

REEF Estimator Design Theory

Z Altitude and Velocity Estimator

The 3-state altitude estimator was developed with the following state vector:

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{z} \\ \dot{\hat{z}} \\ \hat{b} \end{bmatrix}$$

\hat{z} is the vehicle altitude estimate, $\dot{\hat{z}}$ is the derivative, and \hat{b} is the estimate of the accelerometer z bias. Estimation is performed in the North East Down (NED) inertial reference frame. The estimator was set up as a standard Kalman filter which propagates the following continuous time motion model:

$$f(\hat{\mathbf{x}}, u) = \begin{bmatrix} \dot{\hat{z}} \\ \tilde{a}_z + \hat{b} - g \\ 0 \end{bmatrix}$$

\tilde{a}_z is the inertial z accelerometer measurement obtained by rotating all 3 axis measurements into the NED frame using the vehicle attitude, and g is the initial gravity magnitude computed when the estimator starts. The measurement equation is straightforward since vehicle altitude is directly measured (in this case via sonic altimeter):

$$h(\hat{\mathbf{x}}) = H\hat{\mathbf{x}} = \hat{z}$$

If an altimeter measurement is obtained, the following posterior estimate is computed using the standard Kalman gain K :

$$\hat{\mathbf{x}}^+ = f(\hat{\mathbf{x}}^-, u, t) + K(\tilde{z} - \hat{z})$$

Otherwise, the motion model is propagated using the current accelerometer reading.

XY Velocity Estimator

The 6-state XY-plane velocity estimator was developed using the following state vector:

$$\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{v}}_{bl} \\ \hat{\mathbf{b}}_q \\ \hat{\mathbf{b}}_a \end{bmatrix}$$

$\hat{\mathbf{v}}_{bl}$ contains estimates of the vehicle's X and Y velocity in the body level frame (unpitched and unrolled), $\hat{\mathbf{b}}_q$ contains pitch and roll attitude bias estimates, and $\hat{\mathbf{b}}_a$ contains x and y accelerometer bias estimates, comprising 6 total states. The propagation motion model is as follows:

$$f(\hat{\mathbf{x}}, u) = \begin{bmatrix} {}^{bf}_{bl}\hat{\mathbf{C}}^T \hat{\mathbf{v}}_{bf} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

For clarification, ${}^{bf}_{bl}\hat{\mathbf{C}}$ is the rotation matrix from body level to body fixed frame. Due to the presence of sines and cosines in this matrix, the propagation equations become nonlinear and an Extended Kalman Filter (EKF) must be used. As such, the motion model is linearized and its Jacobian is evaluated at each

operating point to recalculate the covariance after each propagation. The measurement model assumes that body fixed frame velocities can be directly measured (by RGBD camera in this case):

$$h(\hat{\mathbf{x}}) = \hat{\mathbf{v}}_{bf}$$

If a velocity measurement is obtained, the following posterior estimate is computed using the standard (EKF) Kalman gain K :

$$\hat{\mathbf{x}}^+ = f(\hat{\mathbf{x}}^-, u, t) + K(\tilde{\mathbf{v}}_{bf} - \hat{\mathbf{v}}_{bf})$$

Otherwise, the motion model is propagated using the current accelerometer reading.