1. Re-write each of the following infix expressions in reverse Polish notation, evaluate them using the stack method and draw evaluation trees representing them.

   (a) $3 + 4$

   (b) $3 + 4 \times 5$

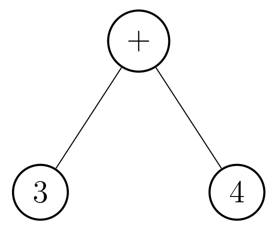
   (c) $(3 + 4) \times 5$

   (d) $5 + 4 \times (6 \div 3)$

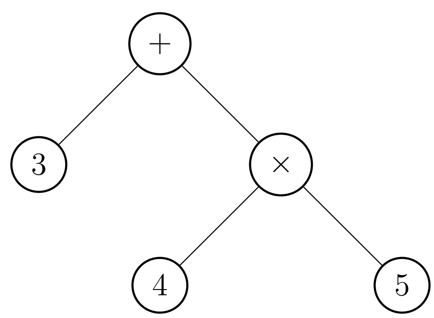
   (e) $((16 \times 2) + 100) \div (11 \times 3) \times 75$

Solution:

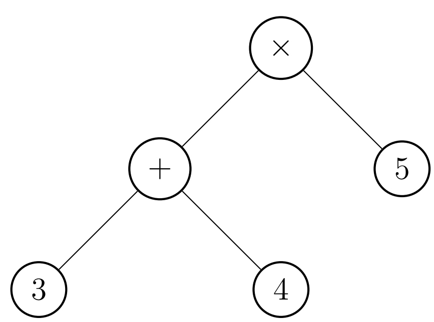
(a) $4 \quad 3 \quad +$



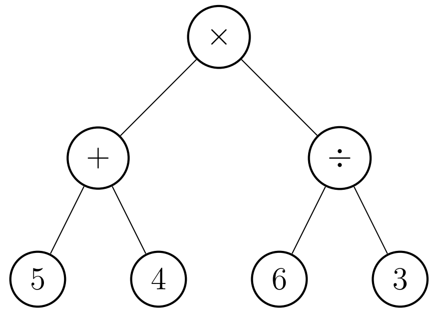
(b) $5 \quad 4 \quad \times \quad 3 \quad +$



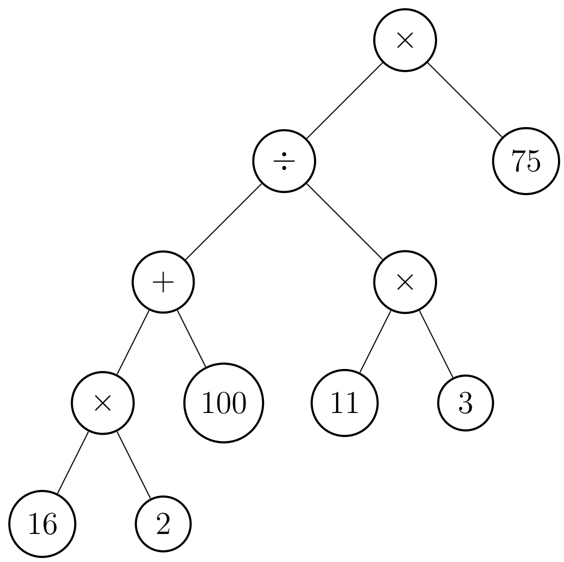
(c) $5 \quad 4 \quad 3 \quad + \quad \times$



(d) $3 \quad 6 \quad \div \quad 4 \quad 5 \quad + \quad \times$



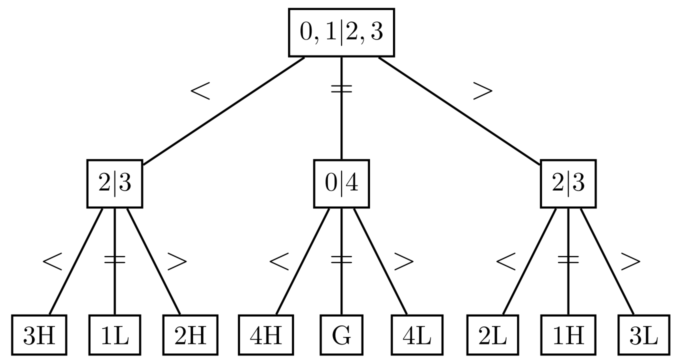
(e) 75 3 11 × 100 2 16 × + ÷ ×



2. Draw a ternary decision tree of height at most two, representing the following problem.

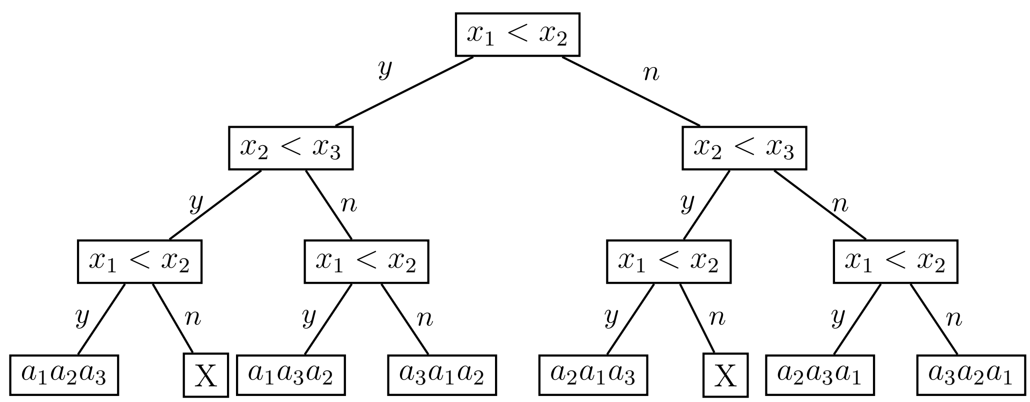
   Suppose you have access to the international prototype kilogram. This is a weight, kept in France, which is the definition of the kilogram. You are presented with four other weights which are supposed to each be exactly one kilogram. However, you suspect that one of them is not a kilogram in weight but you can't tell which one it is. You don't have access to a weighing scales, but you do have access to a balance. This balance can be used to compare the weights of various objects and combinations of objects. [1]

Solution:



3. Draw a decision tree for bubble sort with three elements. [1]

Solution:



4. What is the smallest possible height of the decision tree of an algorithm for sorting four objects using binary comparisons? [1]

Solution: Number of outcomes (leaves) is $l = 4! = 24$. Binary tree, so $m = 2$. So minimum height is $log_2 24 \approx 4.58$. Rounding up, gives smallest height as 5.

5. Calculate the minimum number of binary comparisons needed (in the worst case) when four objects are sorted [1].
   (a) by bubble sort.
   (b) by insertion (sequential method).
   (c) by insertion (bisection method).

Solution:

(a) $3 + 2 + 1 = 6$

(b) $1 + 2 + 3 = 6$

(c) $1 + 2 + 2 = 5$.

References

[1] N. Biggs. *Discrete Mathematics*. Oxford science publications. OUP Oxford, 2002.