SciComp2017 HW2

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1 Problem1

1.1 1

For A_q the condition number can be calculated as $\sqrt{\frac{\lambda_{max}}{\lambda_{min}}}$ where λ is the set of eigenvalues of $A_q^T * A_q$. After calculating the characterisic polinomial and solving for λ we get.

$$k(A_q) = \frac{\max\left(\frac{\sqrt{2}}{2}\sqrt{q^2 - q\sqrt{q^2+4}+2}, \frac{\sqrt{2}}{2}\sqrt{q^2 + q\sqrt{q^2+4}+2}\right)}{\min\left(\frac{\sqrt{2}}{2}\sqrt{q^2 - q\sqrt{q^2+4}+2}, \frac{\sqrt{2}}{2}\sqrt{q^2 + q\sqrt{q^2+4}+2}\right)}$$

1.2 2

If we take

$$b = (10^7, 1), \Delta b = (1, 0), q = 10^7$$

then

$$x = (0, 1), \Delta x = (1, 0)$$

and we get

$$\frac{\|\Delta x\|}{\|x\|} = 1 \geq \frac{10^6}{10^{14}+1} = \frac{\|\Delta b\|}{\|b\|}$$

2 Problem2

The first formula has the first order error

Since,

$$f(x+h) = f(x) + f'(x)h + O(h^2)$$

then

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} = f'(x) + O(h)$$

If function is twice differentiable, with the second formula we get

$$f(x+h) = f(x) + f'(x)h + \frac{f''(x)h}{2} + O(h^3)$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)h}{2} + O(h^3)$$

then

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h} = f'(x) + O(h^2)$$

Experimental results confirm that and ,until precision errors come into play, we can see a steeper decline of the approximation error.

The minimum for the first approximation is achieved at $h10^{-8}$ and equals $2.554135347665465e-08$. For the centered formula the minimum is achieved at 10^{-7} and equals $6.2239102760486276e-12$

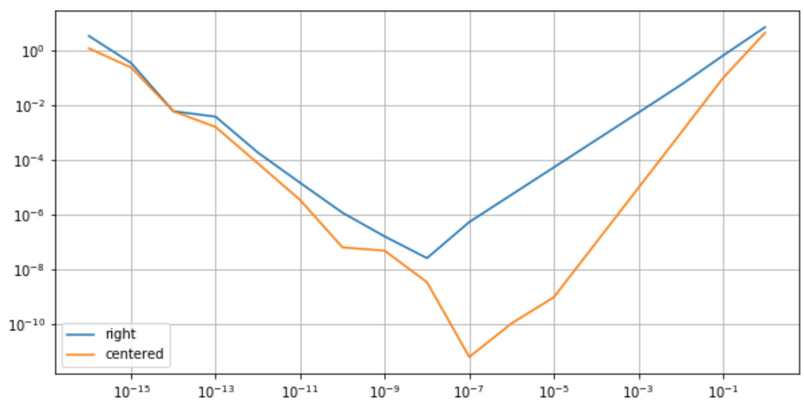

Figure 1: Error graphs

3 Problem3

4 Problem4

By Taylor expansion, the local approximation order is $O(\Delta t^3)$

$$x(t+\Delta t) = x(t) + v\Delta t + \frac{1}{2}f(t)\Delta t^2 + O(\Delta t^3)$$

Substituting first equation into the second one we get

$$x(t + \Delta t) = x(t) + (v(t) + \frac{1}{2} f(t)\Delta t)\Delta t$$

$$x(t + \Delta t) = x(t) + v\Delta t + \frac{1}{2} f(t)\Delta t^2$$

So, subtracting this from the Tailor expansion we get the the desired approximation order. Then, the global approximation order is $\frac{C}{t} * O(\Delta t^3) = O(\Delta t^2)$ Please find the experiments in the corresponding jupyter notebook

5 Problem5

Please see the attached notebook for the implementations and comparison. Algorithm performs grid search in order to find the best regulatization parameter. In terms of error, works similar to $sklearn.linear_model.LinearRegression$ on one dataset, and performs better on the other one.