

# Graph Theory

ian.mcloughlin@gmit.ie

Graphs

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Colouring

Sorting

Searching

Shortest Path

# Graphs

# Seven Bridges of Königsberg



Is it possible to walk through the city crossing each of the seven bridges once and only once?

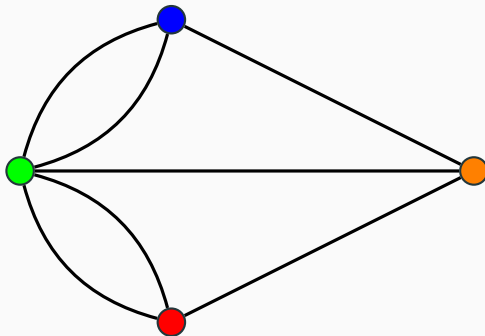
[www.nature.com/nbt/journal/v29/n11](http://www.nature.com/nbt/journal/v29/n11)



- Born 1707 in Basel, Switzerland.
- Euler's identity:  $e^{i\pi} + 1 = 0$ .
- Solved the Bridges of Königsberg problem.
- It's not possible to cross all bridges once and once only.

[https://en.wikipedia.org/wiki/Leonhard\\_Euler](https://en.wikipedia.org/wiki/Leonhard_Euler)

# Graph of Königsberg



## Definition

A *graph* consists of a finite set  $V$  and set  $E$  of 2-subsets of  $V$ .

**Vertices** – the elements of the set  $V$  are called vertices.

**Edges** – the elements of  $E$  are called edges.

$G = (V, E)$  – this is the way we write the graph  $G$  consists of the vertex set  $V$  and the edge set  $E$ .

$$V = \{ \textit{Green}, \textit{Blue}, \textit{Orange}, \textit{Red} \}$$

$$E = \{ \\ \{ \textit{Green}, \textit{Blue} \}, \{ \textit{Green}, \textit{Blue} \}, \{ \textit{Green}, \textit{Red} \}, \\ \{ \textit{Green}, \textit{Red} \}, \{ \textit{Blue}, \textit{Orange} \}, \{ \textit{Green}, \textit{Orange} \}, \\ \{ \textit{Red}, \textit{Orange} \} \\ \}$$



# Defining different types of graphs

## Our definition of a graph

The definition given above for a graph is consistent with looped edges, but not directed edges and not repeated edges. We only need to make small changes to the definition of a graph to allow for directed edges and repeated edges.

**Repeated edges** are edges that start and end at the same nodes.

**Directed edges** are edges where a direction is added.

**Looped edges** begin and end at the same node.

The application will determine the definition we want to use.

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