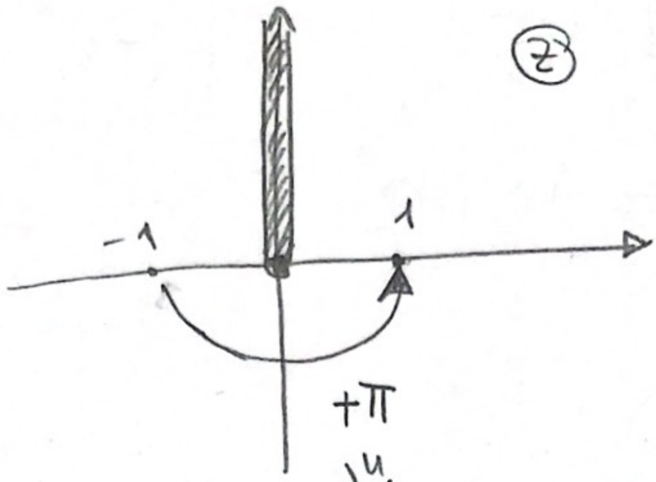
w1

$\varphi(z) = \sqrt[3]{z}$

$z \in [0, i\infty]$

$\varphi(-1) = e^{\frac{i\pi}{3}}$



$\Downarrow$

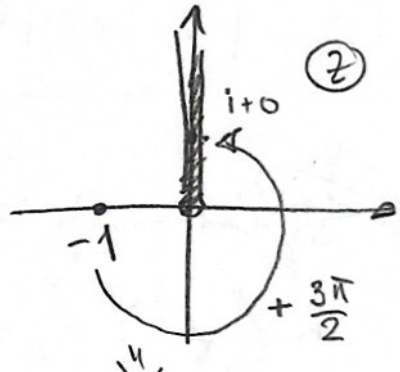
$\Delta \arg \varphi = +\frac{\pi}{3}$.

$$\varphi(1) = \left|\frac{\varphi(1)}{\varphi(-1)}\right| \varphi(-1) e^{i\Delta\arg\varphi}$$

$$\Rightarrow \underline{\varphi(1) = e^{\frac{i\pi}{3}} \cdot e^{+\frac{i\pi}{3}} = e^{\frac{2i\pi}{3}}}$$

$$\left|\frac{\varphi(1)}{\varphi(-1)}\right| = 1$$

$$\varphi(i+0) = \underbrace{\left|\frac{\varphi(i+0)}{\varphi(-1)}\right|}_{=1} \varphi(-1) e^{i\Delta\arg\varphi}.$$



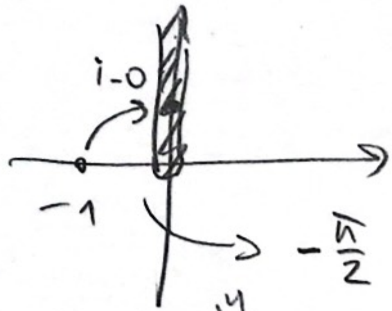
$\Downarrow$

$\Delta \arg \varphi = \frac{\pi}{2}$.

$\Downarrow$

$$\varphi(i+0) = e^{\frac{i\pi}{3}} e^{\frac{i\pi}{2}} = e^{\frac{i\pi 5}{6}}$$

$$\varphi(i-0) = \underbrace{\left|\frac{\varphi(i-0)}{\varphi(-1)}\right|}_{=1} \varphi(-1) e^{i\Delta\arg\varphi}$$



$\Downarrow$

$\boxed{\Delta \arg \varphi = -\frac{\pi}{6}}$

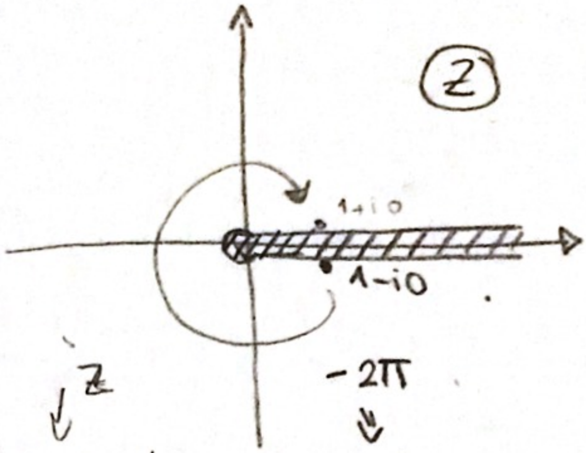
$\Downarrow$

$$\varphi(i-0) = e^{\frac{i\pi}{3}} e^{-\frac{i\pi}{6}} = e^{\frac{i\pi}{6}}$$

$$\varphi(z) = \ln z$$

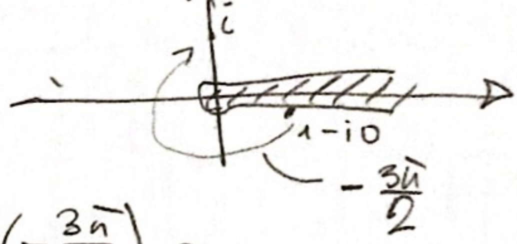
$$z \in [0, +\infty)$$

$$\varphi(1-i0) = 0$$



$$\varphi(1+i0) = \varphi(1-i0) + \ln\left|\frac{1+i0}{1-i0}\right| + i\Delta\arg z =$$

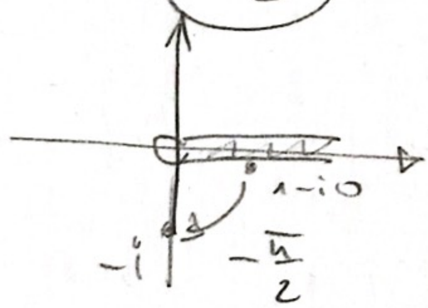
$$= 0 + 0 + (-2i\pi) = -2i\pi$$



$$\varphi(i) = \varphi(1-i0) + \ln\left|\frac{i}{1-i0}\right| + i\left(-\frac{3\pi}{2}\right) =$$

$$= -\frac{3i\pi}{2}$$

$$\varphi(-i) = -\frac{i\pi}{2}$$



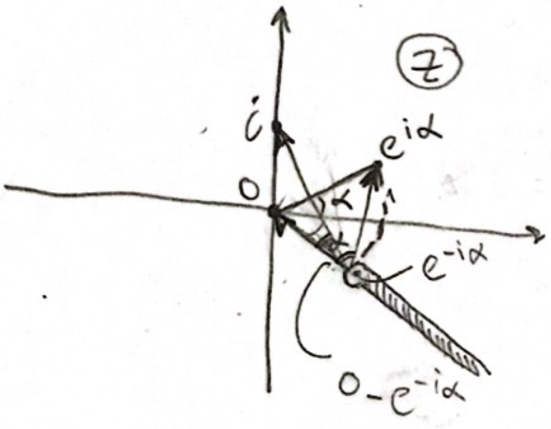
w2

$\alpha \in (0, \frac{\pi}{2})$

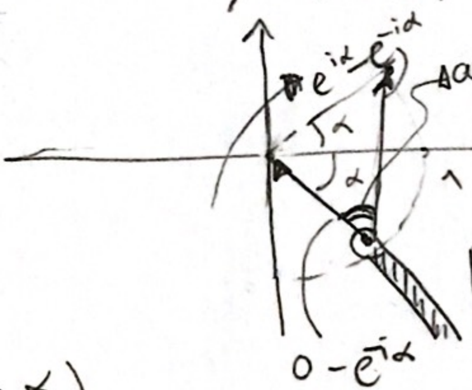
$\varphi_1(z) = \sqrt{z - e^{-i\alpha}}$ $\quad \varphi_1(0) = i e^{-\frac{i\alpha}{2}}$

$\varphi_2(z) = \ln(z - e^{-i\alpha})$ $\quad \varphi_2(0) = -i\pi - i\alpha$

a)



$\varphi_1(e^{i\alpha}) = \left| \frac{\varphi_1(e^{i\alpha})}{\varphi_1(0)} \right| \varphi_1(0) e^{i \Delta \arg \varphi_1} \overset{*}{=}$



$\Delta \arg(z - e^{-i\alpha}) = -\left(\frac{\pi}{2} - \alpha\right)$

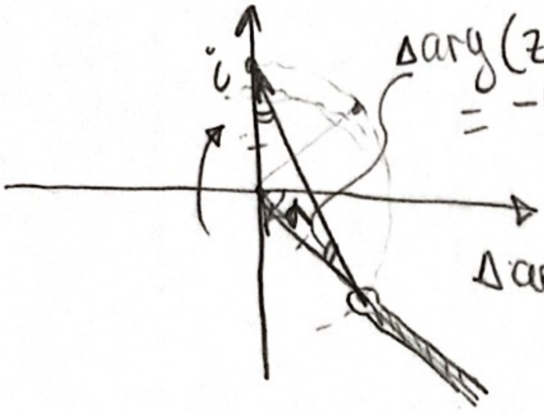
$\Downarrow$

$\boxed{\Delta \arg \varphi_1 = -\frac{\pi}{4} + \frac{\alpha}{2}}$

$\overset{*}{=} \left| \frac{\sqrt{e^{i\alpha} - e^{-i\alpha}}}{i e^{-\frac{i\alpha}{2}}} \right| \cdot i e^{-\frac{i\alpha}{2}} e^{i\left(-\frac{\pi}{4} + \frac{\alpha}{2}\right)} = \left| \frac{\sqrt{2 \sin\alpha}}{i e^{-\frac{i\alpha}{2}}} \right| \cdot i e^{-i\left(\frac{\alpha}{2} + \frac{\pi}{4} - \frac{\alpha}{2}\right)} =$

$= \sqrt{2\sin\alpha}\, e^{\frac{i\pi}{2}} e^{-\frac{i\pi}{4}} = \sqrt{2\sin\alpha}\, e^{\frac{i\pi}{4}}$

$\varphi_1(i)$



$\Delta \arg(z-e^{-i\alpha}) = -\dfrac{(\pi-\frac{\pi}{2}-\alpha)}{2} = -\dfrac{\pi}{4}+\dfrac{\alpha}{2}$

$\Delta \arg \varphi_1 = -\dfrac{\pi}{8}+\dfrac{\alpha}{4}$

$$\varphi_1(i) = \left|\frac{\varphi_1(i)}{\varphi_1(0)}\right| \varphi_1(0)\, e^{i\Delta\arg\varphi_1} = \left|\frac{\sqrt{i-e^{-i\alpha}}}{i e^{-\frac{i\alpha}{2}}}\right| i e^{-\frac{i\alpha}{2}} e^{i\left(-\frac{\pi}{8}+\frac{\alpha}{4}\right)} = *$$

$$i-e^{-i\alpha} = e^{\frac{i\pi}{2}} - e^{-i\alpha} = 2i e^{\frac{i\pi}{4}-\frac{i\alpha}{2}} \underbrace{\left(\frac{e^{\frac{i\pi}{4}+\frac{i\alpha}{2}} - e^{-\frac{i\pi}{4}-\frac{i\alpha}{2}}}{2i}\right)}_{\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)}$$

$$* = \underset{i}{\underbrace{e^{\frac{i\pi}{2}}}} e^{-\frac{i\alpha}{2}-\frac{i\pi}{8}+\frac{i\alpha}{4}} \cdot \sqrt{2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)} = \boxed{e^{\frac{3i\pi}{8}-\frac{i\alpha}{4}}\sqrt{2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)}}$$

$$\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right) = \cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)$$

$$\varphi_2(e^{i\alpha}) = \varphi_2(0) + \ln\left|\frac{e^{i\alpha}-e^{-i\alpha}}{-e^{-i\alpha}}\right| + i\left(-\frac{\pi}{2}+\alpha\right) =$$

$\Delta \arg(z-e^{-i\alpha}) = -\dfrac{\pi}{2}+\alpha$ (ранее было)

$$= \underbrace{-i\pi - i\alpha + \frac{i\pi}{2}}_{-\frac{3i\pi}{2}} + i\alpha + \ln(2\sin(\alpha))$$

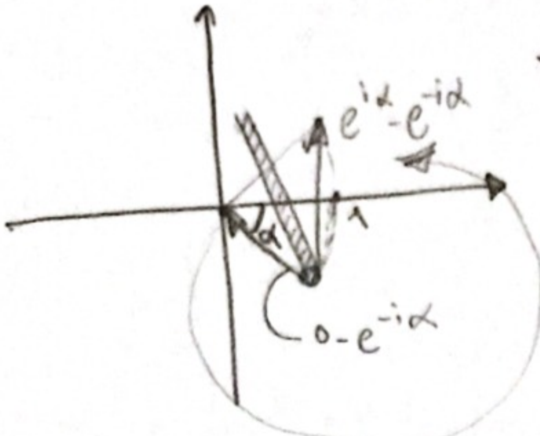
$$\Rightarrow \boxed{-\frac{3i\pi}{2} + \ln(2\sin(\alpha))}$$

$$\varphi_2(i) = \varphi_2(0) + \ln\left|\frac{i-e^{-i\alpha}}{-e^{-i\alpha}}\right| + i\left(-\frac{\pi}{4}+\frac{\alpha}{2}\right) =$$

$$= -i\pi - i\alpha - \frac{i\pi}{4} + \frac{i\alpha}{2} + \ln\left(2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right) =$$

$$= \boxed{-\frac{5i\pi}{4} - \frac{i\alpha}{2} + \ln\left(2\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right)}$$

B)

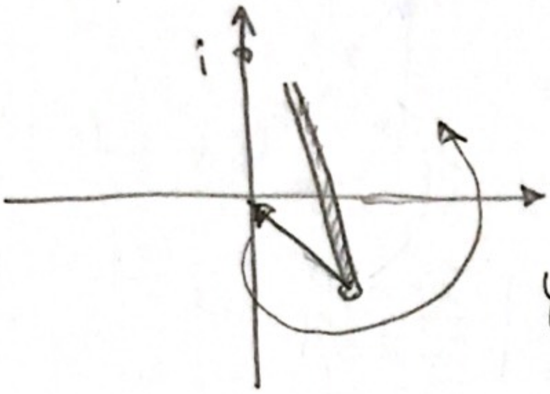


$$\Delta \arg(z-e^{-i\alpha}) = 2\pi - \left(\frac{\pi}{2}-\alpha\right) = \frac{3\pi}{2}+\alpha$$

$$\Delta \arg \varphi_1 = \boxed{\frac{3\pi}{4}+\frac{\alpha}{2}}$$

$$\varphi_1(e^{i\alpha}) = \left|\frac{\sqrt{e^{i\alpha}-e^{-i\alpha}}}{e^{-i\frac{\alpha}{2}}}\right| i e^{-i\frac{\alpha}{2}} e^{i\left(\frac{3\pi}{4}+\frac{\alpha}{2}\right)}$$

$$= \sqrt{2\sin\alpha}\, e^{\frac{i\pi}{2}-\frac{i\alpha}{2}+\frac{3i\pi}{4}+\frac{i\alpha}{2}} = \boxed{\sqrt{2\sin\alpha}\, e^{\frac{5i\pi}{4}}}$$



$$\Delta \arg(z-e^{-i\alpha}) = 2\pi - \left(\frac{\pi}{4}-\frac{\alpha}{2}\right) = \frac{7\pi}{4}+\frac{\alpha}{2}$$

$$\Delta \arg \varphi_1 = \boxed{\frac{7\pi}{8}+\frac{\alpha}{4}}$$

$$\varphi_1(i) = \left|\frac{\varphi_1(i)}{\varphi(0)}\right| \varphi_1(0) e^{i\arg\varphi_1} =$$

$$= \sqrt{2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)}\, e^{\frac{i\pi}{2}} e^{-\frac{i\alpha}{2}} e^{\frac{i7\pi}{8}+\frac{i\alpha}{4}} = \boxed{\sqrt{2\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)}\, e^{\frac{i11\pi}{8}-\frac{i\alpha}{4}}}$$

$$\parallel$$
$$e^{-\frac{i5\pi}{8}-\frac{i\alpha}{4}}$$

$$\varphi_2(e^{i\alpha}) = -i\pi - i\alpha + \ln\left|\frac{e^{i\alpha}-e^{-i\alpha}}{-e^{-i\alpha}}\right| + i\left(\frac{3\pi}{2}+\alpha\right) =$$

$$= \boxed{\frac{i\pi}{2} + \ln(2\sin(\alpha))}$$

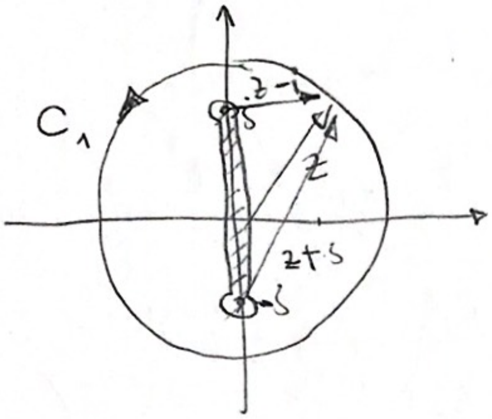
$$\varphi_2(i) = -i\pi - i\alpha + i\left(\frac{7\pi}{4}+\frac{\alpha}{2}\right) + \ln\left(2\sin\left(\frac{\pi}{4}+\frac{\alpha}{2}\right)\right) =$$

$$= \boxed{\frac{i3\pi}{4} - \frac{i\alpha}{2} + \ln\left(2\cos\left(\frac{\pi}{4}-\frac{\alpha}{2}\right)\right)}$$

№3

$$f(z) = (1+z^2)^{\frac{1}{2}} = \sqrt{z+i}\,\sqrt{z-i}$$

а)



$\Delta \arg(z+i) = 2\pi$

$\Delta \arg(z-i) = 2\pi$ $\Rightarrow$ однознач-ность

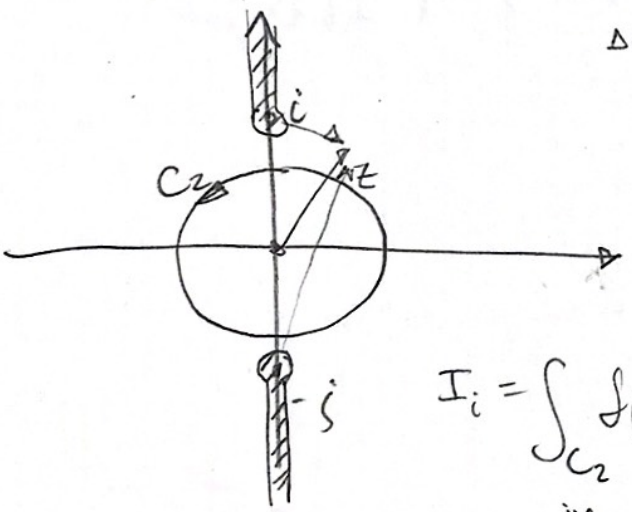
$\Delta \arg f(z) = \frac{1}{2}(2\pi + 2\pi) = 2\pi$

$$I_1 = \int_{C_1} f(z)\,dz = \int_{C_{R\to\infty}} \sqrt{1+z^2}\,dz = 2\pi i \operatorname{Res}_{z\to\infty} f(z) = \pi i$$ Отв

↗ используем контур

$$\sqrt{1+z^2} = z\sqrt{1+\frac{1}{z^2}} \approx z\left(1+\frac{1}{2z^2}\right) = z + \frac{1}{2z}$$

б)



$\Delta \arg(z+i) = 0$

$\Delta \arg(z-i) = 0$ $\Rightarrow$ однознач-ность

$\Delta \arg f(z) = \frac{1}{2}(0+0) = 0.$

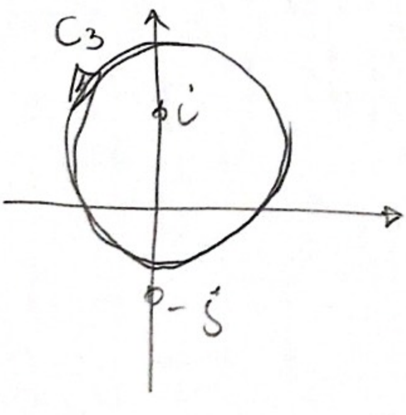
$$I_i = \int_{C_2} f(z)\,dz = \int_{C_{R\to 0}} \sqrt{1+z^2}\,dz = \int_{C_{r\to 0}} \sqrt{1+r^2 e^{2i\varphi}}\, e^{i\varphi} r e^{i\varphi}\,d\varphi$$

$z = re^{i\varphi}$

$dz = re^{i\varphi}d\varphi$

$= 0.$

с)



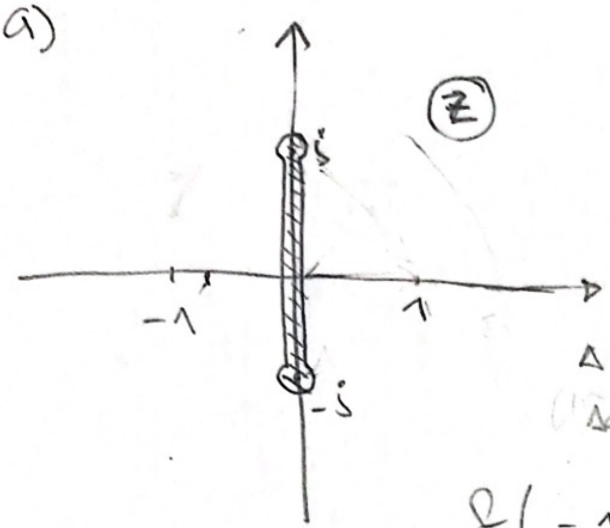
$\Delta \arg(z+i) = 2\pi$

$\Delta \arg(z-i) = 0$ $\Rightarrow$ неоднозначность

$\Delta \arg f(z) = \frac{1}{2}\cdot 2\pi = \pi$

ω3

1)



$f(-1) = -\sqrt{2}$

$f(-z) = \sqrt{1+z^2} = f(z)$

$\Downarrow$

$\boxed{f(1) = \sqrt{2}}$

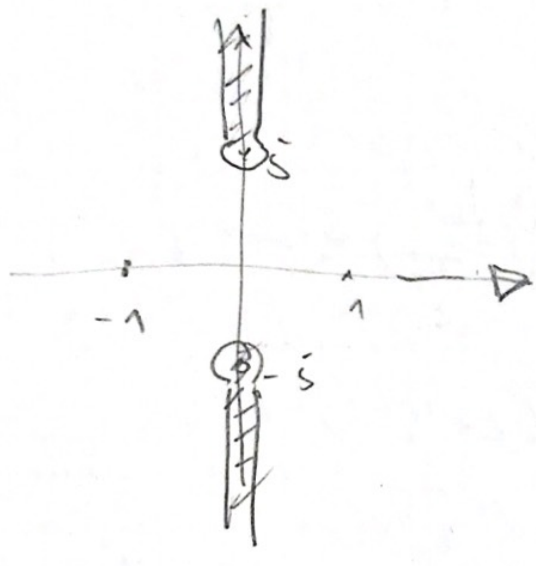
$z = re^{i\alpha}$

$z^2 = r^2 e^{2i\alpha}$

$\Delta \arg(z+i) = \frac{3\pi}{2}$

$\Delta \arg(z-i) = \frac{\pi}{2}$ $\quad \Delta \arg f = \pi$

$f(-1) = \left| \frac{f(-1)}{f(1)} \right| f(1)\, e^{i \arg \varphi} = \sqrt{2}\, e^{i\pi} = -\sqrt{2}$



$\Delta \arg(z-i) = \frac{\pi}{2}$

$\Delta \arg(z+i) = -\frac{\pi}{2}$

$\Delta \arg f = 0$

$f(-1) = \left| \frac{f(-1)}{f(1)} \right| f(1)\, e^{i0} = \sqrt{2}$

w4

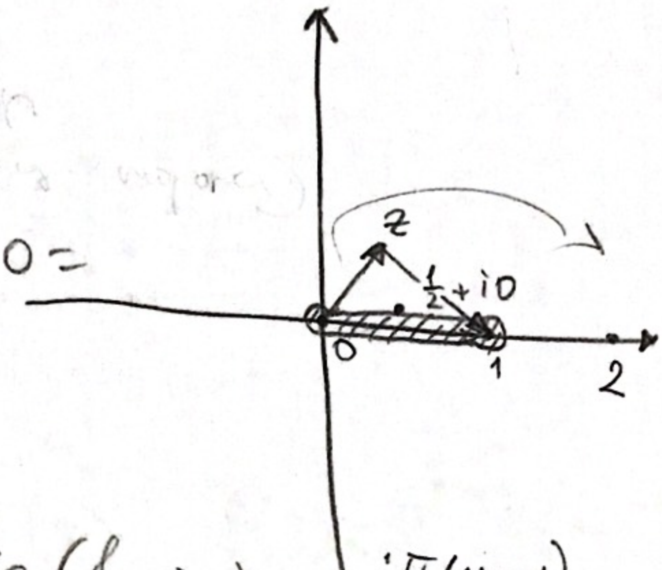
$$\boxed{\varphi(z) = z^{\mu}(1-z)^{1-\mu}}, \quad \varphi\left(\tfrac{1}{2}+i0\right) = \tfrac{1}{2}$$

$$\varphi(2) = ?, \quad \varphi(-1) = ? \qquad \lim_{z\to\infty} \frac{\varphi(z)}{z}$$

$$\Delta \arg z = 0$$

$$\Delta \arg(1-z) = -\pi$$

$$\Delta \arg \varphi = -\pi(1-\mu) + \mu \cdot 0 = \underline{\pi(\mu-1)}$$

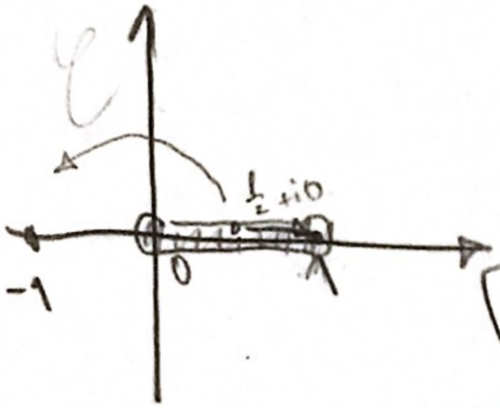


$$\varphi(2) = \left|\frac{\varphi(2)}{\varphi(\frac{1}{2}+i0)}\right| \varphi\left(\tfrac{1}{2}+i0\right) \cdot e^{i\pi(\mu-1)} =$$

$$= \left|\frac{2^{\mu}(1-2)^{1-\mu}}{\frac{1}{2}}\right| \frac{1}{2} e^{i\pi(\mu-1)} = \left|(-1)^{1-\mu} 2^{\mu+1}\right| \cdot \frac{1}{2} e^{i\pi(\mu-1)} = \boxed{2^{\mu} e^{i\pi(\mu-1)}} \qquad e^{-i\pi} = -1 \Rightarrow$$

$$\shortparallel$$

$$\boxed{-2^{\mu} e^{i\pi\mu}}$$



$\Delta \arg z = \pi$

$\Delta \arg(1-z) = 0$

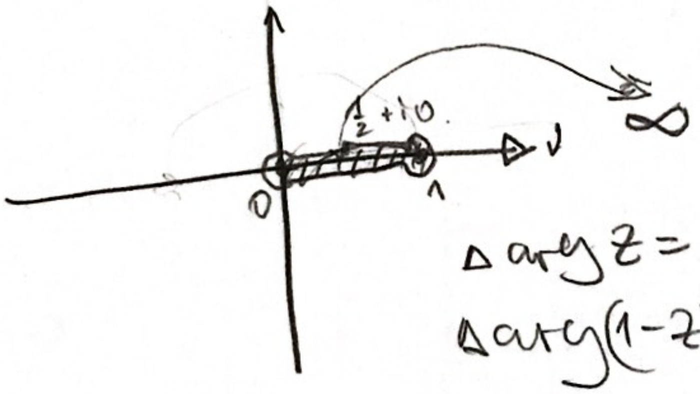
$\boxed{\Delta \arg \varphi = \mu\pi}$

$$\varphi(-1) = \left|\frac{\varphi(-1)}{\varphi(\frac{1}{2}+i0)}\right| \varphi(\tfrac{1}{2}+i0)\cdot e^{i\pi\mu} =$$

$$= \left|\frac{(-1)^{\mu}\, 2^{1-\mu}}{\frac{1}{2}}\right| \frac{1}{2}\, e^{i\pi\mu} = \boxed{2^{1-\mu} e^{i\pi\mu}}$$

$$\lim_{z\to\infty} \frac{\varphi(z)}{z} = \lim_{z\to\infty} f(z) \qquad f(z) = z^{\mu-1}(1-z)^{1-\mu}$$

$$= f(\tfrac{1}{2}+i0) = \frac{\frac{1}{2}}{\frac{1}{2}} = 1.$$



$\Delta \arg z = 0$

$\Delta \arg(1-z) = -\pi$

$\Delta \arg f(z) = -\pi(1-\mu) = \pi(\mu-1)$

$$f(\infty) = \lim_{z\to\infty} \left|\frac{z^{\mu-1}(1-z)^{1-\mu}}{1}\right| 1\cdot e^{i\pi(\mu-1)} =$$

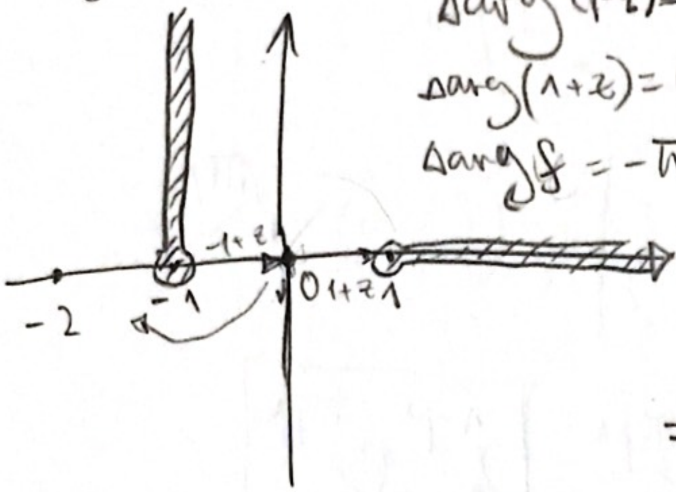
$$= \lim_{z\to\infty} -e^{i\pi\mu} z^{\mu-1}(z-1)^{1-\mu} = \lim_{z\to\infty} -e^{i\pi\mu} z^{\mu-1} z^{1-\mu}\left(1-\frac{1}{z}\right)^{1-\mu} =$$

$$= \boxed{-e^{i\pi\mu}}$$

№5

$$\varphi(z) = \ln(1-z^2) = \ln \underbrace{(1-z)(1+z)}_{f(z)}$$

$$\varphi(0) = -2\pi i$$



$\Delta \arg(1-z) = -\pi$

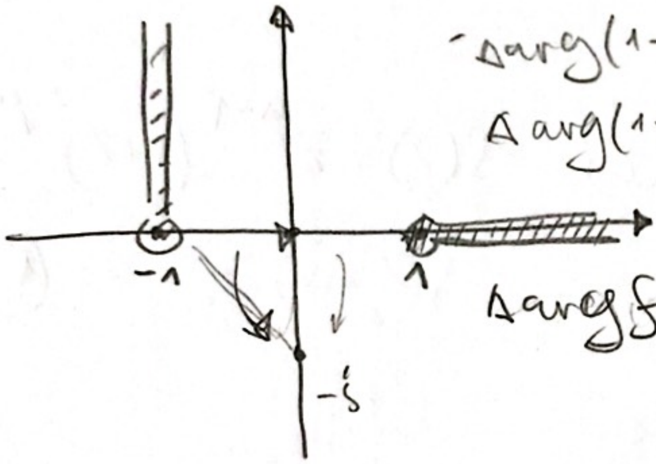
$\Delta \arg(1+z) = 0$

$\Delta \arg f = -\pi$

$$\varphi(-2) = \varphi(0) + \ln\left|\frac{f(-2)}{f(0)}\right| + i(-\pi) =$$

$$= -3\pi i + \ln\left|\frac{-3}{1}\right| =$$

$$= \boxed{-3\pi i + \ln 3}$$



$\Delta \arg(1-z) = -\frac{\pi}{4}$

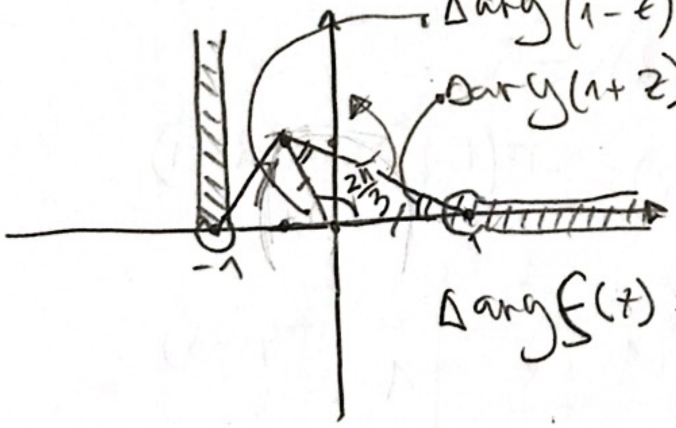
$\Delta \arg(1+z) = \frac{\pi}{4}$

$\Delta \arg f = 0.$

$$\varphi(-i) = \varphi(0) + \ln\left|\frac{f(-i)}{f(0)}\right| + i \cdot 0 =$$

$$= -2\pi i + \ln\left|\frac{2}{1}\right| =$$

$$= \boxed{\ln(2) - 2\pi i}$$



$\Delta \arg(1-z) = \frac{\pi}{3}$

$\Delta \arg(1+z) = -\frac{\pi}{6}$

$\Delta \arg f(z) = \frac{\pi}{6}$

$$\varphi\left(\frac{-1+\sqrt{3}i}{2}\right) = \varphi(0) + \ln\left|\frac{f\left(\frac{-1+\sqrt{3}i}{2}\right)}{f(0)}\right| + i\frac{\pi}{6} =$$

$$= -2\pi i + \ln\left|1 - \frac{(1-2\sqrt{3}i-3)}{4}\right| + i\frac{\pi}{6} =$$

$$\frac{-1+\sqrt{3}i}{2} = \cos\frac{2\pi}{3} + \sin\frac{2\pi}{3} i$$

$$= -\frac{11 i\pi}{6} + \ln\left|1 - \frac{1}{4} + \frac{2\sqrt{3}}{4}i + \frac{3}{4}\right|$$

$$= -\frac{11 i\pi}{6} + \ln\left|\frac{3}{2} + \frac{\sqrt{3}}{2}i\right| = \boxed{-\frac{11 i\pi}{6} + \ln\sqrt{3}}$$

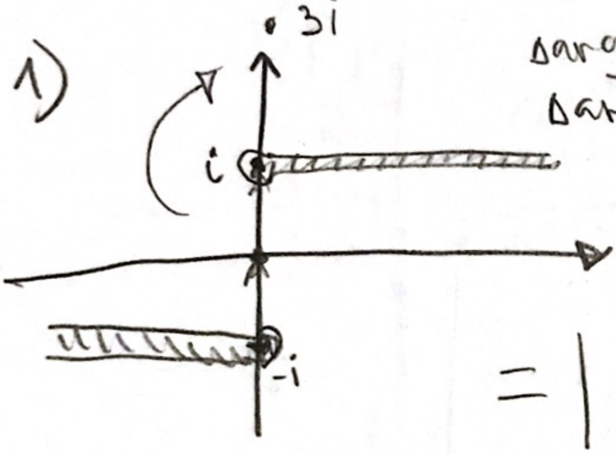
$$\ln\sqrt{3} = \frac{1}{2}\ln 3$$

w6

$$\varphi(z)=\sqrt[3]{1+z^2}=\sqrt[3]{(-i+z)(i+z)}$$

$$\varphi(0)=1$$

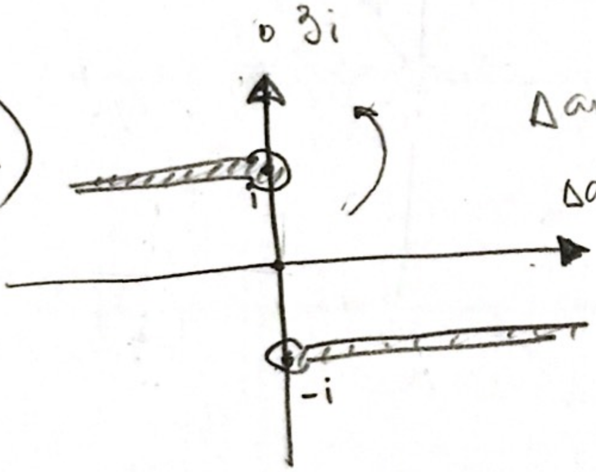
1)



$$\Delta\arg(-i+z)=0$$
$$\Delta\arg(i+z)=-\pi$$
$$\Delta\arg\varphi=-\frac{\pi}{3}$$

$$\varphi(3i)=\left|\frac{\varphi(3i)}{\varphi(0)}\right|\varphi(0)e^{-\frac{i\pi}{3}}=\left|\frac{\sqrt[3]{1-9}}{1}\right|e^{-\frac{i\pi}{3}}=\boxed{2e^{-\frac{i\pi}{3}}}$$

2)



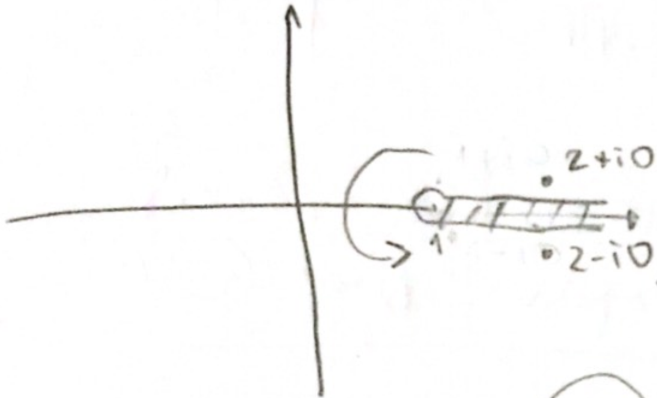
$$\Delta\arg(-i+z)=0$$
$$\Delta\arg(i+z)=\pi$$
$$\Delta\arg\varphi=\frac{\pi}{3}\Rightarrow\varphi(3i)=\boxed{2e^{\frac{i\pi}{3}}}$$

№9

$$f(z) = z^a (z-1)^b$$

1) $f(z) = z(z-1)$

Положим, что точка ветвления — 1:



будем $f(2+i0) = 2$

и повернёмся на $+2\pi$,

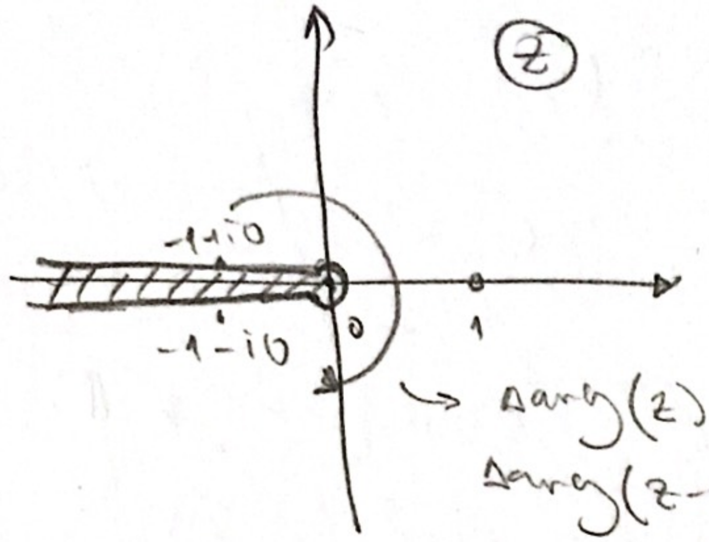
вычислим $f(2-i0)$

сумма т.о. поворот будет 0

$$f(2-i0) = \left|\frac{f(2-i0)}{f(2+i0)}\right| f(2+i0) e^{+i2\pi} =$$

$= 2 \Rightarrow$ это не точка ветвления

Проделаем то же самое с 0 — получим то же самое.



$\to \Delta\arg(z) = -2\pi$

$\Delta\arg(z-1) = 0$

допустим

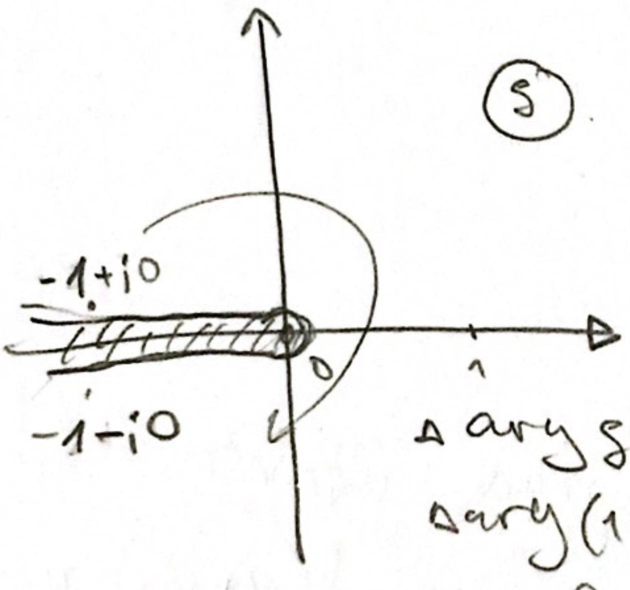
$f(-1+i0) = 2$

$f(-1-i0) = \left|\frac{2}{2}\right| f(-1+i0)\, e^{-1\cdot 2\pi} =$

$= 2$

Осталась точка $z = \infty$.

$\xi = \frac{1}{z}$ $\quad \xi = 0 \quad \Rightarrow$



$f(\xi) = \frac{1}{\xi}\left(\frac{1}{\xi} - 1\right) =$

$= \frac{(1-\xi)}{\xi^2}$

берем $f(\xi = -1+i0) = 2$

$\Delta\arg \xi = -2\pi$

$\Delta\arg(1-\xi) = 0$

$\Delta\arg f = 4\pi$

$f(\xi = -1-i0) =$

$= \left|\frac{2}{2}\right| \cdot 2\, e^{4\pi i} = 2$

$\Downarrow$

0 точек ветвления при $a=1$ $b=1$

ДЗ

2) $f(z) = z(z-1)^{\frac{1}{2}}$

т. ветвл. — (1), т.к. будем иметь при том его и в т. 1:

$$f(2-i0) = \left|\frac{2}{2}\right| \cdot 2e^{i\pi} = -2$$

$\Downarrow$ т. ветвления

в т. — (0) — всё то же самое, что и в т. 1)

$z = \infty$ $\quad f(\zeta) = \frac{1}{\zeta}\sqrt{\left(\frac{1}{\zeta} - 1\right)} = \sqrt{\frac{1}{\zeta^2}\frac{1-\zeta}{\zeta}} = \sqrt{\frac{1-\zeta}{\zeta^3}}$

рассм ζ^3: $\quad f(\zeta = -1+i0) = -\sqrt{2}$

$$f(-1-i0) = \left|\frac{-\sqrt{2}}{-\sqrt{2}}\right| (-\sqrt{2}) \cdot e^{3\pi i} = \sqrt{2} \Rightarrow \text{т. ветвлен.}$$

$\Downarrow$ $\boxed{N\left(1, \frac{1}{2}\right) = 2}$

4) $f(z) = z^{\frac{2}{3}}(z-1)^{\frac{1}{3}}$ — т. ветвления

$\frac{1}{\zeta^{\frac{2}{3}}} \cdot \frac{(\ldots)}{\zeta^{\frac{1}{3}}} = \frac{(\ldots)}{\zeta^{1}} \Rightarrow$

$\frac{2}{3} + \frac{1}{3} = 1$ поворот на 2π

$\Downarrow$ не т. ветвл.

3) $f(z) = z^{\frac{1}{2}}(z-1)^{\frac{1}{3}}$

1: $f(2-i0) = \left|\frac{\sqrt{2}}{\sqrt{2}}\right| \sqrt{2}\, e^{\frac{2i\pi}{3}} \neq \sqrt{2} \Rightarrow$ т. ветвления

0: $f(-1+i0) = \left|\frac{-i\sqrt[3]{2}}{-i\sqrt[3]{2}}\right| i\sqrt[3]{-2}\, e^{-i\pi} \Rightarrow$ т. ветвления

т. ветвлен. $\Rightarrow N\left(\frac{1}{2}, \frac{1}{3}\right) = 3$

∞: $f(\zeta = -1+i0) = \quad \Rightarrow$ т. ветвлен.

поворот на $\frac{5}{3}\pi$ функции

$$\frac{1}{\zeta^{\frac{1}{2}}} \cdot \frac{(\ldots)}{\zeta^{\frac{1}{3}}} = \frac{(\ldots)}{\zeta^{\frac{5}{6}}}$$

№8

а) $f(z) = \ln z \qquad \varsigma = \frac{1}{z} \Rightarrow f(\varsigma) = -\ln \varsigma$

запишем значение в окрестности ς, допустим для $f(\varsigma_0)$:

$$f(\varsigma) = -\ln \varsigma_0 + \ln\left|\frac{\varsigma}{\varsigma_0}\right| + i\Delta\arg \varsigma$$

при повороте на 2π вокруг 0 — $f(\varsigma)$ — меняется $\Rightarrow$

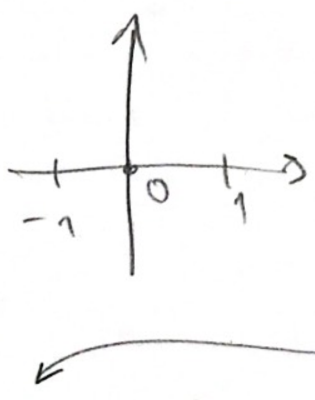
$z = \infty$ — т. ветвления

б) $f(z) = \ln\left(\frac{z-1}{z+1}\right) \qquad f(\varsigma) = \ln\underbrace{\left(\frac{\frac{1}{\varsigma}-1}{\frac{1}{\varsigma}+1}\right)}_{\varphi(\varsigma)} =$

$$= \ln\left(\frac{1-\varsigma}{1+\varsigma}\right)$$

$$f(\varsigma) = -\ln\left(\frac{1-\varsigma_0}{1+\varsigma_0}\right) + \ln\left|\frac{\varphi(\varsigma)}{\varphi(\varsigma_0)}\right| + \underbrace{i\Delta\arg\left(\frac{1-\varsigma}{1+\varsigma}\right)}_{=0}$$

$$\ln\left|\frac{\varphi(\varsigma)}{\varphi(\varsigma_0)}\right| = \ln\left|\frac{(1-\varsigma)(1+\varsigma_0)}{(1+\varsigma)(1-\varsigma_0)}\right|$$



при ς и ς_0 отлич. поворотом на 2π — $= 1 \Rightarrow$ $\ln 1 = 0$.

$\Rightarrow$ это не т. ветвления.

в) $f(z) = \ln(z^2 - 1)$

$$f(\xi) = \ln\left(\frac{1}{\xi^2} - 1\right)$$

$$f(\xi) = \ln\left(\frac{1 - \xi_0^2}{\xi_0^2}\right) + \ln\left|\frac{(1 - \xi^2)\,\xi_0^2}{\xi^2(1 - \xi_0^2)}\right| + i\Delta\arg\left(\frac{1 - \xi_0^2}{\xi_0^2}\right)$$

есть вклад

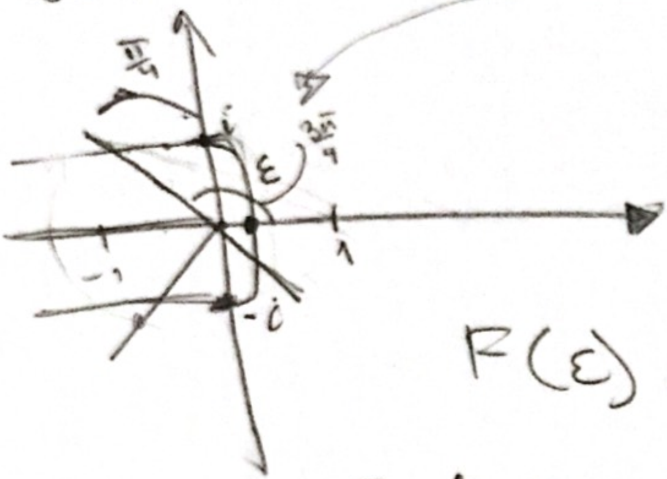
$\Rightarrow$ $\xi_0 = 0$ — т. ветвления

г) $f(z) = \sqrt{z^2 - 1}$

$$f(\xi) = \sqrt{\frac{1}{\xi^2} - 1} = \sqrt{\frac{1 - \xi^2}{\xi^2}}$$

видно, что функция меняется однозначно.

№7



$$f(x) = \ln\Big(\big(\overset{(-i+x)(i+x)}{1+x^2}\big)^{\frac{1}{2}}\Big) = F(x)$$

$$f(\pm 1) = \ln(\sqrt{2}) \Rightarrow$$

$$F(\varepsilon) = F(\pm 1) + \ln\left|\frac{(1+\varepsilon^2)^{\frac{1}{2}}}{\sqrt{2}}\right| + i\cdot 0 =$$

$$= \ln\sqrt{2} - \ln\sqrt{2} = 0 \quad (\varepsilon \to 0)$$

$$F(\varepsilon e^{\frac{3\pi i}{4}}) = F(\pm i) + \ln\left|\frac{(1+\varepsilon^2 e^{\frac{6\pi i}{4}})^{\frac{1}{2}}}{\sqrt{2}}\right| + i\Delta\arg e =$$

$$= \underbrace{\ln\sqrt{2} + \ln\frac{\sqrt{1+\varepsilon^2 e^{\frac{3\pi i}{2}}}}{\sqrt{2}}}_{0\ (\varepsilon \to 0)} + \frac{1}{2}i\left(+\frac{\pi}{4} + \frac{7\pi}{4}\right) = \pi i$$

на корень.

$$F(\varepsilon e^{-\frac{3\pi i}{4}}) = 0 + \frac{1}{2}i\left(-\frac{\pi}{4} - \frac{7\pi}{4}\right) = -\pi i$$