

# Spectral GNNs vs GATs

## When attention is (not all you need) the "only" option

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# Summary

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# Motivation

- Growing importance of efficient last-mile delivery logistics.
- Use of drones (Unmanned Aerial Vehicles, UAVs) for overcoming traffic constraints, reducing delivery times, and lowering operational costs.
- Significant increase in literature on Delivery Drones in recent years [1].

# Introduction to Last-Mile Delivery Drones

**Figure:** Drones Congestion  
in a high-traffic Last Mile  
Delivery context.

Source: [4]



# Introduction to Last-Mile Delivery Drones

## Last Mile Delivery Drones (LMDD)

- Heterogeneous research area:
  - Combining drones and trucks.
  - Linear integer modeling.
  - Fuzzy logic for uncertainties.
  - Multi-objective optimization.
  - Exclusive drone-based solutions.
- **Complex Systems Decentralized Approach:**
  - Tradable permit model for multi-agent airspace use [5].

# Related Work and Centralized Control

- **Necessity of Air Traffic Management:**
  - Most centralized models don't address collision avoidance [1].
  - Ensuring optimal path planning and efficient airspace control.
- **Centralized Control and UTM:**
  - **Centralized Control:**
    - Federal Aviation Administration (FAA) and NASA's Unmanned Aircraft System Traffic Management (UTM) [2].
    - Ensures organized, legislative-backed airspace control.
  - **Decentralized Models:**
    - Novel but complex in scalability and regulatory compliance.

# Proposed Approach

- **Aispace Control and MAPF Approach:**
  - Multi-Agent Path Finding (MAPF) is a solution for addressing spatial characteristics and collision avoidance.
- **Proposed Strategy:**
  - Employing MAPF strategy for Last Mile Delivery Drone problem.
  - Three approaches: MILP, heuristic and hybrid.
  - Use of prioritized planning [10] and conflict-based search [3] to manage computational complexity in the heuristic.
  - Comparing the MILP with the heuristic.
  - Qualitative comparison between centralized and decentralized approaches.

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# LMDD as MAPF

- Tasks for each agent (drone) described by tuples:

$$\text{task}_i := \{(x_{\text{start}_i}, y_{\text{start}_i}), (x_{\text{goal}_i}, y_{\text{goal}_i})\}, \forall \text{ drone } d_i$$

- Grid bounds:  $x \leq X, y \leq Y$ .
- Goal: find paths  $P_{d_i}$  making the path length as short as possible.

# Path Planning and Constraints

- Drones limited to four principal movements: upward  $(x, y + 1)$ , downward  $(x, y - 1)$ , rightward  $(x + 1, y)$ , and leftward  $(x - 1, y)$ .
- Adjacency:  $(x_1, y_1)$  is adjacent to  $(x_2, y_2) \iff |x_2 - x_1| + |y_2 - y_1| = 1$ .
- New decision variable:  $t_{\text{begin}}$  (arrival time).

# LMDD Goals

- Given time  $T$ , minimize the sum of distances of all drones.
- Allow drones to wait in cells and choose entry time.
- Non-weighted distances make the problem easier than standard MAPF.

# Network Flow Problem

- MAPF is equivalent to multi-commodity minimum cost maximum flow problem [9].
- Visualized as Network Flow problem.
- The number of drones is a max flow in the time-expanded net

# MAPF as Network Flow Problem

- Transformation into a time-expanded network [8].

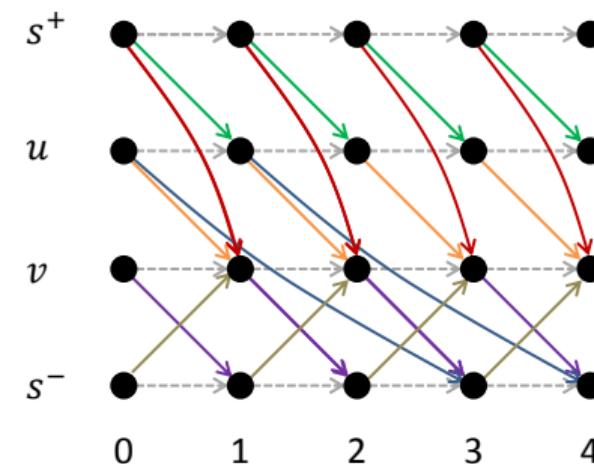
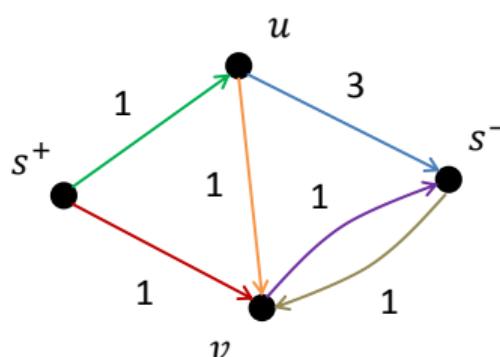


Figure: Time-expanded network representation (sourced from [7])

# Multi-Commodity Flow Formulation

- Each drone is a separate commodity flowing from start to goal node.
- Ensuring each drone reaches its goal within a given time horizon  $T$ .

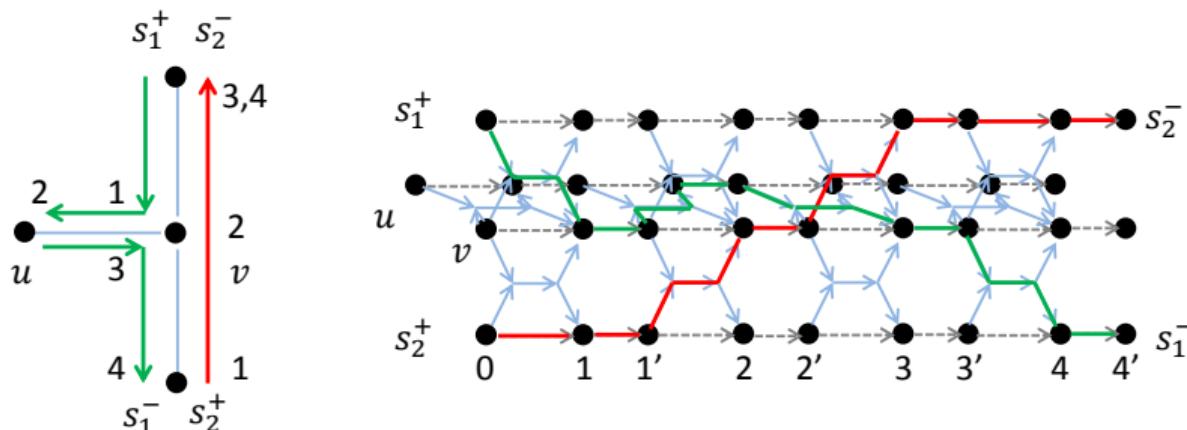


Figure: Equivalence of MAPF to multi-commodity network flow (sourced from [7])

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# MILP Model - Graph Modelling

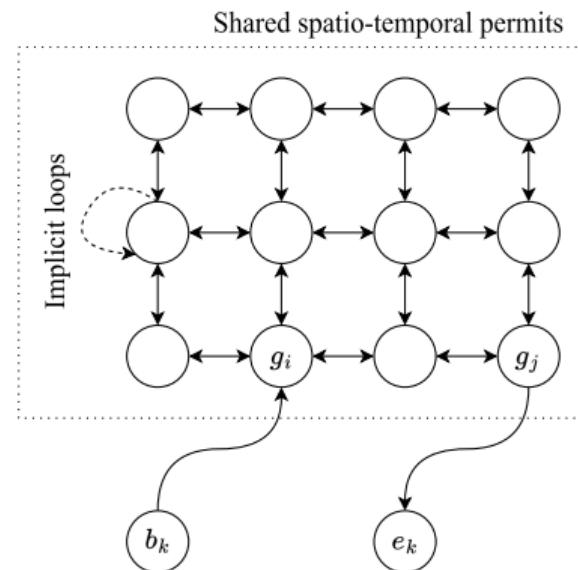
- Proposal of a Mixed-Integer Linear Programming (MILP) model to solve the problem.
- Graph construction representing spatio-temporal permits shared by the drones.
- Modeled as a directed graph (digraph)  $G = (V, A)$ .
- $V$ : set of nodes representing airspace and virtual drone locations.
- $A$ : set of directed arcs representing permitted transitions between nodes.

# MILP Model - Virtual Nodes

- Source node  $b_k$ : Starting point of drone  $k$ 's mission.
- Sink node  $e_k$ : Ending point of drone  $k$ 's mission.
- Focus on spatial topology, omitting temporal component.

Figure: Graph modelling.

Source: The authors.



# MILP Model - Parameters

- $T$ : Maximum time allowed for the mission.
- $b_k$ : Initial virtual vertex representing the initial position of drone  $k$ .
- $e_k$ : Final virtual vertex representing the final position of drone  $k$ .
- $\mathcal{R}$ : Set of drones.
- $\mathcal{G}$ : Digraph  $(\mathcal{V}, \mathcal{A})$  representing the airspace.
- $\mathcal{V}$ : Set of vertices of  $\mathcal{G}$ .
- $\mathcal{B} \subset \mathcal{V}$ : Set of initial virtual vertices  $b_k$ .
- $\mathcal{E} \subset \mathcal{V}$ : Set of final virtual vertices  $e_k$ .
- $\mathcal{S}$ : Set  $\mathcal{V} \setminus (\mathcal{B} \cup \mathcal{E})$ .
- $\mathcal{A}$ : Set of arcs  $(i, j) \in \mathcal{A}$  of  $\mathcal{G}$ .

# MILP Model - Variables

- Decision Variables:
  - $x_{i,j,t}^k = 1 \iff$  drone  $k$  jumps from  $i$  to  $j$  at time  $t$ .
- Indices:
  - $k$ : Drone  $\implies k \in \mathcal{R}$ .
  - $t$ : Time  $\implies 1 \leq t \leq T$ .
  - $i, j, l$ : Vertices  $\implies i, j, l \in \mathcal{V}$ .

# MILP Model - Objective Function

- Minimize the total sum of the number of drone movements:

$$\min \sum_{k \in \mathcal{R}} \sum_{t=1}^T \sum_{(i,j) \in \mathcal{A}: j \notin (\mathcal{E} \cup \mathcal{B})} x_{i,j,t}^k$$

- Minimize the total number of drone movements, counting  $n - 1$  jumps for each drone that performs  $n$  jumps.

# MILP Model - Constraints

- Ensure each drone starts its mission:

$$\sum_{t=1}^T \sum_{j \in \mathcal{S}} x_{b_k, j, t}^k = 1, \quad \forall k \in \mathcal{R}$$

- Flow conservation:

$$\sum_{j \in \mathcal{V}} x_{i, j, t-1}^k = \sum_{l \in \mathcal{V}} x_{j, l, t}^k, \quad \forall j \in \mathcal{V}, \forall k \in \mathcal{R}, \forall t \in \{2, \dots, T\}$$

# MILP Model - Constraints (Cont.)

- Border condition at time  $t = 0$ :

$$x_{i,j,0}^k = \begin{cases} 1, & \text{if } i = b_k \wedge j = b_k, \\ 0, & \text{otherwise.} \end{cases} \quad \forall k \in \mathcal{R}, \forall (i,j) \in A$$

- Mutual exclusion of vertex occupation:

$$\sum_{k \in \mathcal{R}} \sum_{j \in \mathcal{V}} x_{i,j,t}^k \leq 1, \quad \forall j \in \mathcal{V}, \forall t \in \{1, \dots, T\}$$

# MILP Model - Mission Accomplishment

- Ensure each drone completes its mission:

$$\sum_{t=1}^T \sum_{i \in \mathcal{S}} x_{i,e_k,t}^k \geq 1, \quad \forall k \in \mathcal{R}$$

# Heuristic Approach

- Utilize distance measure as heuristic metric [6].
- Organize drones in ascending order (prioritized planning) based on start and end points.
- Employ iterative Breadth-First Search (BFS) on temporal graph.
- Dynamic constraints update of occupied positions (conflict-based search).
- Combine heuristic sorting and iterative BFS for efficient path planning and adaptability.

# Algorithm Notation

Table: Notation used in the Algorithm.

Notation	Definition
$\mathcal{V}$	Set of vertices in the graph : $(i, j, t)$
$\mathcal{E}$	Set of edges
$\mathcal{D}$	Set of drones
$\mathcal{S}$	Set of already scheduled vertices
$\mathcal{P}_d \subseteq \mathcal{V}$	Path of drone $d$
$\mathcal{G} = (\mathcal{V}, \mathcal{E})$	Temporal Graph

# Heuristic Algorithm Steps

- ① **Drones Sorting:** Ascending sort using Euclidean Distance.

$$\mathcal{D}_{\text{sorted}} = \text{sort}(\mathcal{D}, \text{heuristic}) \quad (1)$$

- ② **Path for Each Drone:** Compute path  $P_d$  using BFS on graph  $\mathcal{G}$ .

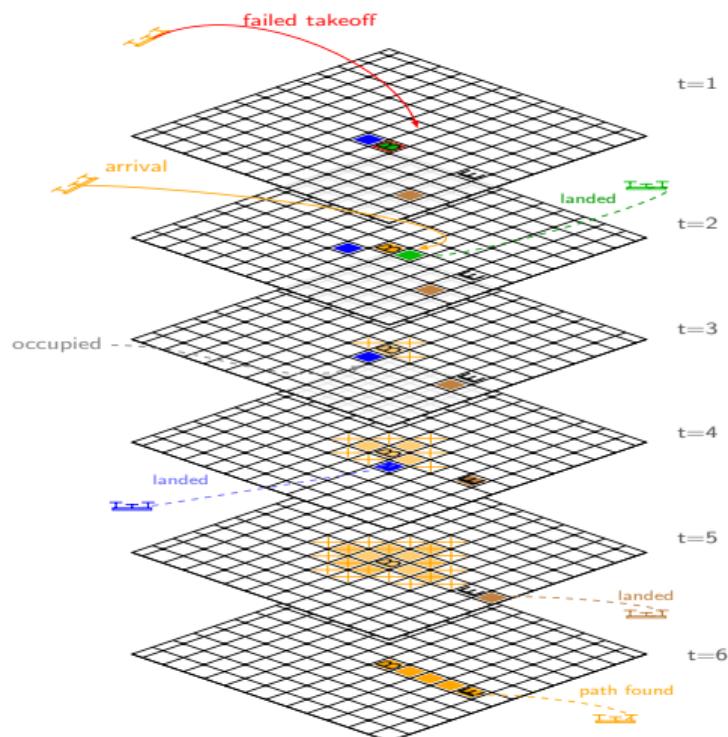
$$\forall d \in \mathcal{D}_{\text{sorted}} : \quad \mathcal{P}_d = \text{BFS}(\mathcal{G}, d) \quad (2)$$

- ③ **Constraints Update:** Update set of already scheduled vertices  $\mathcal{S}$ .

$$\mathcal{S} = \mathcal{S} \cup \bigcup_{d \in \mathcal{D}_{\text{sorted}}} \mathcal{P}_d \quad (3)$$

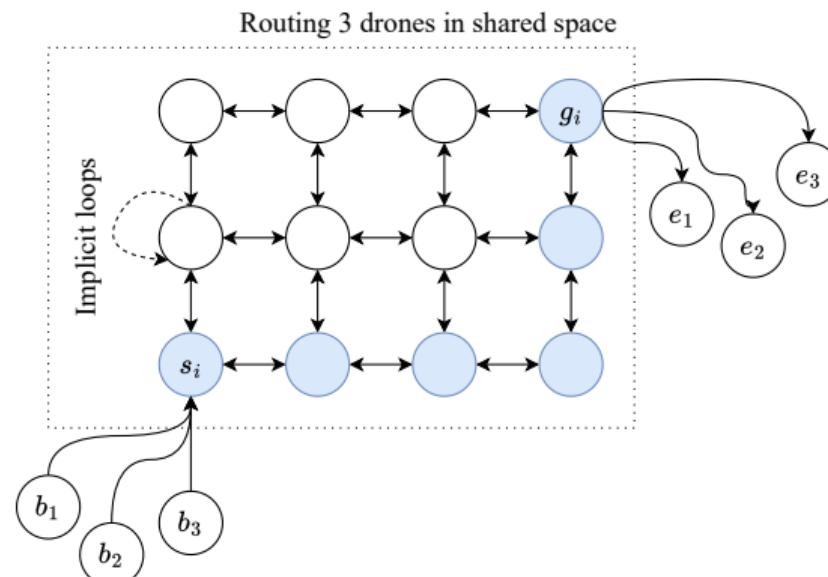
# Algorithm Visualization

Figure: Algorithm visualization.  
Source: The authors.



# Complexity Analysis and Boundedness

- Worst-case complexity:  $\mathcal{O}((N + M)KNM \log((N + M)KNM))$ .
- Approximation:  $\mathcal{O}(N^3K \log(N^3K))$  for square grids.



# Hybrid Methodology

- **Heuristic Solution Generation**
  - Quickly generates an initial feasible solution.
  - Determines a plausible time horizon  $T_{\text{heuristic}}$ .
- **MILP Model Refinement**
  - Uses  $T_{\text{heuristic}}$  and initial feasible solution from heuristic.
  - Refines the solution to ensure global optimality.
- **Advantages**
  - Combines computational speed with solution accuracy.
  - Skips multiple iterations to determine  $T$ , reducing computational expense.

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# Results

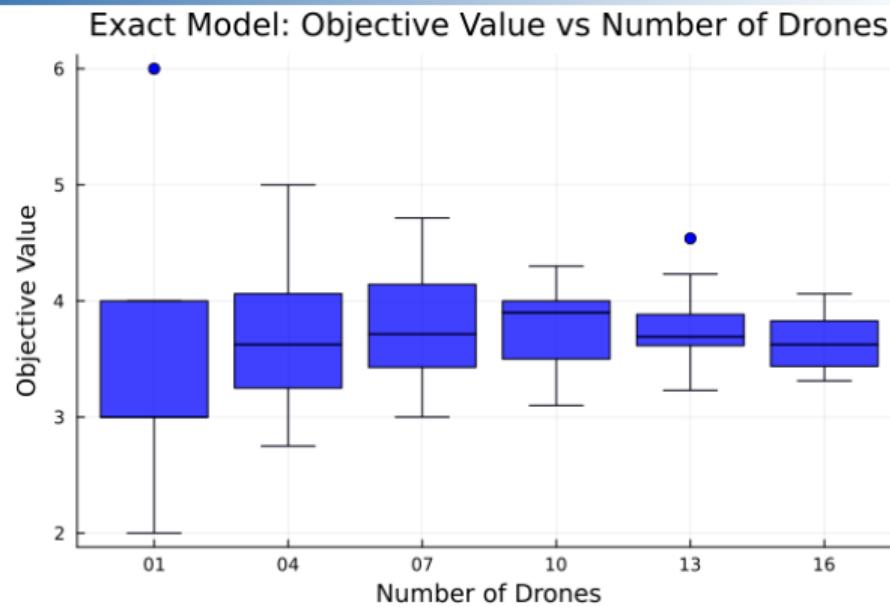


Figure: Exact Model Objective. Source: The authors.

# Results

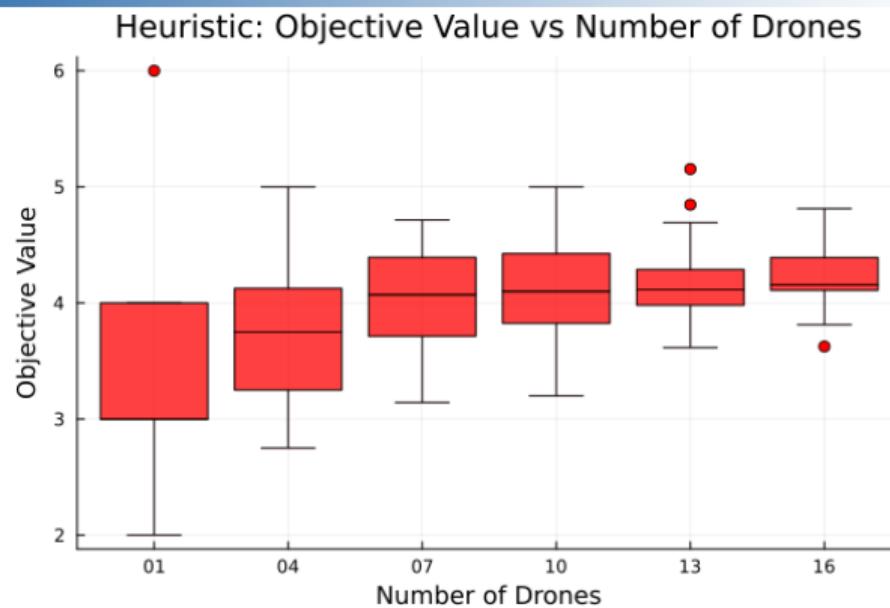


Figure: Heuristic Objective. Source: The authors.

# Results

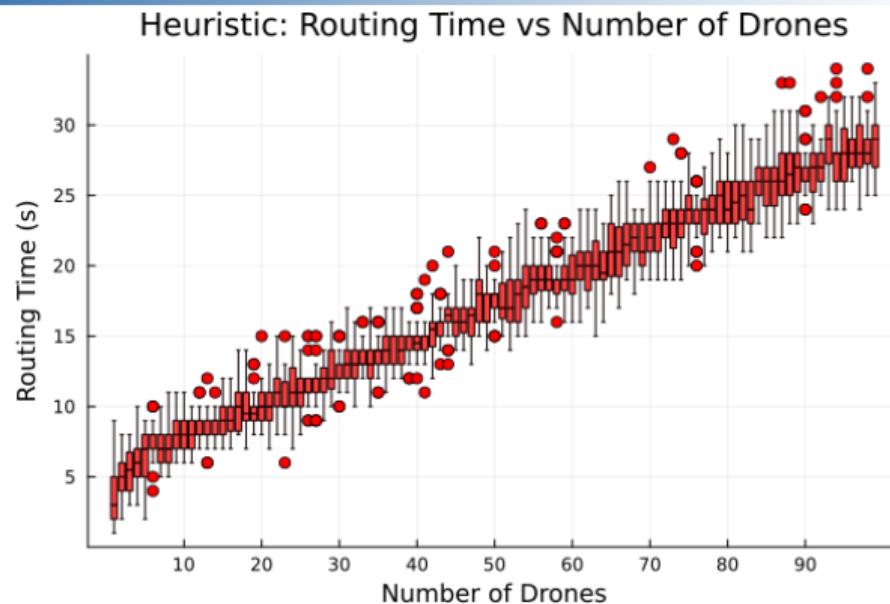


Figure: Heuristic Routing Time ( $T$ ) corresponding to Figure ???. Source: The authors.

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## Advancements

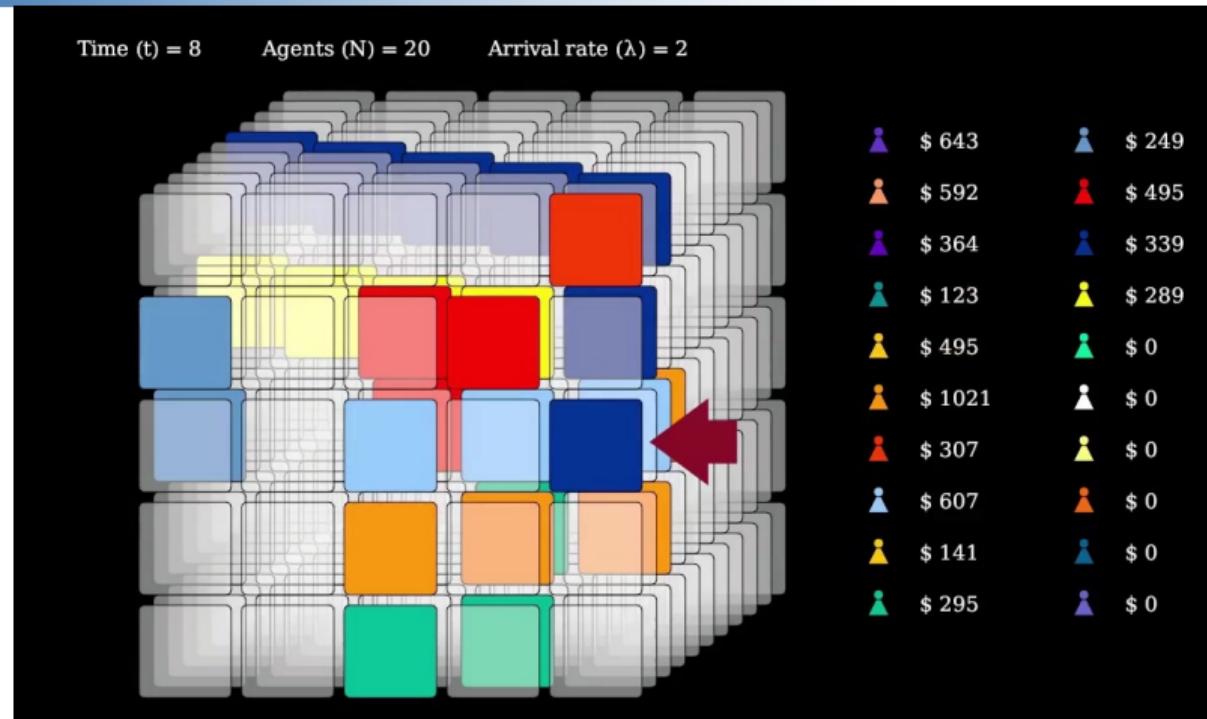
- Addressing the Last Mile Delivery Drones problem through a MAPF approach ;
- Ensuring collision avoidance and efficient airspace control.
- Significant advancement over previous algorithms with guaranteed finite and bounded polynomial time convergence for the heuristic

## Limitations

- Does not address real world problems like energy, fuel and speed of the drones;
- The experiment using binary search to find the  $T$  for the MILP in  $\mathcal{O}(\log(T))$  is not conducted;
- Small grid sizes in experiments;

# Future Works

Figure: Tradable Permit Model in the Decentralized LMDD.  
Source: [5]



# Future Works

## Decentralized Model from [5]

- naive agents
- greedy random choices

## Decentralized Model with Intelligence

- Reinforcement Learning
- Graph Dynamical System
  - Complex Networks
  - Graph Neural Networks

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