

Graph Theory

ian.mcloughlin@gmit.ie

Graphs

Trees

Paths and Cycles

Colouring

Sorting

Searching

Shortest Paths

Graphs

Seven Bridges of Königsberg



Is it possible to walk through the city crossing each of the seven bridges once and only once?

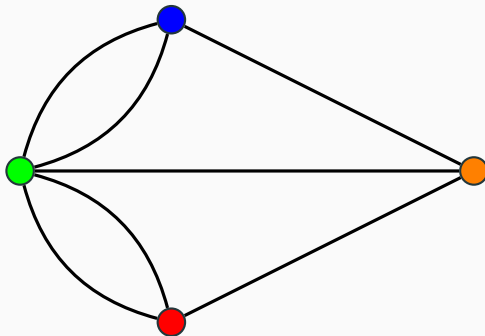
www.nature.com/nbt/journal/v29/n11



- Born 1707 in Basel, Switzerland.
- Euler's identity: $e^{i\pi} + 1 = 0$.
- Solved the Bridges of Königsberg problem.
- It's not possible to cross all bridges once and once only.

https://en.wikipedia.org/wiki/Leonhard_Euler

Graph of Königsberg



Definition

A *graph* consists of a finite set V and a set E of 2-subsets of V .

Vertices – the elements of the set V are called vertices.

Edges – the elements of E are called edges.

$G = (V, E)$ – this is the way we write the graph G consists of the vertex set V and the edge set E .

$$V = \{ \textit{Green}, \textit{Blue}, \textit{Orange}, \textit{Red} \}$$

$$E = \{ \\ \{ \textit{Green}, \textit{Blue} \}, \{ \textit{Green}, \textit{Blue} \}, \{ \textit{Green}, \textit{Red} \}, \\ \{ \textit{Green}, \textit{Red} \}, \{ \textit{Blue}, \textit{Orange} \}, \{ \textit{Green}, \textit{Orange} \}, \\ \{ \textit{Red}, \textit{Orange} \} \\ \}$$

Adjacency list

Green	Blue	Orange	Red
Blue	Green	Blue	Green
Orange	Orange	Green	Orange
Red		Red	

Defining different types of graphs

Our definition of a graph

The definition given above for a graph is consistent with looped edges, but not directed edges and not repeated edges. We only need to make small changes to the definition of a graph to allow for directed edges and repeated edges.

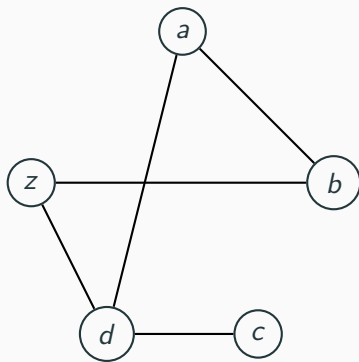
Repeated edges are edges that start and end at the same vertices.

Directed edges are edges where a direction is added.

Looped edges begin and end at the same vertex.

The application will determine the definition we want to use.

A better example



Exercise

Determine the vertex set, edge set and adjacency list of this graph.

global.oup.com/booksites/content/9780198507185/

Degree of a vertex

Definition

The degree of a vertex is the number of edges that contain it.



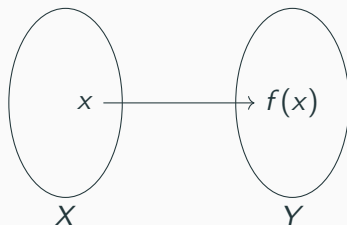
The degree of the vertex a is 2.

Exercise

For each of the vertices on the previous slide, determine its degree.

Definition

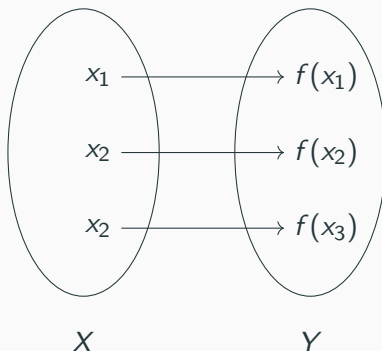
Suppose that X and Y are sets. We say we have a function f from X to Y if for each x in X we can specify a unique element in Y , which we denote by $f(x)$.



Definition

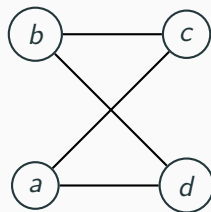
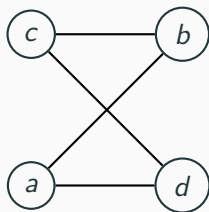
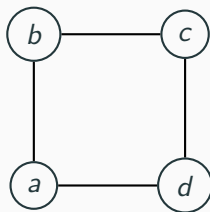
A bijection is function f from a set X to a set Y where both of the following are true:

- every y in Y is a value $f(x)$ for at most one x in X .
- every y in Y is a value $f(x)$ for at least one x in X .



Definition

Two graphs G_1 and G_2 are said to be isomorphic when there is a bijection α for the vertex set V_1 of G_1 to the vertex set V_2 of G_2 such that $\{\alpha(x), \alpha(y)\}$ is an edge of G_2 if and only if (x, y) is an edge of G_1 .



Trees

Paths and Cycles

Colouring

Sorting

Searching

Shortest Paths