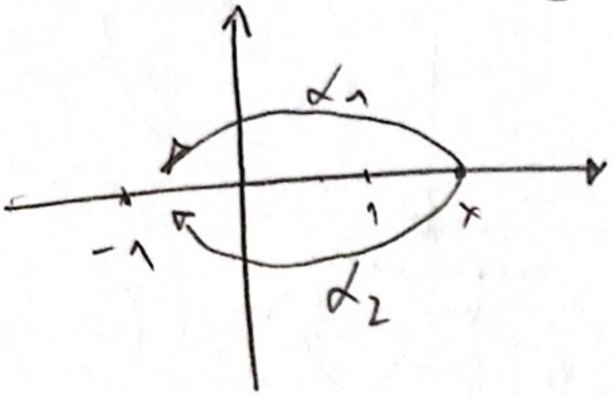
№1

$$I(x)=\int_{\infty}^{x}\frac{e^{i\pi x}\,dx}{x^2-1}, \quad x>1$$

f



1) $I_{\alpha_1}(x)-I_{\alpha_2}(x)=$

$$=\int_{\alpha_1+\alpha_2}\frac{e^{i\pi x}dx}{x^2-1}=2\pi i\,\mathrm{Res}\,f(1)=$$

интеграл по замкнутому контуру

$$=2\pi i\,\frac{e^{i\pi x}}{x+1}\Big|_{x=1}=2\pi i\,\frac{e^{i\pi}}{2}=-\pi i$$

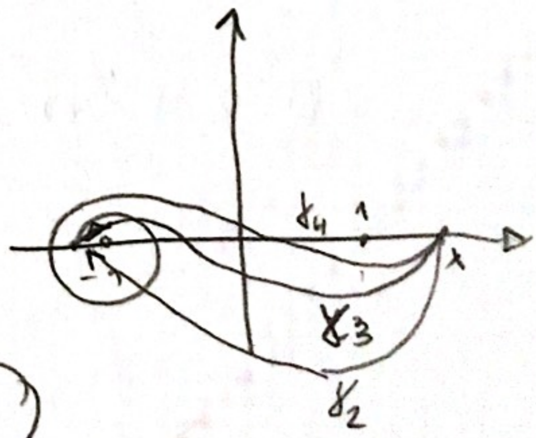
$$I_{\gamma_1}(x)-I_{\gamma_2}(x)=2\pi i\left(\mathrm{Res}\,f(1)+\mathrm{Res}\,f(-1)\right)=$$

$$=2\pi i\left(\frac{e^{i\pi x}}{x+1}\Big|_{x=1}+\frac{e^{i\pi x}}{x-1}\Big|_{x=-1}\right)=$$

$$=2\pi i\left(-\frac{1}{2}+\frac{-1}{-2}\right)=0$$

2) $I_{\gamma_3}(x)-I_{\gamma_2}(x)=2\pi i\,\mathrm{Res}\,f(-1)=$

$$=2\pi i\cdot\frac{e^{i\pi x}}{x-1}\Big|_{x=-1}=\pi i$$



$I_{\gamma_4}(x)-I_{\gamma_3}(x)=2\pi i\,\mathrm{Res}\,f(-1)=\pi i$

w6

$$I=\int_0^{\infty}\frac{\ln x\,dx}{x^2+1}=\int_0^{\infty}\frac{\ln e^t\,e^t dt}{e^{2t}+1}=\int\frac{t\,e^t dt}{e^{2t}+1}=\int_0^{\infty}\frac{t\,dt}{e^t+e^{-t}}$$

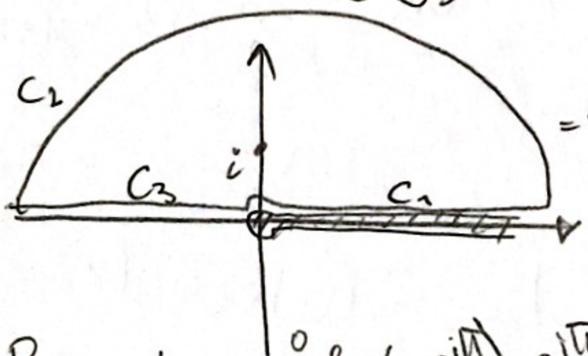
$x=e^t$

$dx=e^t dt$

1) $$\int_0^{+\infty}\overbrace{\frac{\ln x\,dx}{x^2+1}}^{f(x)} = 2\pi \qquad I_{1+2+3}=2\pi i\operatorname{Res} f(i)=2\pi i\left.\frac{\ln x}{(x+i)}\right|_{x=i}=\frac{2\pi i}{2i}\ln i=\pi\ln e^{\frac{i\pi}{2}}=\frac{i\pi^2}{2}$$

$x^2=-1$

$x=\pm i$ в контуре только i



2) $$I_1=\int_0^{\infty}\frac{\ln x\,dx}{x^2+1}\qquad I_3=\int_{-\infty}^{0}\frac{\ln x\,dx}{x^2+1}=\int_{-\infty}^{0}\frac{\ln(re^{i\pi})\,e^{i\pi}dr}{r^2e^{2i\pi}+1}=$$

$x=re^{i\pi}$

$dx=e^{i\pi}dr$

$$=-\int_{-\infty}^{0}\frac{(\ln r+i\pi)dr}{r^2e^{2i\pi}+1}=\int_0^{+\infty}\frac{\ln r\,dr}{r^2e^{2i\pi}+1}+\int_0^{+\infty}\frac{i\pi\,dr}{r^2e^{2i\pi}+1}$$

" или $x=re^{i\pi}$

" I_1

3) $$I_3=I_1+\int_0^{+\infty}\frac{i\pi\,dr}{r^2e^{2i\pi}+1}=I_1+\int_0^{+\infty}\frac{i\pi\,dx}{x^2+1}$$

$$I_{1+2+3}=I_1+I_3=2I_1+i\pi\arctan(x)\Big|_0^{+\infty}\Rightarrow\frac{i\pi^2}{2}=2I_1+\frac{\pi}{2}\cdot i\pi\Rightarrow$$

$$\boxed{I_1=0}$$