

Graph Theory

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Graphs

Seven Bridges of Königsberg



Is it possible to walk through the city crossing each of the seven bridges once and only once?

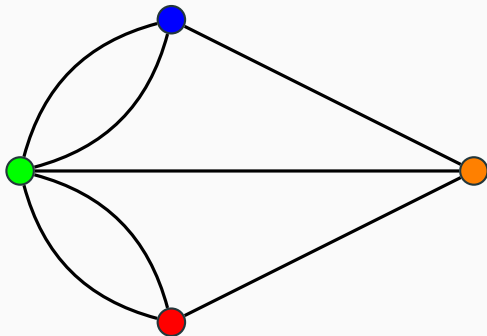
www.nature.com/nbt/journal/v29/n11



- Born 1707 in Basel, Switzerland.
- Euler's identity: $e^{i\pi} + 1 = 0$.
- Solved the Bridges of Königsberg problem.
- It's not possible to cross all bridges once and once only.

https://en.wikipedia.org/wiki/Leonhard_Euler

Graph of Königsberg



Definition

A *graph* consists of a finite set V and a set E of 2-subsets of V .

Vertices – the elements of the set V are called vertices.

Edges – the elements of E are called edges.

$G = (V, E)$ – this is the way we write the graph G consists of the vertex set V and the edge set E .

$$V = \{ \textit{Green}, \textit{Blue}, \textit{Orange}, \textit{Red} \}$$

$$E = \{ \\ \{ \textit{Green}, \textit{Blue} \}, \{ \textit{Green}, \textit{Blue} \}, \{ \textit{Green}, \textit{Red} \}, \\ \{ \textit{Green}, \textit{Red} \}, \{ \textit{Blue}, \textit{Orange} \}, \{ \textit{Green}, \textit{Orange} \}, \\ \{ \textit{Red}, \textit{Orange} \} \\ \}$$

Adjacency list

| Green | Blue | Orange | Red |
|--------|--------|--------|--------|
| Blue | Green | Blue | Green |
| Orange | Orange | Green | Orange |
| Red | | Red | |

Defining different types of graphs

Our definition of a graph

The definition given above for a graph is consistent with looped edges, but not directed edges and not repeated edges. We only need to make small changes to the definition of a graph to allow for directed edges and repeated edges.

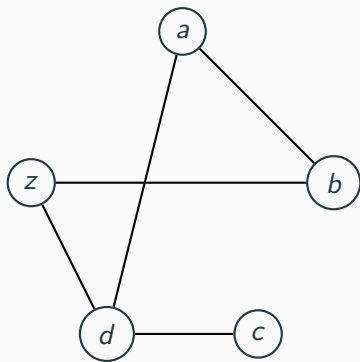
Repeated edges are edges that start and end at the same vertices.

Directed edges are edges where a direction is added.

Looped edges begin and end at the same vertex.

The application will determine the definition we want to use.

A better example



Exercise

Determine the vertex set, edge set and adjacency list of this graph.

global.oup.com/booksites/content/9780198507185/

Degree of a vertex

Definition

The degree of a vertex is the number of edges that contain it.



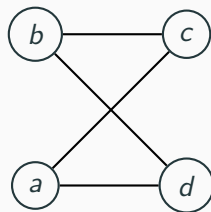
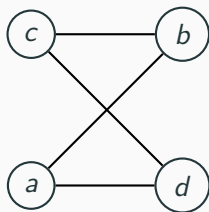
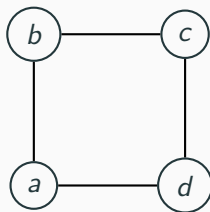
The degree of the vertex a is 2.

Exercise

For each of the vertices on the previous slide, determine its degree.

Definition

Two graphs G_1 and G_2 are said to be isomorphic when there is a bijection α for the vertex set V_1 of G_1 to the vertex set V_2 of G_2 such that $\{\alpha(x), \alpha(y)\}$ is an edge of G_2 if and only if (x, y) is an edge of G_1 .



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