Two d mass transfer:

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right) + 0$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{1}{r}\left(\frac{\partial T}{\partial r} + r\frac{\partial^2 T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right)$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{1}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right)$$

found in: 3 nodes, 3 nodes

$z = 0$ Bottom $T = 165°C$

$z = L$

$r = 0$, $\left.\frac{\partial T}{\partial r}\right|_{z, r=0} = 0$

$r = R$, $T = 25°C$

$t = 0$, $T|_{r,z} = 25°C$

$$\frac{\partial T}{\partial t} = \frac{T^{l+1}_{(r,z)} - T^{l}_{(r,z)}}{\Delta t}$$

$$\frac{\partial T}{\partial r} = \frac{T^{l+1}_{(r+1,z)} - T^{l+1}_{(r-1,z)}}{2\Delta r}$$

$$\frac{\partial^2 T}{\partial r^2} = \frac{\left[T_{(r+1,z)} - 2T_{(r,z)} + T_{(r-1,z)}\right]}{\Delta r^2}$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{\left[T^{l+1}_{(r,z-1)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r,z+1)}\right]}{\Delta z^2}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\left(\frac{1}{r}\left(\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial r} + \frac{\partial^2 T}{\partial z^2}\right)$$

$$T^{l+1}_{(r,z)} = T^{l}_{(r,z)} + \frac{k}{\rho c_p}\left(\left(\frac{1}{r}\right)\left(\frac{T^{l+1}_{(r+1,z)} - T^{l+1}_{(r-1,z)}}{2\Delta r}\right) + \frac{\left[T^{l+1}_{(r+1,z)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r-1,z)}\right]}{\Delta r^2} + \frac{\left[T^{l+1}_{(r,z-1)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r,z+1)}\right]}{\Delta z^2}\right.$$

$$T^{l+1}_{(r,z)} = T^{l}_{(r,z)} + \frac{k}{\rho c_p}\left[\left[\left(\frac{1}{r(2\Delta r)} + \frac{1}{\Delta r^2}\right)T^{l+1}_{(r+1,z)} - \frac{2}{\Delta r^2}T^{l+1}_{(r,z)} - \left(\frac{1}{r(2\Delta r)} - \frac{1}{\Delta r^2}\right)T^{l+1}_{(r-1,z)}\right] + \frac{\left[T^{l+1}_{(r,z-1)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r,z+1)}\right]}{\Delta z^2}\right]$$

This must = 0 for all r for a valid solution to take place

$$T^{l+1}_{(r,z)} = T^{l}_{(r,z)} + \frac{k}{\rho c_p}\left[\left[\left(\frac{1}{r(2\Delta r)} + \frac{1}{\Delta r^2}\right)T^{l+1}_{(r+1,z)} - \frac{2}{\Delta r^2}T^{l+1}_{(r,z)} - \left(\frac{1}{r(2\Delta r)} - \frac{1}{\Delta r^2}\right)T^{l+1}_{(r-1,z)}\right] + \left[\frac{T^{l+1}_{(r,z-1)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r,z+1)}}{\Delta z^2}\right]\right]$$

$$T_{(r,z)} = T^{l}_{(r,z)} + \frac{\Delta t\, k}{\rho c_p}\left[\frac{T^{l+1}_{(r+1,z)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r-1,z)}}{\Delta r^2} + \left[\frac{T^{l+1}_{(r,z-1)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r,z+1)}}{\Delta z^2}\right]\right]$$

$$Q_{w_{avg}} = \frac{hA(T_s - T_\infty)}{2}$$

$$Q_{w_{avg}} = \frac{hA(140^\circ C - 25^\circ C)}{2}$$

$$\boxed{Q_{w_{avg}} = \frac{115\,hA}{2}}$$

$$T_{(r,z)}^{l+1} = T_{(r,z)}^{l} + \frac{\Delta t k}{\rho c_p}\left[\frac{T_{(r+1,z)}^{l+1} - 2T_{(r,z)}^{l+1} + T_{(r-1,z)}^{l+1}}{\Delta r^2} + \frac{\left[T_{(r,z-1)}^{l+1} - 2T_{(r,z)}^{l+1} + T_{(r,z+1)}^{l+1}\right]}{\Delta z^2}\right]$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=0} = 0$$

$$\therefore T_{(r,z+1)} - T_{(r,z-1)} = 0$$

$$T_{(r,-1)}^{l} = T_{(r,1)}^{l}$$

Same for $r=0$

$$T_{(-1,z)}^{l} = T_{(1,z)}^{l}$$

$$\therefore T_{(r,z)}^{l+1} = T_{(r,z)}^{l} + \frac{\Delta t k}{\rho c_p}\left[\frac{2T_{(r+1,z)}^{l} - 2T_{(r,z)}^{l}}{\Delta r^2} + \frac{\left[2T_{(r,z+1)}^{l} - 2T_{(r,z)}\right]}{\Delta z^2}\right]$$

for $(0,0)$

knowns

$$\therefore \overset{l+1}{T}(r,z) = \overset{l}{T}_{(r,z)} + \frac{\Delta t k}{\rho c_p}\left[\frac{2\overset{l+1}{T}_{(r+1,z)} - 2\overset{l+1}{T}_{(r,z)}}{\Delta r^2}\right] + \left[\frac{2\overset{l+1}{T}_{(r,z+1)} - 2\overset{l+1}{T}_{(r,z)}}{\Delta z^2}\right]$$

knowns

3 knowns

for (0, 0, l) knowns

for (r, z, l) knowns

$$\therefore \overset{l+1}{T}(r,z) = \overset{l}{T}_{(r,z)} + \frac{\Delta t}{\rho c_p}\left[\frac{\overset{l}{T}_{(r+1,z)} - 2\overset{l}{T}_{(r,z)} + T_{(r-1,z)}}{\Delta r^2}\right] + \left[\frac{\overset{l}{T}_{(r,z+1)} - 2T_{(r,z)} + T_{(r,z-1)}}{\Delta z^2}\right]$$

known

q + 2δ°(

for (2, z, l)

$$\therefore \overset{l+1}{T}(r,z) = \overset{l}{T}_{(r,z)} + \frac{k\Delta t}{\rho c_p}\left[\left[\frac{\overset{l+1}{T}_{(r+1,z)} - 2\overset{l+1}{T}_{(r,z)} + \overset{l+1}{T}_{(r-1,z)}}{\Delta r^2}\right] + \left[\frac{\overset{l+1}{T}_{(r,z+1)} - 2\overset{l+1}{T}_{(r,z)} + \overset{l+1}{T}_{(r,z-1)}}{\Delta z^2}\right]\right]$$

known

known

for [; , 2 , l]

known

$$\therefore T^{l+1}_{(r,z)} = T^{l}_{(r,z)} + \frac{k\Delta t}{\rho c_p}\left[\left[\frac{T^{l+1}_{(r+1,z)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r-1,z)}}{\Delta r^2}\right] + \left[\frac{T^{l+1}_{(r,z+1)} - 2T^{l+1}_{(r,z)} + T^{l+1}_{(r,z-1)}}{\Delta z^2}\right]\right]$$

$$(0.0063977)\left(\frac{1}{0.0833^2} + \frac{1}{0.05^2}\right)$$

(0,0) (1,0) (2,0) (0,1) (1,1) (2,1) (0,2) (1,2) (2,2)

25   25   165   165   190

(2,0) (1,0) (0,0)

. . . (9,1)

. . . (9,2)