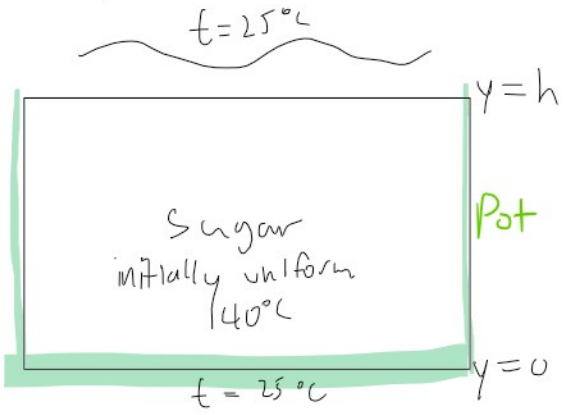
Project

March 10, 2022 8:17 PM

Simplest case



Assumptions:
No generation
Uniform temperature of 140 C at t=0
Room temperature of 25 C, glass underneath pot is also 25
Heat transfer only in y direction
Insulated pot walls
Pot height of 25 cm=0.25 m

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)$$

$T(y,0) = 140$

$T(0,t) = 25$

$T(h,t) = 25$

Let $\theta = T - 25$, $\alpha = \frac{k}{\rho c_p}$ = 0.145/(1578.42*1435.9)=6.0E-8

$\theta(y,0) = 115$

$\theta(0,t) = 0$

$\theta(h,t) = 0$

$$\frac{\partial \theta}{\partial t} = \alpha \frac{\partial^2 \theta}{\partial y^2}$$

Separate variables

$$\theta(y,t) = e^{-\alpha a^2 t}\left(A \cos ax + B \sin ax\right)$$

BCs

$\theta(0,t) = 0 = e^{-\alpha a^2 t} A' \rightarrow A' = 0$

$\theta(y,t) = B' e^{-\alpha a^2 t} \sin ay$

$\theta(h,t) = 0 = B' \sin ah \rightarrow a = \frac{n\pi}{h}$

$$\theta(y,t) = \sum_{n=1}^{\infty} B' e^{\left(-\alpha \left(\frac{n\pi}{h}\right)^2 t\right)} \sin\left(\frac{n\pi}{h} y\right)$$

∞

$$\theta(y,t) = \sum_{n=1} B' e^{(\quad)t} \sin\left(\frac{n\pi}{h} y\right)$$

$$\theta(y,0) = 115 = \sum_{n=1}^{\infty} B'_n \sin\left(\frac{n\pi}{h} y\right)$$

$$115\, y\Big]_0^h = \int_0^h \sum_{n=1}^{\infty} B'_n \sin\left(\frac{n\pi}{h} y\right) \sin\left(\frac{m\pi}{h} y\right) dy$$

$$115\, h = B'_n \int_0^h \sin^2\left(\frac{m\pi}{L} x\right) dy = B'_n \int_0^L \frac{1}{2}\left(1 - \cos\frac{2m\pi}{h} y\right) dy$$

$$115\, h = B'_n h \qquad B'_n = 115$$

$$\theta(y,t) = T(y,t) - 25 = \sum_{n=1}^{\infty} 230 \exp\left(-6\times10^{-8}\left(\frac{n\pi}{0.25}\right)^2 t\right) \sin\left(\frac{n\pi}{0.25} y\right)$$