supplied power into a form suitable for use in computers.

3.1 Computer power requirements

Computers require a variety of DC voltages, the most common being:

3.3 V (orange)

5 V (red)

12 V (yellow).

These are normally supplied relative to a common ground (black).

Negative −12 V (blue) is often available.

A PSU also permanently supplies 5 V (purple) and will turn on when the green terminal is shorted to ground.

3.2 Power supply tasks

A computer's power supply unit (PSU) has two key jobs:

Rectify the mains-supplied AC to a steady DC supply.

Step down the voltage (230 V) to the required level(s).

The precise methods and order that these tasks are performed in will vary, and are outside the scope of our discussion.

3.3 Capacity

A power supply will usually have a rated capacity:

- This will either be given in terms of power (watts) or in current (amps).
- On a supply providing multiple voltages, there will usually be a limit on each rail as well as possibly an overall limit.

3.4 Three-phase supply

Mains power is generated and distributed in three-phase form, with 3 live conductors and one neutral conductor. The sine wave is shifted by 120 degrees, or $\frac{2\pi}{3}$ radians in any phase relative to one of the two other phases.

Let $v_n(t)$ be the voltage in phase n of a three-phase supply. We can use Equation 17 for the first phase. Phase 2 must lags phase 1 by 120 degrees. Similarly, phase 3 leads phase 1 by 120 degrees.

$$v_1(t) = V_{\text{max}} \sin(2\pi f t) \tag{37}$$

$$v_2(t) = V_{\text{max}} \sin\left(2\pi f t - \frac{2\pi}{3}\right) \tag{38}$$

$$v_3(t) = V_{\text{max}} \sin\left(2\pi f t + \frac{2\pi}{3}\right) \tag{39}$$

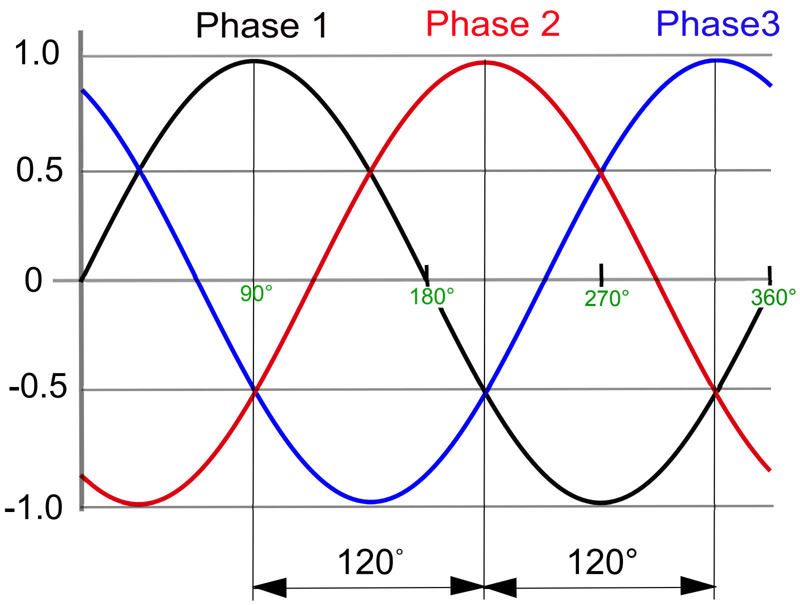

Figure 2: 3-phase waveform (Wikipedia)

3.4.1 Line and phase voltages

When dealing with three-phase power, we actually have two voltages to consider:

Phase voltage is the voltage between *any* phase and neutral.

Line voltage is the voltage measured between *any* two phases.

The line and phase voltages are related mathematically by $\sqrt{3}$:

$$V_{\text{line}} = \sqrt{3} \times V_{\text{phase}} \tag{40}$$

$$\Rightarrow V_{\text{phase}} = \frac{V_{\text{line}}}{\sqrt{3}} \tag{41}$$

Example 10 (Line to phase voltage)**.** A three phase power supply has a phase voltage of 220 V. Calculate the line voltage.

$$V_{\text{line}} = \sqrt{3} \times V_{\text{phase}} \tag{42}$$

$$= \sqrt{3} \times 220 \tag{43}$$

$$= 381\,\text{V} \tag{44}$$

Example 11 (Phase to line voltage)**.** A three phase power supply has a line voltage of 400 V. Calculate the phase voltage.

$$V_{\text{phase}} = \frac{V_{\text{line}}}{\sqrt{3}} \tag{45}$$

$$= \frac{400}{\sqrt{3}} \tag{46}$$

$$= 231\,\text{V} \tag{47}$$

4 Distribution path

Figure 3 shows the basic power distribution hierarchy in a data centre.

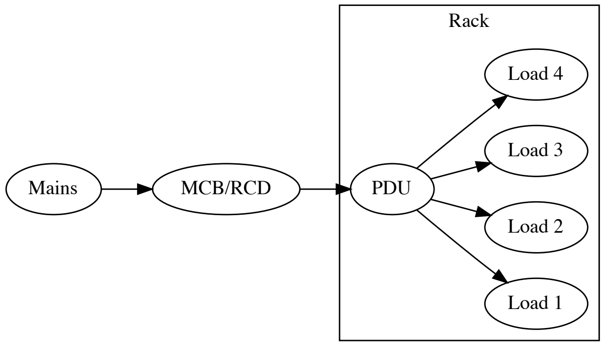

Figure 3: Power distribution schematic

We will consider the distribution path looking *backwards* from the IT equipment towards the incoming mains.

4.1 Power distribution unit (PDU)

Each rack normally has a power distribution unit, which is no more complicated than a multiplug adapter:

- PDU can be mounted vertically in the back of rack or in a rack space (facing in or out).
- PDUs are available with various combinations of input and output connectors.
- PDUs may or may not have surge protection and switches.
- Maximum current will usually be dependent on the connector type on the input to the PDU.
- "Smart" PDUs are available that can measure power demand and turn on/off sockets remotely. (See later on!)

4.2 Common connectors

Table 2 shows the most common mains power connectors (plugs and sockets) that will be encountered in a data centre environment.

Type	I_{max}	Male	Female
BS 1363	13 A		
IEC C13/14	10 A	C14	C13
IEC C19/20	16 A	C20	C19
IEC 60309	16 A		

Table 2: Common connector types

5 Disturbances

Our IT equipment expects clean power with its key parameters (voltage, frequency) maintained within allowable tolerences (230 V RMS, 50 Hz). Waveform must be a *clean* sine wave, not *distorted*.

5.1 Disturbance types

Blackout: total loss of power.

Surge/sag: **short-term** (0.5 of a cycle up to 1 minute) voltage variations:

- **Surge or spike** is a short-term high-voltage condition more than 110 % of the nominal value.
- **Sag** is a short-term low-voltage condition.

Over and under-voltage conditions that **persist for time periods** ranging from minutes to days:

- **Over-voltage** is increased mains voltage.
- **Under-voltage** is reduced mains voltage. (Formerly: brownout)

Frequency fluctuations away from 50 Hz for long and short periods.

Waveform distortion when the mains voltage waveform no longer is a sinusoid. This can manifest in a number of ways: offsets, harmonics, notching and noise.

5.2 CBEMA curve

Undesirable conditions are generally less disruptive the shorter that they persist for. This includes disturbances in the power supply. The Computer Business Equipment Manufacturers Association (CBEMA) in the 1970s generated a curve that partitioned voltage events and times into acceptable and unacceptable region. The Information Technology Industry Council (ITIC) adapted the curve in the 1990s, which was most recently updated in 2000, Figure 4

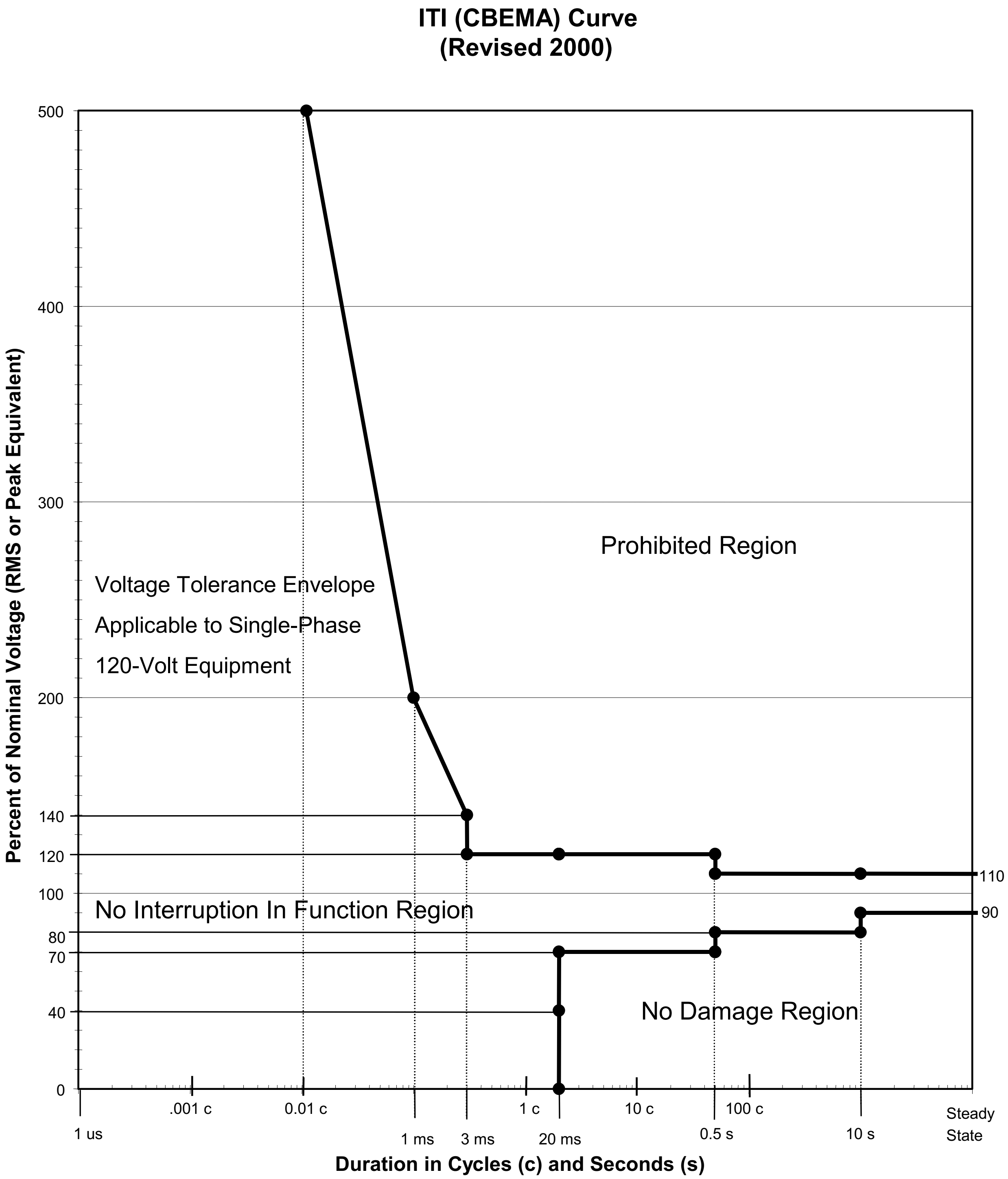

Figure 4: ITI CBEMA Curve 2000

6 Uninterruptible Power Supplies (UPS)

6.1 Key Components

UPS units are complex devices but contain a number of key building blocks you should know:

Battery as an energy storage medium. Common types include: Lead-Acid, Nickel-Cadmium (NiCd), Nickel Metal Hydride (NiMH), Lithium-Ion.

Rectifier to convert mains AC to DC for battery charging.

Inverter to take DC and convert it to AC at a given voltage and frequency.

Transfer switch to swap between two sources of power. Can be a mechanical relay or contactor[1] but is more usually solid-state, called a *static transfer switch* or STS.

Surge suppressor: a solid-state device that reduces voltage by letting current flow to earth when a voltage exceeds the so-called *let through* voltage.

6.2 Form factors

UPS units are available in various form factors:

Freestanding / tower similar to PC powering a single device or multiple devices via a PDU. Usually located adjacent to the IT equipment.

Rackmount powering a single device or multiple devices via a PDU. Usually co-located inside the same rack as the IT equipment.

Floor-standing UPS devices located within the IT environment itself or in another part of the facility. These normally supply multiple IT loads and are often managed by facilities rather than IT personnel.

[1] A contactor is the conventional name used for a large relay able to switch many amperes of current.

6.3 UPS types

There are three main categories of UPS: standby, line-interactive and double conversion. All UPS devices will protect against blackout (for as long as their batteries last).

6.3.1 Standby

A standby UPS normally just passes the utility through to the output, while performing basic surge suppression, Figure 5.

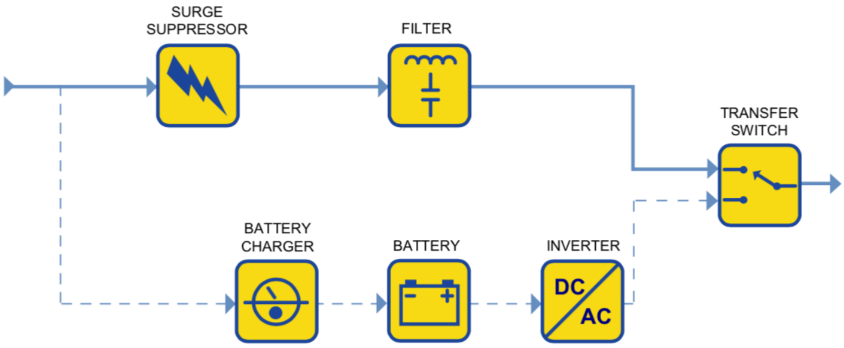

Figure 5: Standby UPS schematic (APC)

The battery is charged from the mains. Under failure of the mains supply, the UPS will use its inverter to generate AC. The transfer switch changes the output from utility to inverter.

The standby UPS protects against blackouts and small surges/sags whilst remaining online. It will transfer to inverter supply in the case of under/over voltage conditions.

6.3.2 Line interactive

A line interactive UPS is capable of correcting reasonably small under/over voltage conditions **whilst remaining online**, Figure 6.

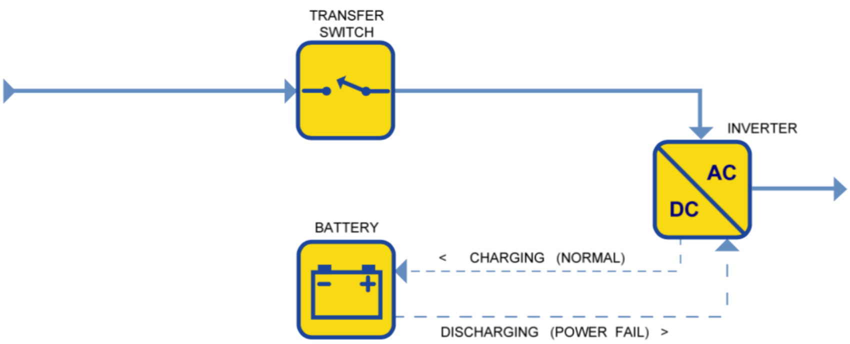

Figure 6: Line interactive UPS schematic (APC)

The line interactive UPS will correct surges/sags and under/over voltage whilst remaining online. It will use transfer to battery power to correct issues with AC frequency and waveform quality.

6.3.3 Double conversion

The double-conversion UPS differs from the standby and line-interactive UPS in that it doesn't differentiate between online/offline modes of operation. Double conversion UPS units correct both voltage and frequency disturbances. Figure 7.

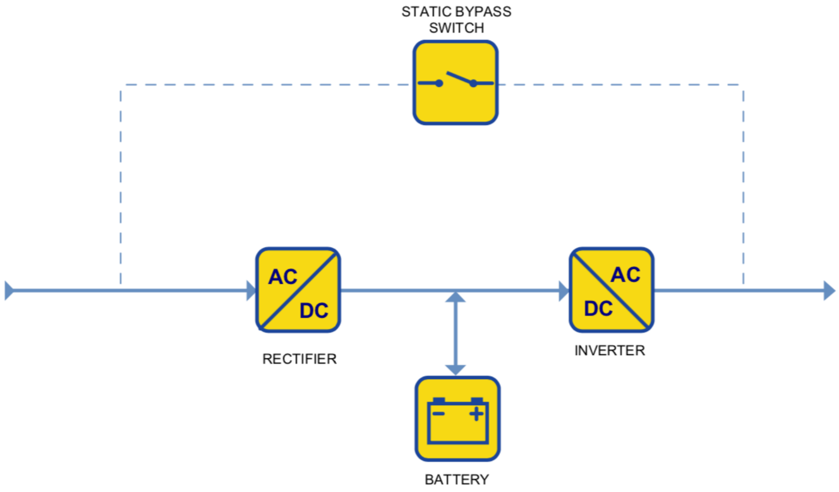

Figure 7: Double conversion UPS schematic (APC)

They consist of a battery bank charged by the mains, from which an inverter generates a clean AC waveform at the right voltage and frequency,

7 UPS sizing

UPS sizing needs to consider how much power the UPS is expected to supply (determines inverter size), and for how long (determines battery size). The reactive / apparent power requirements need to be considered.

Specifying a UPS is a somewhat inexact process. A undersized unit will not work, but an oversized unit will. Therefore, we normally pad calculated requirements by a 20 % buffer.

7.1 Real power requirements

The UPS power required, with padding, can be calculated:

$$P_{\text{total}} = \left(\sum_{\text{devices}} P_{\text{device}} \right) \times 1.2 \tag{48}$$

7.2 Apparent power

When sizing power distribution components, we need to consider the so-called Volt-Amp rather than the Watt. So far we know that the watt is the unit of power. From the power relation we first met in Equation 10, $P = V \cdot I$, we might reasonably expect that $1\,\mathrm{W} = 1\,\mathrm{V\,A}$. However, in real life this isn't the case.

7.2.1 Reactive power

In simple loads like incandescent lights and heaters, regardless of size, the current and voltage will be perfectly in phase. However, with many real-world loads the current wave will lead or more usually lag the voltage wave, becuase of the dynamical nature of the circuits.

Inductive loads will cause the current wave to *lag* the voltage wave.

Capacitive loads will cause the current wave to *lead* the voltage wave.

These loads are the electrical equivalents of a hose or balloon filled with pressurised water, or a large heavy flywheel.

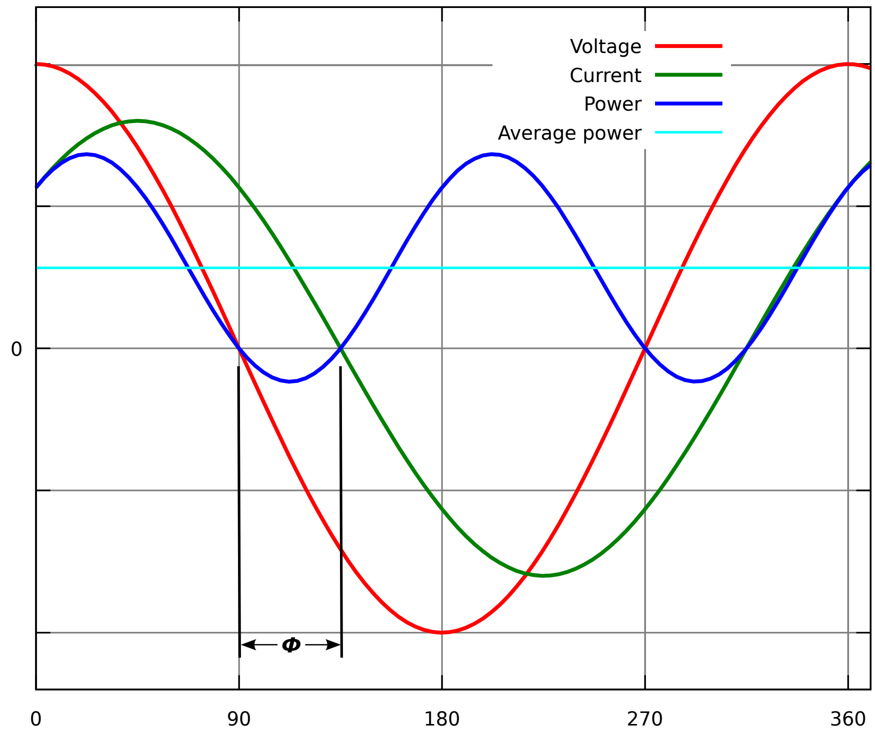

Figure 8: Reactive load showing current lagging voltage (Wikipedia)

Let P be the real power, and Q be the reactive power. The imaginary unit j is such that $j^2 = -1$. (Some textbooks, including leaving cert maths use i as the imaginary unit, but it is prone to confusion with i meaning current.) We define the apparent power S by:

$$S = P + jQ \tag{49}$$

This is a complex number, which we can think of as the so-called power triangle.

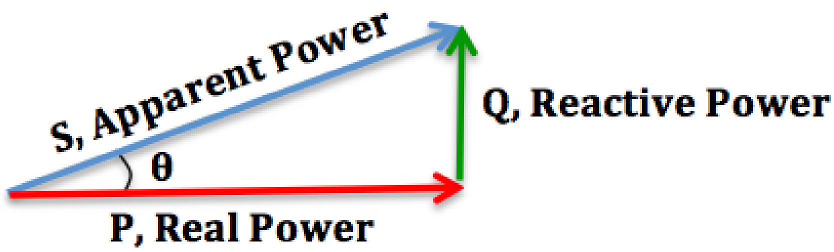

Figure 9: Power triangle (Wikipedia)

The apparent power for sizing purposes of a UPS is then simply the magnitude, $|S|$.

$$|S|^2 = P^2 + Q^2 \tag{50}$$

$$|S| = \sqrt{P^2 + Q^2} \tag{51}$$

7.2.2 Power factor

Looking back at the power triangle, Figure 9, the angle θ encodes the relative breakdown between real and reactive power. Assuming we know P and Q, the angle θ is simply:

$$\theta = \tan^{-1}\frac{Q}{P} \tag{52}$$

We normally consider not the angle θ, but the cosine of it. This gives us the ratio of real power to apparent power, and it is called the **power factor**:

$$\cos\theta = \frac{P}{|S|} \tag{53}$$

In terms of its interpretation:

- Power factor is a dimensionless number between -1 and 1.
- Negative power factors imply a device generating real power, not consuming it. We will assume the power factor here is between 0 and 1.
- Power factor does not tell if current is leading/lagging the voltage. Assumed lagging unless specified.

7.2.3 Apparent power for a single device

If you have the voltage and amps **directly** specified for a particular piece of equipment, you can just multiply to get S:

$$|S_{\text{device}}| = V \times I \tag{54}$$

Example 12 (VA calculation from voltage and current)**.** A server's power supply has a rating plate claiming that it consumes up to 3.5 A when connected to a 230 V supply. Determine the VA. Here, we simply use the V and I ratings as given.

$$|S| = V \times I \tag{55}$$

$$= 230 \times 3.5 \tag{56}$$

$$= 805\,\mathrm{V\,A} \tag{57}$$

If you have the power drawn by the device in watts, and you know the power factor, determine $|S|$ by calculating:

$$|S_{\text{device}}| = \frac{P}{\cos\theta} \tag{58}$$

If you're not given a power factor, $\cos\theta = 0.8$ will usually work, but state that assumption.

Example 13 (VA calculation from power)**.** The specification sheet for a server shows that it consumes up to 250 W. Determine the VA requirement. Given the information we have, we will assume a power factor of 0.8.

$$|S| = \frac{P}{\cos\theta} \tag{59}$$

$$= \frac{250}{0.8} \tag{60}$$

$$= 312.5\,\mathrm{V\,A} \tag{61}$$

7.2.4 Apparent power for multiple devices

Sum up the VA requirements, remembering to apply the power factor (if not uniform) to each device before summation. The padding is normally added post summation.

$$|S|_{\text{total}} = \left(\sum_{\text{devices}} |S_{\text{device}}| \right) \times 1.2 \tag{62}$$

7.3 Runtime

Runtime for a UPS is normally determined using the sizing chart on the specification sheet.

Runtime can often be extended by adding additional battery packs.

7.3.1 Battery capacity

The capacity is specified in units of A h. This means that the battery can supply the given number of amperes of current for one hour.

$$\text{hours available} = \frac{\text{capacity}}{\text{current}} \tag{63}$$

Alternatively it can trade off the amount of current delivered against the time period.

Example 14 (Battery capacity calculation)**.** A 12 V battery has a capacity of 80 A h. It is to supply a load that requires a constant 5 A. Calculate how many hours would this battery last assuming it was 100% charged when connected to the load.

$$\text{hours available} = \frac{80}{5} \tag{64}$$

$$= 16\,\text{h} \tag{65}$$