1. Construct all (non-isomorphic) trees of heights from one to four, that have five vertices. [1]. Note that there is one of height one, four of height two, three of height three, and one of height four.

2. Construct two non-isomorphic rooted trees both having twelve vertices, six leaves, and height four [1].

3. Calculate the minimum height of a ternary rooted tree with ten leaves.

4. Draw a ternary decision tree of height at most two, representing the following problem.

   Suppose you have access to the international prototype kilogram. This is a weight, kept in France, which is the definition of the kilogram. You are presented with four other weights which are supposed to each be exactly one kilogram. However, you suspect that one of them is not a kilogram in weight but you can't tell which one it is. You don't have access to a weighing scales, but you do have access to a balance. This balance can be used to compare the weights of various objects and combinations of objects. [1]

5. Draw a decision tree for bubble sort with three elements. [1]

6. What is the smallest possible height of the decision tree of an algorithm for sorting four objects using binary comparisons? [1]

7. Calculate the minimum number of binary comparisons needed (in the worst case) when four objects are sorted [1].
   (a) by bubble sort.
   (b) by insertion (sequential method).
   (c) by insertion (bisection method).

References

[1] Norman Biggs, *Discrete Mathematics*, Oxford University Press, 2nd edition, 2002.