Graph Theory

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Topics

Graphs

Trees

Paths and Cycles

Colouring

Sorting

Searching

Shortest Paths

Graphs

Seven Bridges of Königsberg

Is it possible to walk through the city crossing each of the seven bridges once and only once?

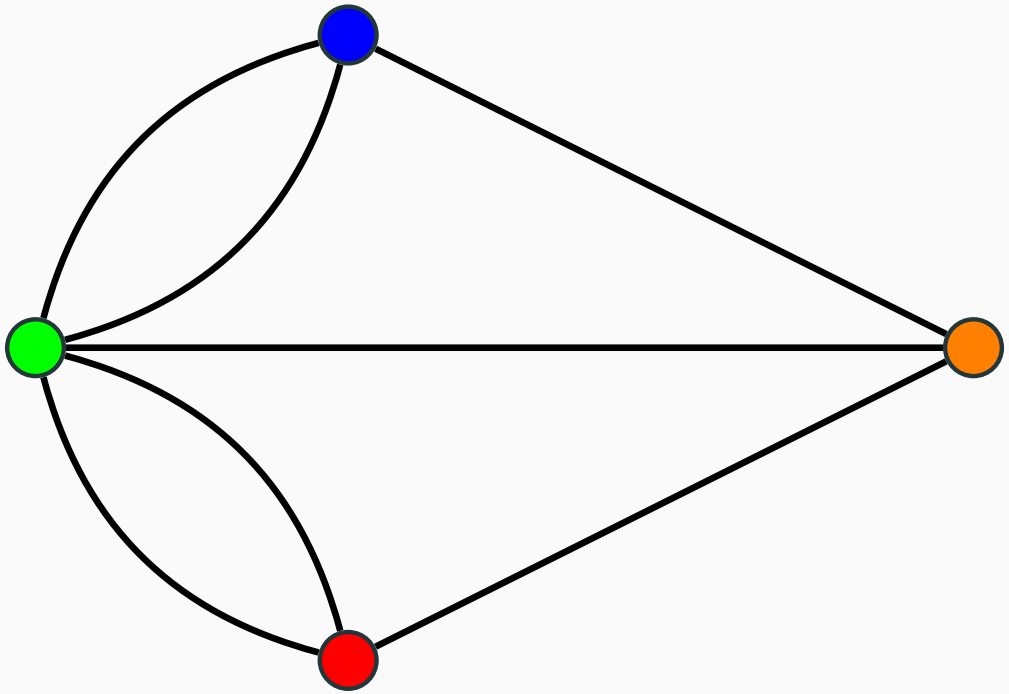
www.nature.com/nbt/journal/v29/n11

Leonhard Euler

- Born 1707 in Basel, Switzerland.
- Euler's identity: $\mathrm{e}^{i\pi} + 1 = 0$.
- Solved the Bridges of Königsberg problem.
- It's not possible to cross all bridges once and once only.

https://en.wikipedia.org/wiki/Leonhard_Euler

Graph of Königsberg



Graph definition

Definition

A *graph* consists of a finite set V and a set E of 2-subsets of V.

Vertices – the elements of the set V are called vertices.

Edges – the elements of E are called edges.

$G = (V, E)$ – this is the way we write the graph G consists of the vertex set V and the edge set E.

Sets of Königsberg

$$V = \{Green, Blue, Orange, Red\}$$

$$\begin{aligned} E = \{ & \\ & \{Green, Blue\}, \{Green, Blue\}, \{Green, Red\}, \\ & \{Green, Red\}, \{Blue, Orange\}, \{Green, Orange\}, \\ & \{Red, Orange\} \\ \} & \end{aligned}$$

Defining different types of graphs

Our definition of a graph

The definition given above for a graph is consistent with looped edges, but not directed edges and not repeated edges. We only need to make small changes to the definition of a graph to allow for directed edges and repeated edges.

Repeated edges are edges that start and end at the same vertices.

Directed edges are edges where a direction is added.

Looped edges begin and end at the same vertex.

The application will determine the definition we want to use.

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