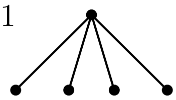
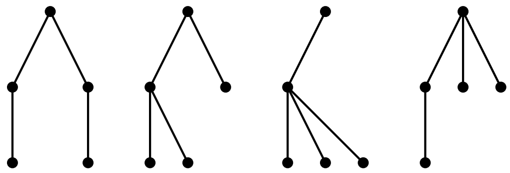
1. Construct all (non-isomorphic) rooted trees of heights from one to four, that have five vertices [1]. Note that there is one of height one, four of height two, three of height three, and one of height four.

Solution:

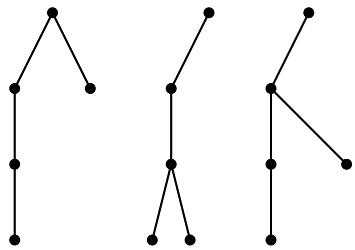
$h = 1$



$h = 2$



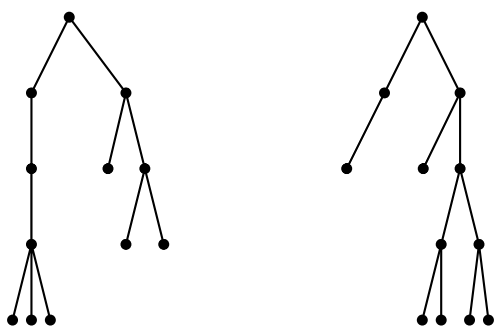
$h = 3$



$h = 4$

2. Construct two non-isomorphic rooted trees both having twelve vertices, six leaves, and height four [1].

Solution:

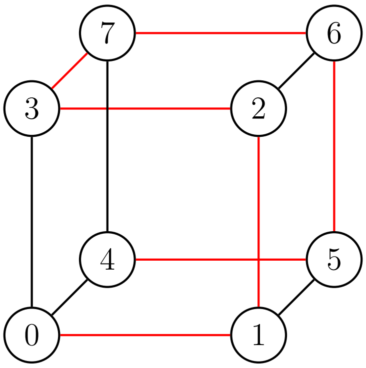


3. Calculate the minimum height of a ternary rooted tree with eleven leaves.

Solution: We have that $h \leq \log_m l$. Now, $\log_3 11 \approx 2.18$. Since the minimum height must be a natural number, and this is a lower bound, we have that the minimum height is 3.

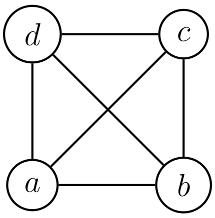
4. Consider a graph with eight vertices and twelve edges connected such that it can be drawn as a cube. Draw this graph, find a spanning tree of it and then high-light the edges on the drawing that are part of the tree [1].

Solution: The following is a picture of the graph. The edges included in the spanning tree are in red.

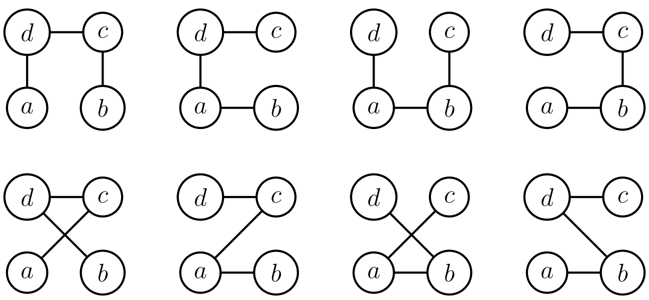


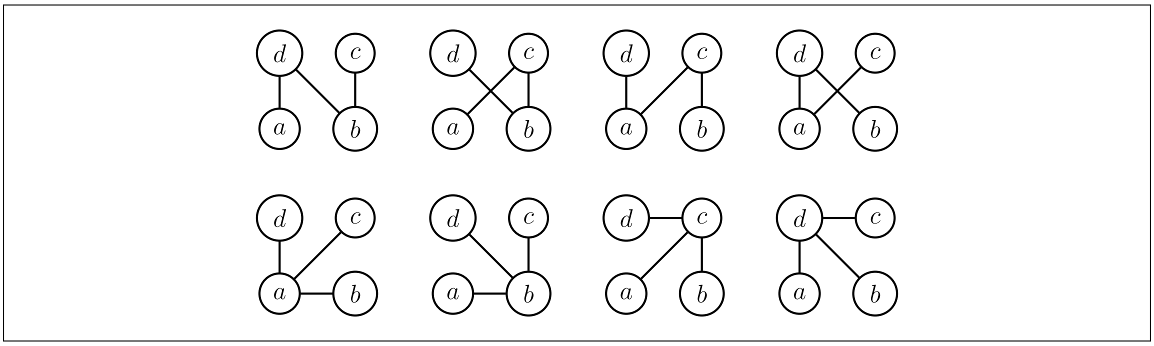
5. Sketch all sixteen distinct spanning trees of the complete graph K_4.

Solution: The complete graph is as follows:



The spanning trees are then:





6. Re-write each of the following infix expressions in reverse Polish notation, draw evaluation trees representing them and evaluate them using the stack method.
   (a) $3 + 4$
   (b) $3 + 4 \times 5$
   (c) $(3 + 4) \times 5$
   (d) $5 + 4 \times (6 \div 3)$
   (e) $((16 \times 2) + 100) \div (11 \times 3) \times 75$

References

[1] N. Biggs. *Discrete Mathematics.* Oxford science publications. OUP Oxford, 2002.