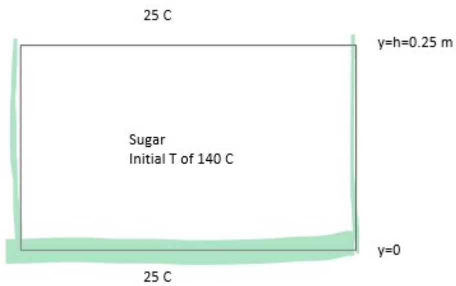
Project

March 10, 2022 8:17 PM

Simplest case



Assumptions:
No generation
Uniform temperature of 140 C at t=0
Room temperature of 25 C, glass underneath pot is also 25
Heat transfer only in y direction
Insulated pot walls
Pot height of 25 cm=0.25 m

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)$$

$$T(y,0)=140$$

$$T(0,t)=25$$

$$T(h,t)=25$$

Let $\theta = T-25$, $\alpha = \frac{k}{\rho c_p}$ = 0.145/(1.57842*1435.9)=6.0E-5

$$\theta(y,0)=115$$

$$\theta(0,t)=0$$

$$\theta(h,t)=0$$

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial y^2}$$

Separate variables

$$\theta(y,t) = e^{-\alpha a^2 t}\left(A'\cos ax + B'\sin ax\right)$$

BCS

$$\rightarrow \theta(0,t) = 0 = e^{-\alpha a^2 t}A' \rightarrow A'=0$$

$$\theta(y,t) = B' e^{-\alpha a^2 t} \sin ay$$

$$\rightarrow \theta(h,t) = 0 = B'\sin ah \rightarrow a = \frac{n\pi}{h}$$

$$\theta(y,t) = \sum_{n=1}^{\infty} B' e^{\left(-\alpha\left(\frac{n\pi}{h}\right)^2 t\right)} \sin\left(\frac{n\pi}{h}y\right)$$

$$\theta(y,0) = 115 = \sum_{n=1}^{\infty} B'_n \sin\left(\frac{n\pi}{h}y\right)$$

$$115y\Big]_0^h = \int_0^h \sum_{n=1}^{\infty} B'_n \sin\left(\frac{n\pi}{h}y\right)\sin\left(\frac{m\pi}{h}y\right)dy$$

$$115h = B'_n \int_0^h \sin^2\left(\frac{m\pi}{L}x\right)dy = B'_n \int_0^L \frac{1}{2}\left(1-\cos\frac{2m\pi}{h}y\right)dy$$

$$115h = B'_n h \qquad B'_n = 115$$

$$\theta(y,t) = T(y,t) - 25 = \sum_{n=1}^{\infty} 115 \exp\left(-6\times10^{-5}\left(\frac{n\pi}{0.25}\right)^2 t\right)\left(\sin\left(\frac{n\pi}{0.25}y\right)\right)$$

Python

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}}{\Delta y^2}$$

$$\frac{\partial T}{\partial t} = \frac{T_i^{l+1} - T_i^{l}}{\Delta t}$$

$$\frac{T_i^{l+1} - T_i^{l}}{\Delta t} = \frac{k}{\rho c_p}\,\frac{T_{i+1}^{l} - 2T_i^{l} + T_{i-1}^{l}}{\Delta y^2}$$