

Datascience: A New Field or Just a Rebadging Exercise?

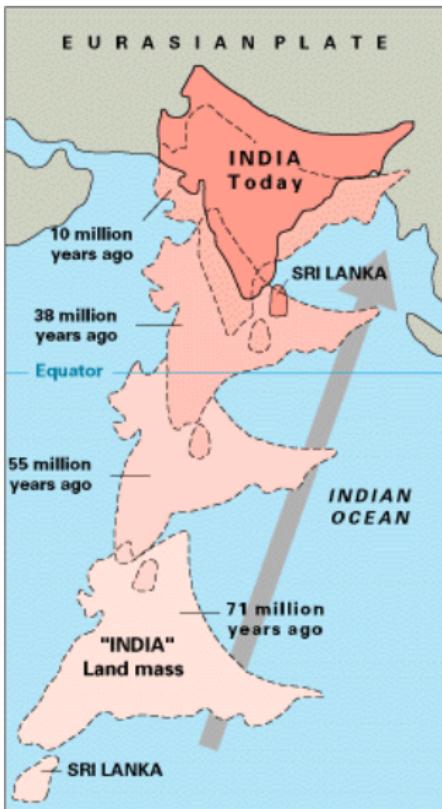
Neil D. Lawrence

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Department of Computer Science, University of Sheffield,
U.K.

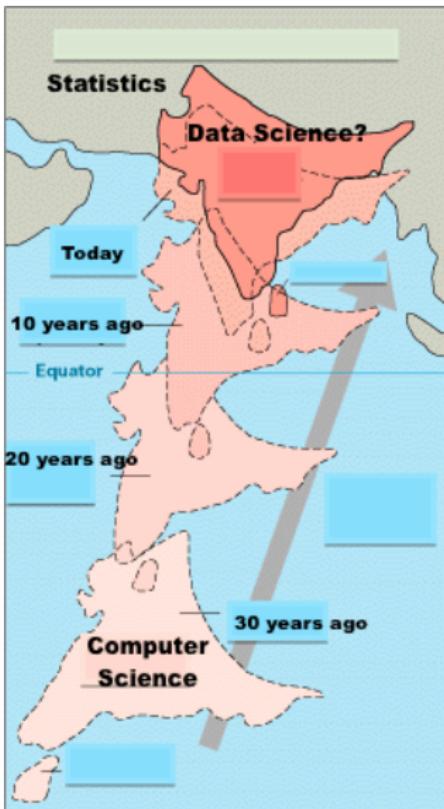
Warwick Statistics

26th November 2014

Shifting Landscapes



Shifting Landscapes



Shifting Landscapes



Shifting Landscapes



Outline

Introduction

Nonparametrics

Process Composition

Conclusions

Box Quote

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*... the scientist must be alert to what is importantly wrong.
It is inappropriate to worry about mice when there are tigers
abroad.* (Box, 1976)

An Incorrect Model

- ▶ Write down our data ...

$$\mathbf{Y} \in \Re^{n \times p}$$

An Incorrect Model

- ▶ Write down our data ...

$$\mathbf{Y} \in \Re^{n \times p}$$

... this is WRONG!

Is this Separation a Historical Anachronism?

- ▶ A presumption: there is something special and separate about indices over n and p .
- ▶ The subtle difference between features and data points.
- ▶ In practice both n and p could be uncountably large!
- ▶ Standard approach seems to assume that p is fixed.
- ▶ A historic anachronism from the days of collating statistical information?

There is nothing special about p ...

- ▶ Rather ... let's assume each data is indexed by the type of data, as well as location, time, etc.
- ▶ So $y_{17,234}$ is price of a hamburger from McDonald's in Leicester square on 13th April 1984 at 13:34 and $y_{239,201}$ is the price of a chicken wrap from Pret a Manger in Cambridge on 27th December 2001 at 14:34.
- ▶ Further $y_{734,124}$ might be the brand of car my mother currently drives.

Prediction

The answer to any prediction problem is a probability distribution. (Peter McCullagh via Peter Diggle)

- ▶ We assume that we are interested in predicting something about our variables (the likely cost of a burger given the cost of a chicken wrap).

Factorizations

- ▶ Often researchers write down the resulting factorization without a second thought:

$$p(\mathbf{Y}|\theta) = \prod_{i=1}^n p(\mathbf{y}_{i,:}|\theta)$$

- ▶ This means that all our information about different data is stored in the parameters.
- ▶ If model is complex, and number of parameters is large, then they will be badly determined when data is few.
- ▶ For me: interesting *research* problems are defined by needing (more) complex models.

Data and Modelling

- ▶ “The Unreasonable Effectiveness of ...
 - ▶ ... Mathematics” (Wigner, 1960)
 - ▶ ...Data” (Halevy et al., 2009)
- ▶ This is a *false* dichotomy.
- ▶ Both are needed for challenging problems of the future.
 - ▶ The relative importance of each is dependent on application.
 - ▶ Norvig also accepts this (see Nando’s question: <http://www.youtube.com/watch?v=yvDCzhbjYWs&t=54m40s>).
- ▶ Prediction requires model (mathematics) and data.
- ▶ Having better models is particularly important when there’s *uncertainty*.

Open Data

- ▶ Automatic data curation: from curated data to curation of publicly available data.
- ▶ Open Data: <http://www.openstreetmap.org/?lat=53.38086&lon=-1.48545&zoom=17&layers=M>.

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- ▶ Social network data, music information (Spotify), exercise.

Not Wrong ... Just Useless

- ▶ Here's a model that's not wrong ...

Not Wrong ... Just Useless

- ▶ Here's a (graphical) model that's not wrong ...



Not Wrong ... Just Useless

- ▶ Here's a model that's not wrong ...



... it's just useless.

Not Wrong ... Just Useless

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... it's just useless.

- ▶ Does that imply all models that are not wrong are useless?
- ▶ What is the minimum we can say about our data to get something useful?

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The TT Channel

- ▶ Objective: predict test data, \mathbf{y}^* , given training data, \mathbf{y} .
- ▶ Parametric models assume

$$p(\mathbf{y}^*|\mathbf{y}) = \int p(\mathbf{y}^*|\boldsymbol{\theta})p(\boldsymbol{\theta}|\mathbf{y})d\boldsymbol{\theta}$$

for some fixed dimensional vector parameters $\boldsymbol{\theta}$.

- ▶ This looks like a communication channel between training and test data (TT Channel).
- ▶ Capacity of channel given by dimensionality of $\boldsymbol{\theta}$.

Massively Missing Data

- ▶ Michael Goldstein's Maid (via Tony O'Hagan).
- ▶ Let me tell you something unusual about myself ...
- ▶ Large amounts of weak information can give a strong picture.
- ▶ But we must deal with uncertainty when this info isn't present.
- ▶ In real life almost all data is missing almost always.

Kolmogorov Consistency

- ▶ **Claim:** To be ‘not wrong’ my model must be ‘Kolmogorov Consistent’.

Kolmogorov Consistency

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- ▶ Kolmogorov consistency says regardless of future observations, my current marginal model of the data is correct. If $\mathbf{y}^* \in \Re^{n^* \times 1}$ then

$$p(\mathbf{y}|n^*) = \int p(\mathbf{y}, \mathbf{y}^*) d\mathbf{y}^*$$

But if the model is Kolmogorov consistent, $p(\mathbf{y}|n^*) = p(\mathbf{y})$.

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But if the model is Kolmogorov consistent, $p(\mathbf{y}|n^*) = p(\mathbf{y})$.

- ▶ Here: \mathbf{y} is past observations, \mathbf{y}^* is all possible *future* observations (in either p or n).
- ▶ Models of this type allow us to deal with *massive* missing data because \mathbf{y}^* can even be infinite dimensional.
- ▶ To these models missing data is equivalent to test data.

Nonparametric TT Channel

- ▶ In a non parametric model:

$$p(\mathbf{y}^*|\mathbf{y})$$

Cannot be written as

$$\int p(\mathbf{y}^*|\boldsymbol{\theta})p(\boldsymbol{\theta}|\mathbf{y})d\boldsymbol{\theta}$$

for fixed dimensional $\boldsymbol{\theta}$.

The TT Channel

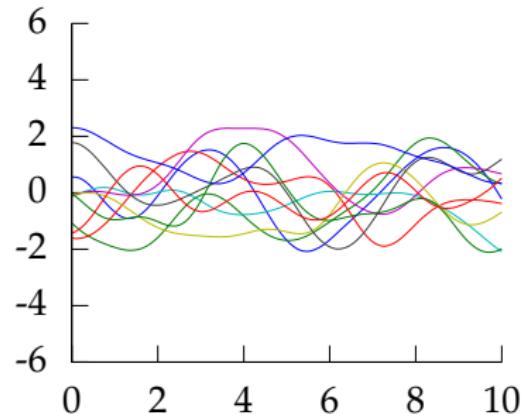
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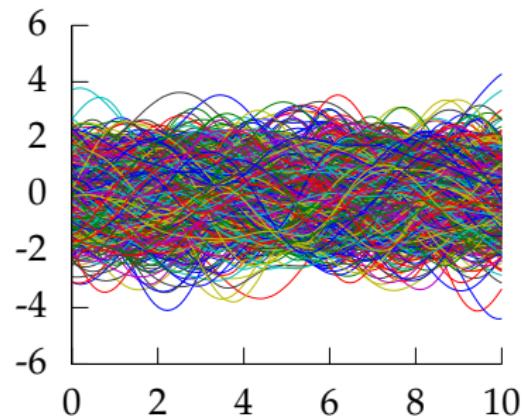
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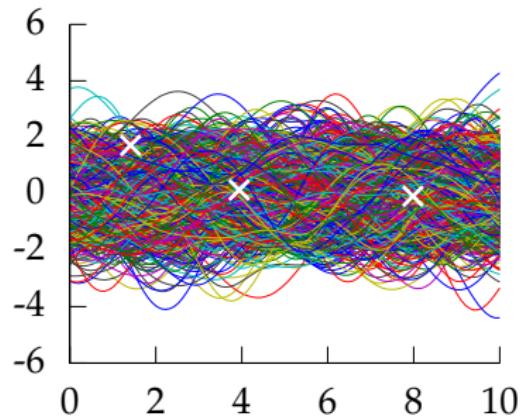
Gaussian Processes: Extremely Short Overview



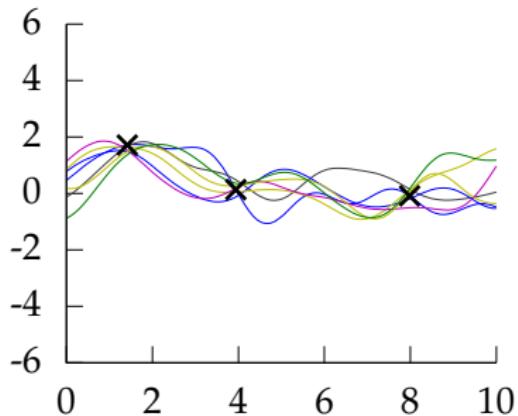
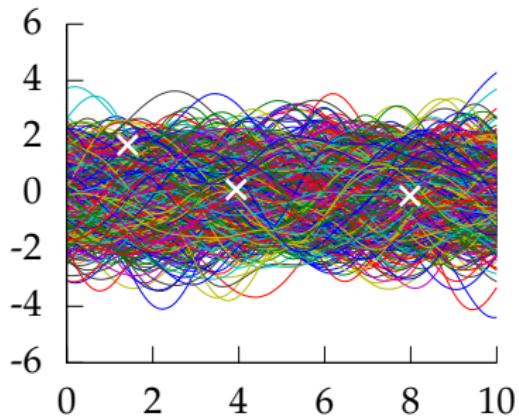
Gaussian Processes: Extremely Short Overview



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Gaussian Processes: Extremely Short Overview



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Mathematically

- ▶ Composite *multivariate* function

$$g(x) = f_5(f_4(f_3(f_2(f_1(x)))))$$

Why Deep?

- ▶ Gaussian processes give priors over functions.
- ▶ Elegant properties:
 - ▶ e.g. *Derivatives* of process are also Gaussian distributed (if they exist).
- ▶ For particular covariance functions they are ‘universal approximators’, i.e. all functions can have support under the prior.
- ▶ Gaussian derivatives might ring alarm bells.
- ▶ E.g. a priori they don’t believe in function ‘jumps’.

Process Composition

- ▶ From a process perspective: *process composition*.
- ▶ A (new?) way of constructing more complex *processes* based on simpler components.

Note: To retain *Kolmogorov consistency* introduce IBP priors over latent variables in each layer (Zhenwen Dai).

Analysis of Deep GPs

- ▶ Duvenaud et al. (2014) Duvenaud et al show that the derivative distribution of the process becomes more *heavy tailed* as number of layers increase.

Inducing Variable Approximations

- ▶ Date back to (Williams and Seeger, 2001; Smola and Bartlett, 2001; Csató and Opper, 2002; Seeger et al., 2003; Snelson and Ghahramani, 2006). See Quiñonero Candela and Rasmussen (2005) for a review.
- ▶ We follow variational perspective of (Titsias, 2009).
- ▶ This is an augmented variable method, followed by a collapsed variational approximation (King and Lawrence, 2006; Hensman et al., 2012).

Augmented Variable Model: Not Wrong but Useful?

Augment standard model with a set
of m new inducing variables, \mathbf{u} .

$$p(\mathbf{y}) = \int p(\mathbf{y}, \mathbf{u}) d\mathbf{u}$$



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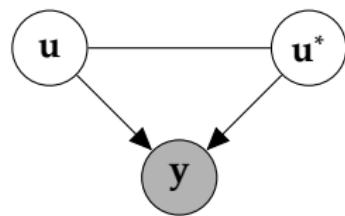
$$p(\mathbf{y}) = \int p(\mathbf{y}|\mathbf{u})p(\mathbf{u})d\mathbf{u}$$



Augmented Variable Model: Not Wrong but Useful?

Important: Ensure inducing variables are *also* Kolmogorov consistent (we have m^* other inducing variables we are not *yet* using.)

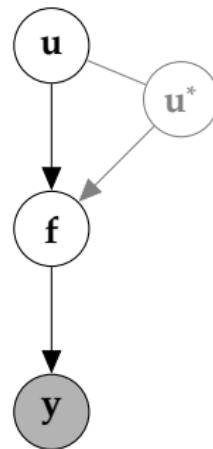
$$p(\mathbf{u}) = \int p(\mathbf{u}, \mathbf{u}^*) d\mathbf{u}^*$$



Augmented Variable Model: Not Wrong but Useful?

Assume that relationship is through \mathbf{f} (represents ‘fundamentals’—push Kolmogorov consistency up to here).

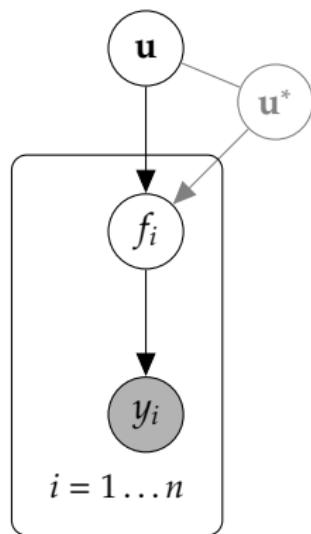
$$p(\mathbf{y}) = \int p(\mathbf{y}|\mathbf{f})p(\mathbf{f}|\mathbf{u})p(\mathbf{u})d\mathbf{f}d\mathbf{u}$$



Augmented Variable Model: Not Wrong but Useful?

Convenient to assume factorization
(*doesn't* invalidate model—think delta
function as worst case).

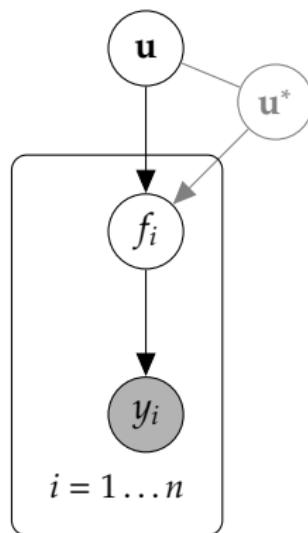
$$p(\mathbf{y}) = \int \prod_{i=1}^n p(y_i|f_i) p(\mathbf{f}|\mathbf{u}) p(\mathbf{u}) d\mathbf{f} d\mathbf{u}$$



Augmented Variable Model: Not Wrong but Useful?

Focus on integral over \mathbf{f} .

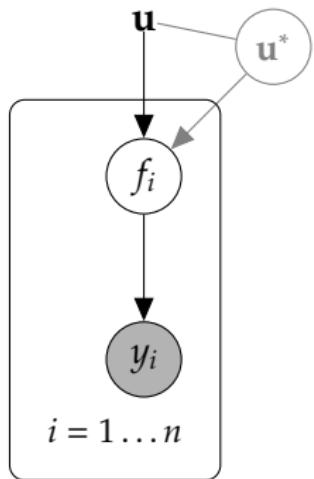
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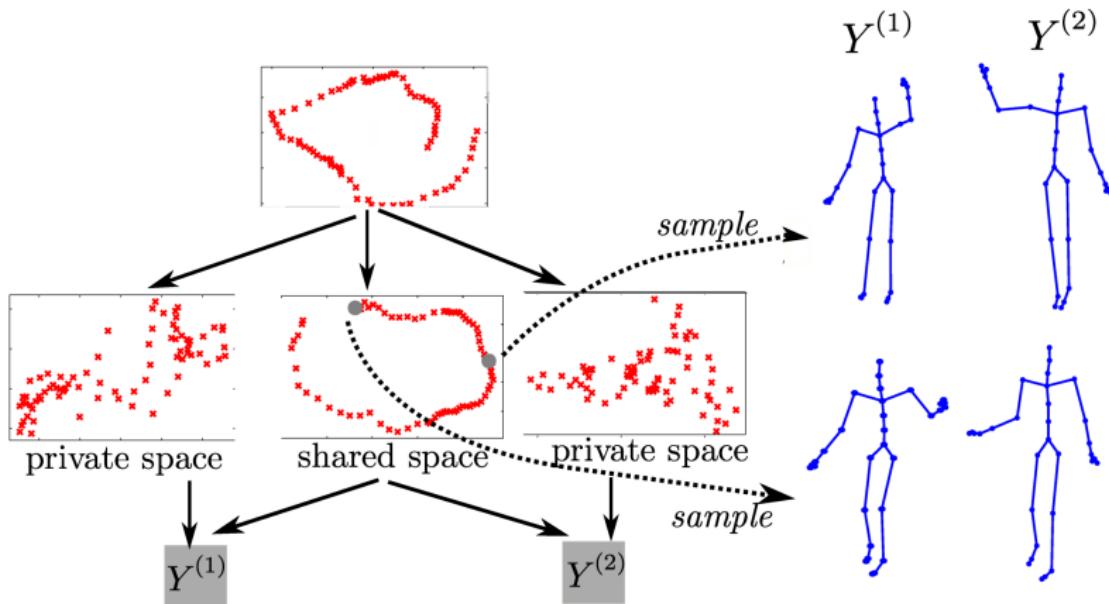
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Motion Capture

- ▶ ‘High five’ data.
- ▶ Model learns structure between two interacting subjects.

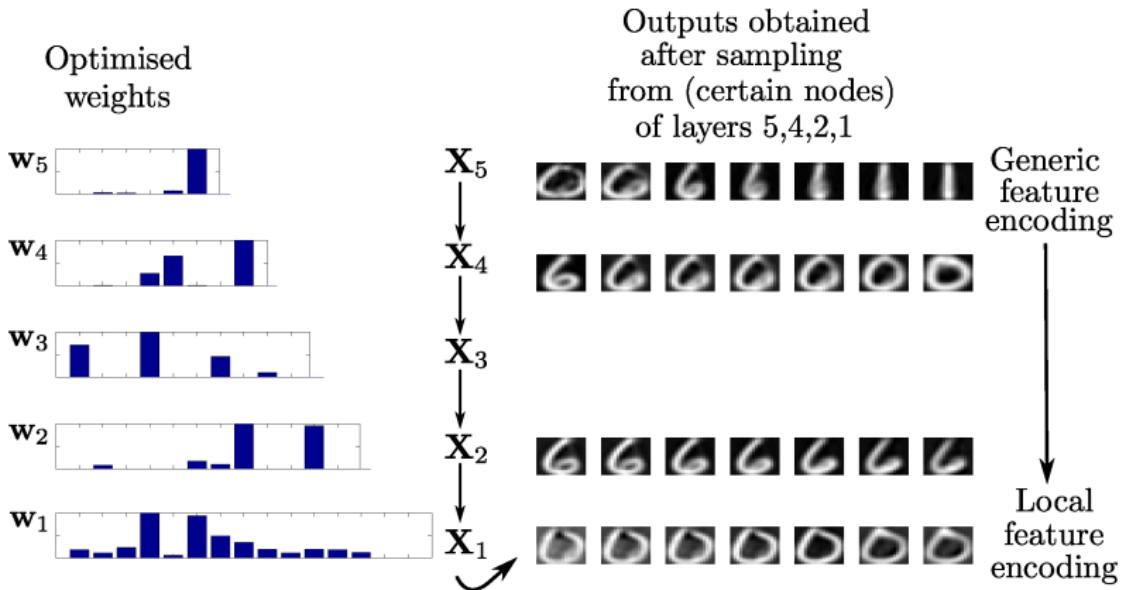
Deep hierarchies – motion capture



Digits Data Set

- ▶ Are deep hierarchies justified for small data sets?
- ▶ We can lower bound the evidence for different depths.
- ▶ For 150 6s, 0s and 1s from MNIST we found at least 5 layers are required.

Deep hierarchies – MNIST



What Can We Do that Internet Giants Can't?

- ▶ Google's resources give them access to volumes of data (or Facebook, or Microsoft, or Amazon).
- ▶ Is there anything for Universities to contribute?
- ▶ Assimilation of multiple views of the patient: each perhaps from a different patient.
- ▶ This may be done by small companies (with support of Universities).
- ▶ A Facebook app for your personalised health.
- ▶ These methodologies are part of that picture.

Challenges for Companies

- ▶ Trying to dominate the modern interconnected data market (e.g. Amazon, Google, Facebook) — buying up talent and competitors.
- ▶ or trying to exploit current 'data silos' (e.g. Tescos clubcard, Experian) — monetising our data today (limited shelf life?)
- ▶ or trying to understand their own systems (the internal google search)
- ▶ or new companies with new ideas that will generate data.

Challenges for Companies

- ▶ How do they break the natural data monopoly?
- ▶ How do they access the necessary expertise?

Challenges in Science

Data sharing is more widely accepted but:

- ▶ Most analysis is simple statistical tests or explorative modelling with PCA or clustering.
- ▶ Few scientists understand these methodologies, apply them as black box.
- ▶ There is an understanding gap between the data & scientist and the data scientist.

Challenges in Health

- ▶ Ensure the privacy of patients is respected.
- ▶ Leverage the wide range of data available for wider societal benefit.

International Development

- ▶ Exploit new telecommunications infrastructure to develop a leap-frog developed countries.
- ▶ Needs mechanisms for data sharing that retain the individual's control.
- ▶ Widespread education of *local* talent in code and model development.

Common Strands

- ▶ Improving access to data whilst balancing against individual's right to privacy against societal needs to advance.
- ▶ Advancing methodologies: development of methodologies needed to characterize large interconnected complex data sets.
- ▶ Analysis empowerment: giving scientists, clinicians, students, commercial and academic partners ability to analyze their own data with latest methodologies.

Open Data Science: A Magic Bullet?

- ▶ Make new methodologies available as widely and rapidly as possible with as few conditions on their use as possible.
- ▶ Educate commercial, scientific and medical partners in use of these methodologies.
- ▶ Act to achieve a balance between data sharing for societal benefit and right of an individual to own their own data.

Achieving This

- ▶ Use BSD-like licenses on software.
- ▶ Educate our partners (summer schools, courses etc).
- ▶ Act to achieve a balance between data sharing for societal benefit and rights of the individual.

Make Analysis Available

The screenshot shows a web browser window displaying a Jupyter notebook from nbviewer.ipynb.org. The URL in the address bar is `nbviewer.ipynb.org/github/SheffieldML/notebook/blob/master/index.ipynb`. The page title is "Sheffield ML Notebooks". Below the title is a list of links:

- Computational Biology and Bioinformatics
- Sheffield Data Science Meet Ups
- GPy Examples
- Lab Classes for Teaching

At the bottom of the page, there is a note: "This web site does not host notebooks, it only renders notebooks available on other websites." To the right, there is information about the service: "Thanks to Rackspace for hosting. nbviewer GitHub repository. nbviewer license." and "nbviewer version: f89be8b IPython version: 3.0.0-dev (ae1c8e0) Rendered 2 minutes ago".

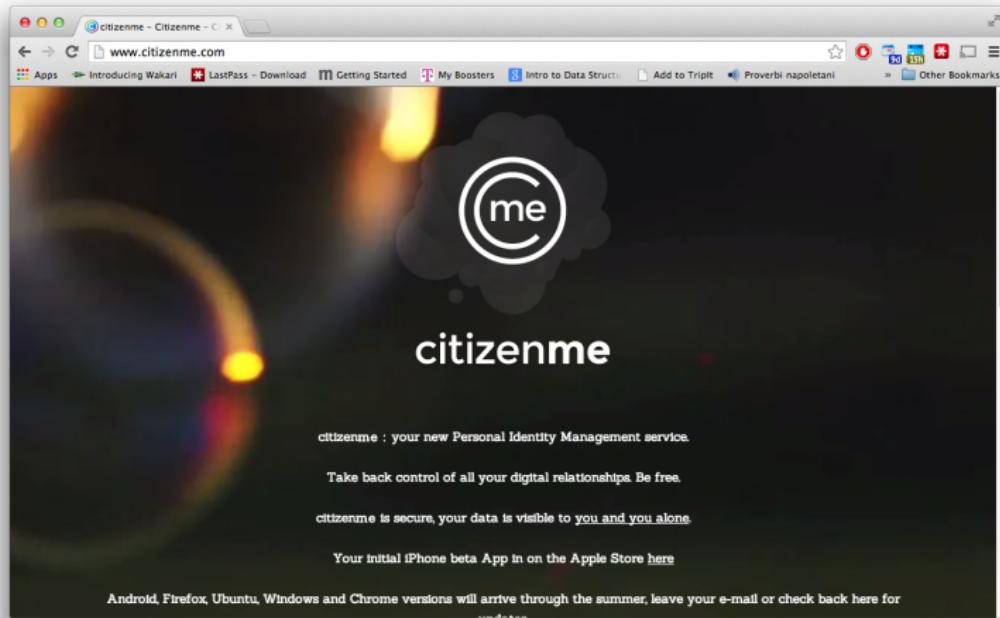
Educating

The screenshot shows a web browser window with the following details:

- Title Bar:** Gaussian Processes Summer School
- Address Bar:** ml.dcs.shef.ac.uk/goss/past_meetings.html
- Toolbar:** Apps, Introducing Wakari, LastPass – Download, Getting Started, My Boosters, Intro to Data Structures, Add to Tript, Proverbi napoletani, Other Bookmarks.
- Content Area:**
 - Logo:** A circular logo for "GAUSSIAN PROCESS SUMMER SCHOOL" featuring a stylized flower design.
 - Title:** GAUSSIAN PROCESS SUMMER SCHOOL
IUDICIJUM POSTERIUM DISCIPULUS EST PRIORIS
 - Navigation Links:** Philosophy, Target, History, News.
 - Breadcrumbs:** You are here: Machine Learning Group / Gaussian Process Summer School
 - Section:** Past Meetings
 - Gaussian Process Road Show, Pereira, Colombia, February 2014
 - Workshop on Spatiotemporal Modelling with Gaussian Processes, Sheffield, January, 2014
 - Gaussian Process Winter School, Sheffield, January, 2014
 - Gaussian Process Road Show, Kampala, Uganda, August 2013
 - Latent Force Model Workshop, Sheffield, June, 2013
 - Gaussian Process Summer School, Sheffield, June, 2013
 - Gaussian Processes in Practice, 2006
 - Gaussian Process Round Table, 2005
 - Other Links:** Home, Accommodation, Getting There, Registration, History, Facebook Page.
 - Page Footer:** This document last modified Tuesday, 18-Mar-2014 06:17:51 GMT

But we need to do much more!

Digital Identity and Data Ownership



Data Warehousing



Blog Post

The screenshot shows a web browser window with the following details:

- Title Bar:** Open Data Science | Inverse Probability
- Address Bar:** inverseprobability.com/2014/07/01/open-data-science/
- Toolbar:** Includes links for Apps, Introducing Wakari, LastPass – Download, Getting Started, My Boosters, Intro to Data Structures, Add to Tripti, Proverbi napoletani, and Other Bookmarks.
- Header:** Shows the WordPress logo, the site title Inverse Probability, and user navigation links for Follow, Like, Reblog, and New Post.
- Content Area:**
 - ## Inverse Probability

Neil Lawrence's thoughts on machine learning, academia and research.
 - HOME ABOUT
 - ### Open Data Science
 - [Leave a reply](#)
 - Not sure if this is really a blog post, it's more of a 'position paper' or a proposal, but it's something that I'd be very happy to have comment on, so publishing it in the form of a blog seems most appropriate.
 - We are in the midst of the information revolution and it is being driven by our increasing ability to monitor, store, interconnect and analyse large interacting sets of data. Industrial mechanisation required a combination of coal and heat engine. Informational mechanisation requires the combination of data and *data engines*. By analogy with a heat engine, which takes high entropy heat energy, and converts it to low entropy, actionable, kinetic energy, a *data engine* is powered by large unstructured data sources and converts them to actionable knowledge. This can be achieved through a combination of mathematical and computational modelling and the combination of required skill sets falls across traditional academic boundaries.

Modern Tools: Github

The screenshot shows a web browser displaying the GitHub interface for the "Sheffield Machine Learning Software (ML@SITrAN)" repository group. The URL is <https://github.com/SheffieldML/>. The page features a header with the GitHub logo, search bar, and navigation links for Explore, Features, Enterprise, and Blog. A "Sign up" and "Sign in" button are also present.

Members: 7 >

Profile Picture	Username
	GPy
	notebook
	vargplvm
	sheffieldml
	mls
	mls
	mls

GPy
Gaussian processes framework in python
Updated 9 hours ago

notebook
Collection of IPython notebooks for demonstrating software.
Updated 11 hours ago

vargplvm
Bayesian GPLVM in MATLAB and R
Updated 8 days ago

Modern Tools: Reddit

AMA: Yann LeCun : Machin... X

www.reddit.com/r/MachineLearning/comments/25lnbt/ama_yann_lecun/

Introducing Wakan | LastPass - Download | Getting Started | My Boosters | Intro to Data Structures | Add to Tripit | Proverbi napoletani | Other Bookmarks

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reddit MACHINELearning comments related other discussions (4)

AMA: Yann LeCun (self.MachineLearning) submitted 1 month ago by ylecuon · stickied post

My name is Yann LeCun. I am the Director of Facebook AI Research and a professor at New York University.

Much of my research has been focused on deep learning, convolutional nets, and related topics. I joined Facebook in December to build and lead a research organization focused on AI. Our goal is to make significant advances in AI. I have answered some questions about Facebook AI Research (FAIR) in several press articles: Daily Beast, KDruggets, Wired.

Until I joined Facebook, I was the founding director of NYU's Center for Data Science.

I will be answering questions Thursday 5/15 between 4:00 and 7:00 PM Eastern Time.

I am creating this thread in advance so people can post questions ahead of time. I will be announcing this AMA on my Facebook and Google+ feeds for verification.

287 comments share

top 200 comments show all 287

sorted by: best ▾

[-] Helloworld 46 points 1 month ago

What is your team at Facebook like?

How is it different than your team at NYU?

In your opinion, why have most renowned professors (e.g. yourself, Geoff Hinton, Andrew Ng) in deep learning attached themselves to a company?

Can you please offer some advice to students who are involved with and/or interested in pursuing deep learning? permalink

[-] ylecuon [S] 68 points 1 month ago

My team at Facebook AI Research is fantastic. It currently has about 20 people split between Menlo Park and New York, and is growing quickly. The research activities focus on learning methods and algorithms (supervised and unsupervised), deep learning + structured prediction, deep learning with sequential/temporal signals, applications in image recognition, face recognition, natural language understanding. An important component is ML software platform and infrastructure. We are using Torch7 for many projects (as does Deep Mind and several groups at Google) and will be contributing to the public version.

My group at NYU used to work a lot on applications in vision/robotics/speech (and other domains) when the purpose was to demonstrate the research community that deep learning actually works. Although we still work on vision, speech and robotics, now that deep learning has taken off, we are doing more work on theoretical stuff (e.g. optimization), new methods (e.g. unsupervised learning) and connections with computational neuroscience and visual psychophysics.

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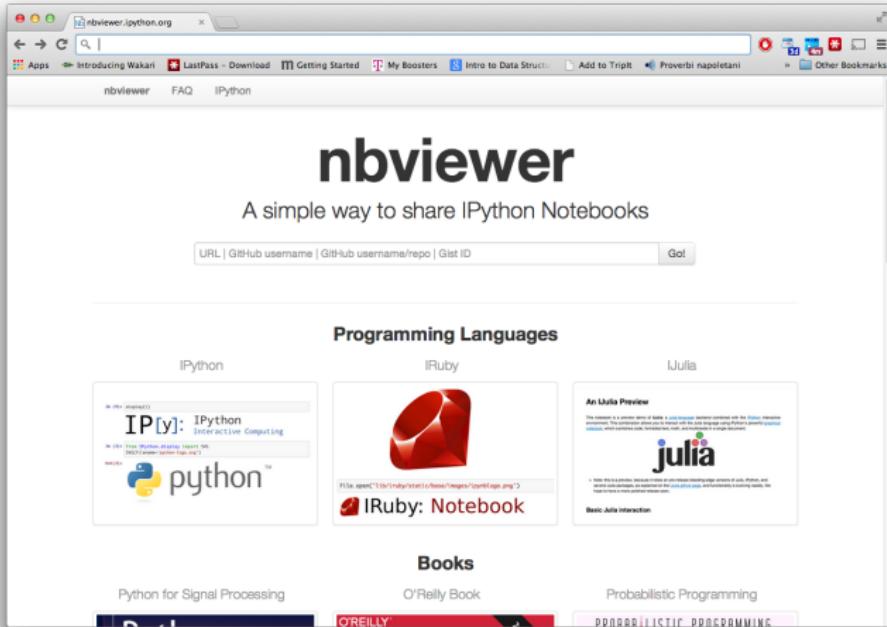
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Please have a look at our [FAQ](#) and [Link-Collection](#)

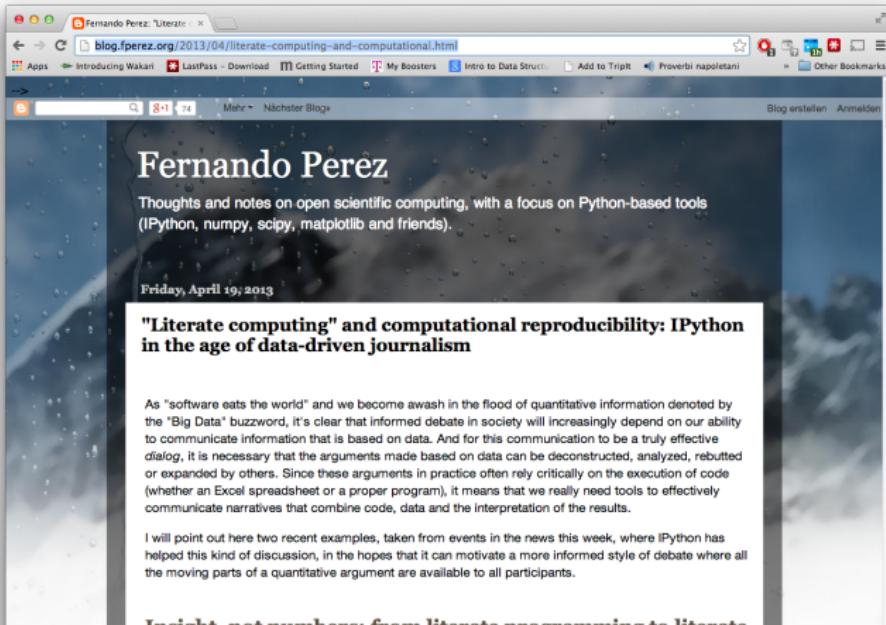
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- Statistics
- Computer Vision

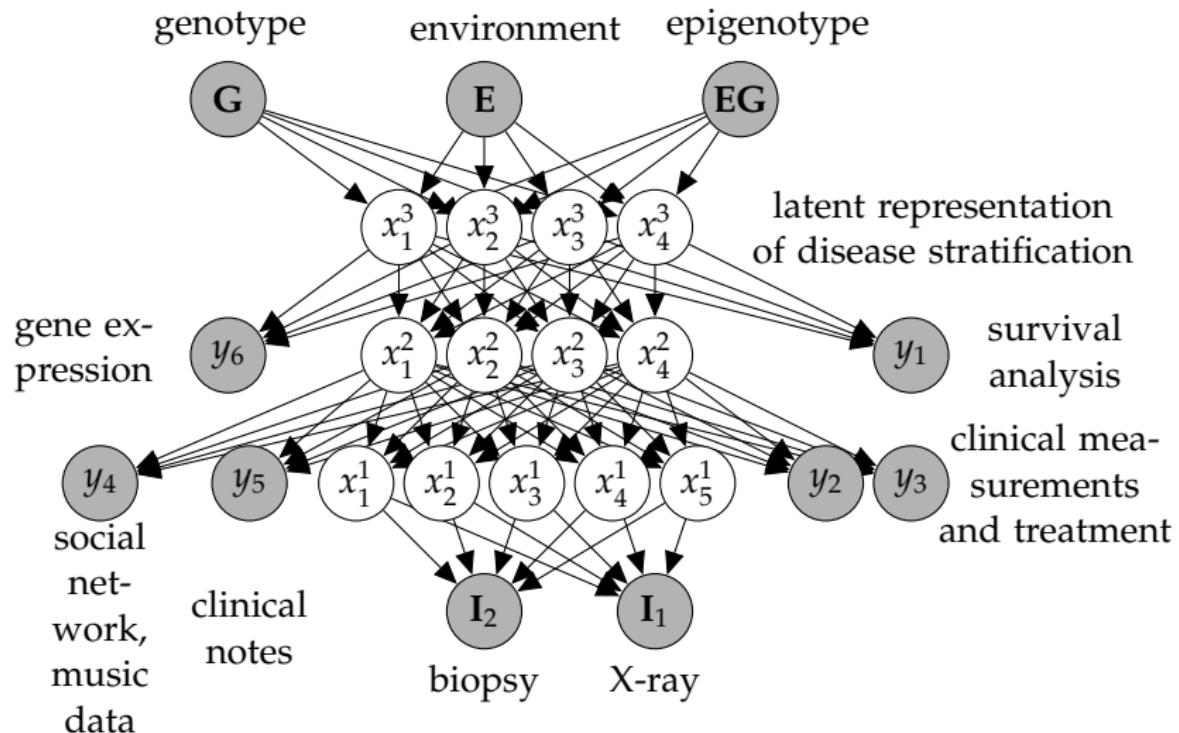
Modern Tools: IPython Notebook



Literate Computing



Deep Health



Summary

- ▶ ‘Big Data’ and simple models only takes us so far.
- ▶ Key question: what do we do when ‘Big Data’ is *small*.
- ▶ Examples include computational biology and personalised health.
- ▶ Our approach is *process composition* (e.g. (Damianou and Lawrence, 2013)).
- ▶ Developing approximate inference algorithms that scale for these models (e.g. (Hensman et al., 2013)).
- ▶ Intention is to deploy these models for assimilating a wide range of data types in personalized health (text, survival times, images, genotype, phenotype).
- ▶ Requires population scale models with millions of features.

References I

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