

Research Review

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Selected paper: Game Tree Searching by Min/Max Approximation

Paper goals or techniques introduced

This paper introduces an iterative method for searching minimax game trees based on the idea of approximating the min and max operators by generalized mean-valued operators. This approximation is used to guide the selection of the next leaf node to expand.

The method of min/max approximation attempts to focus the computer's attention on the important lines of play. The key idea is to approximate the min and max operators with generalized mean-value operators. These are good approximations to the min/max operators, but have continuous derivatives with respect to all arguments. This allows us to define the expandable tip upon whose value the backed-up value at the root most heavily depends in a nontrivial manner. This tip is the next one to be expanded, using our heuristic.

One of the ideas is that by using the generalized mean values to approximate the min and max functions, leaf in a game tree upon whose value the value at the root depends most strongly. This is done by taking derivatives of the generalized mean value functions at each node and using the chain rule. This leaf will be the one to expand next.

The technique is a specific instance of a general method for choosing which leaf to expand in an iterative method. The min/max approximation heuristic is special case of the penalty-based search method, where the penalties are defined in terms of the derivatives of the approximating functions.

Paper results

Experimental results from almost 1,000 games of Connect-Four suggest that this scheme is superior to minimax search with alpha-beta pruning, for the same number of calls to the move routine. Min/max approximation has the potential for improved play since the min/max approximation will favor a move whose min/max value can be achieved in several ways over a move whose min/max value can be achieved in only one way. The min/max approximation pays attention to good backup or secondary moves.

Based on time usage alone, alpha-beta seems to be superior to the implementation of the min/max approximation approach. However, if the comparison is based on move-based resource limits, the story is reversed: min/max approximation is definitely superior. The number of distinct positions considered by alpha-beta was approximately three times larger than the number of distinct positions considered by min/max when a time bound was in effect.

The implementation of minimax search with alpha-beta pruning called the move operator approximately 3500 times per second, while the implementation of the min/max heuristic called the move operator approximately 800 times per second.

As a conclusion, based on approximating the min and max functions by suitable generalized mean-value functions, experimental results indicate that the scheme outplays alpha-beta with iterative deepening.