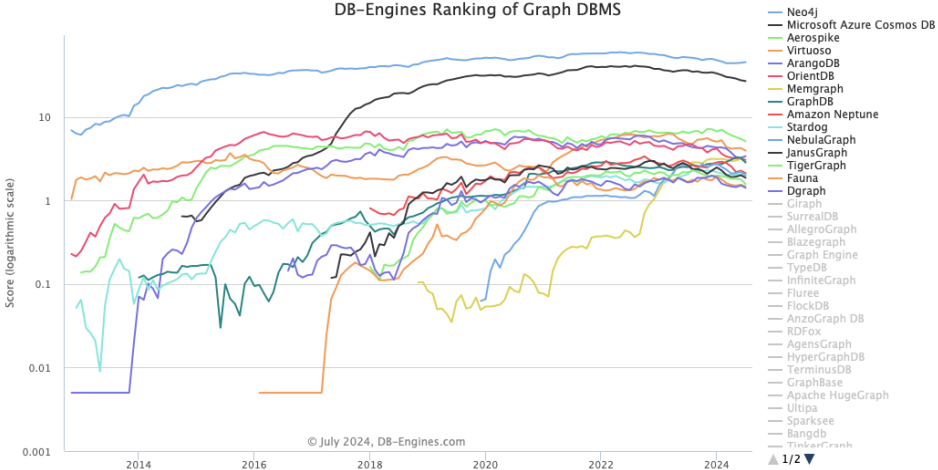
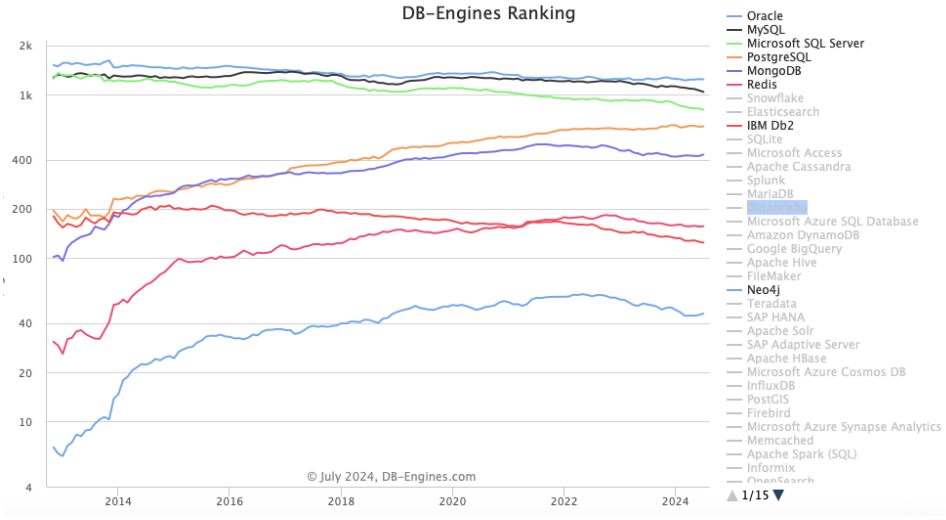

DB-Engines Ranking of Graph DBMS
Score (logarithmic scale)
10
1
0.1
0.01
0.001
2014
2016
2018
2020
2022
2024
© July 2024, DB-Engines.com
Neo4j
Microsoft Azure Cosmos DB
Aerospike
Virtuoso
ArangoDB
OrientDB
Memgraph
GraphDB
Amazon Neptune
Stardog
NebulaGraph
JanusGraph
TigerGraph
Fauna
Dgraph
Giraph
SurrealDB
AllegroGraph
Blazegraph
Graph Engine
TypeDB
InfiniteGraph
Fluree
FlockDB
AnzoGraph DB
RDFox
AgensGraph
HyperGraphDB
TerminusDB
GraphBase
Apache HugeGraph
Ultipa
Sparksee
Bangdb
TinkerGraph
1/2

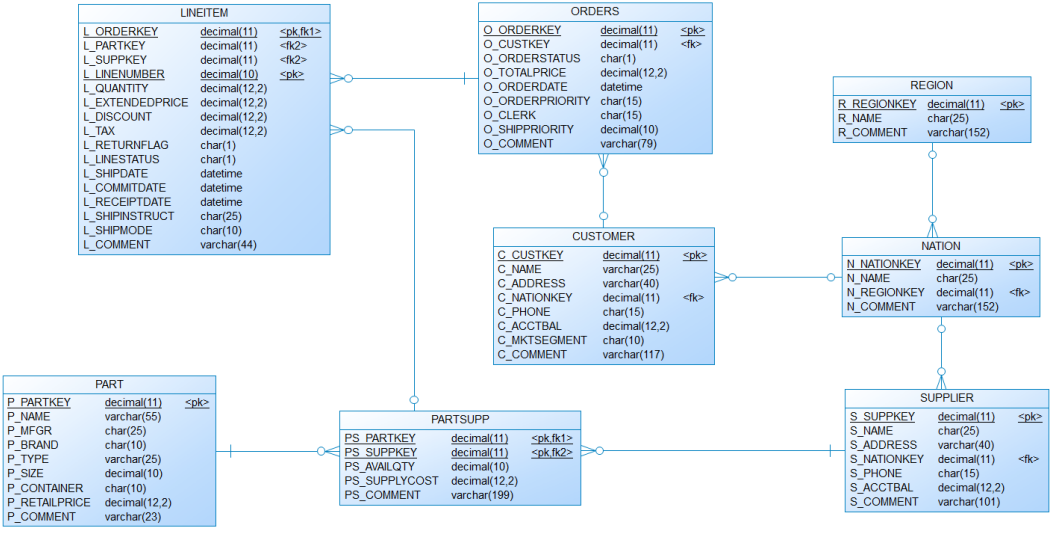
Why are graph databases not so popular yet, apart from historic reasons?



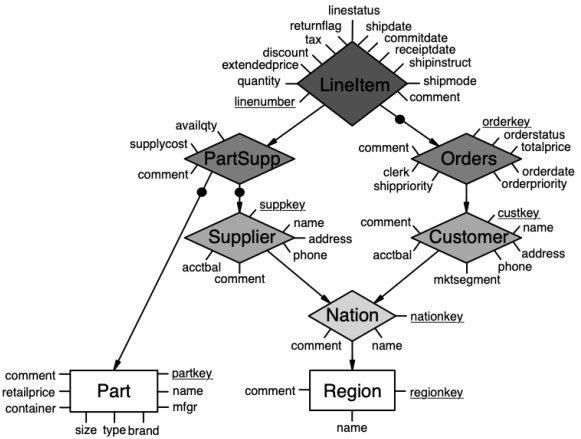
How can we design graph databases well?

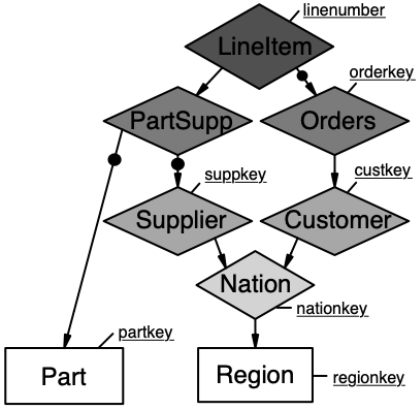
How can we design graph databases well?

The TPC-H Data Model in UML



The TPC-H Data Model as E/R Diagram





Observations

Entity/Relationship diagrams are graphs
Entity/Relationship models capture well-designed databases

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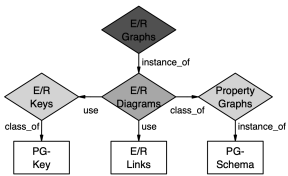
Entity/Relationship diagrams are graphs
Entity/Relationship models capture well-designed databases

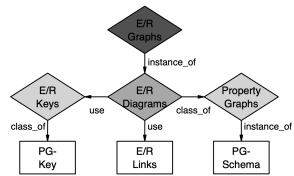
Approach

Explore what E/R modeling has to offer for graph data
Also explore what graph data has to offer for E/R modeling

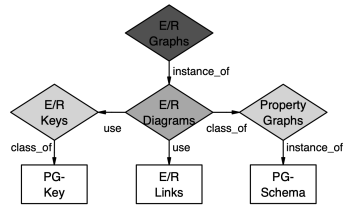
Entity/Relationship Graphs

Principled Design, Modeling, and Data Integrity Management of Graph Databases





1. Background
   - E/R Modeling
   - PG-Key and PG-Schema
2. Contributions: "Lean and mean"
   - E/R Models for Principled Graph Database Design
   - E/R Graphs as Instances of E/R Diagrams
   - Unify Conceptual, Logical and Graph Modeling
   - Entity and Referential Integrity Management
   - Relational/Mixed/Graph Semantics of E/R Graphs
3. Experiments
   - Entity Integrity
   - Referential Integrity
   - Queries
4. Summary



Background: Entity/Relationship Modeling

Peter P. Chen: **The Entity-Relationship Model - Toward a Unified View of Data**. *ACM Trans. Database Syst.* 1(1): 9-36 (1976)

Bernhard Thalheim: **Entity-Relationship Modeling - Foundations of Database Technology**. Springer 2000, ISBN 978-3-540-65470-4, pp. I-XII, 1-627

E/R Schemata and their Diagrams

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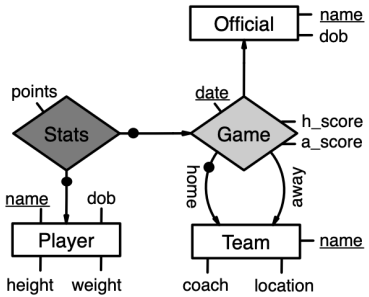

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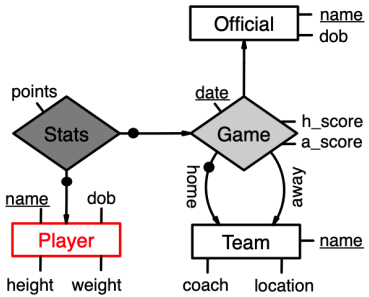

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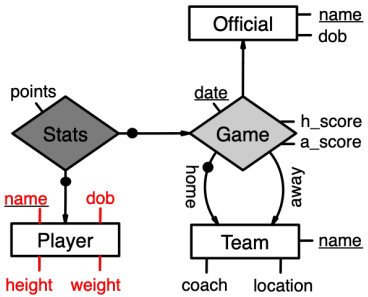

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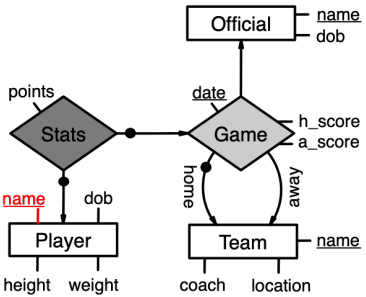

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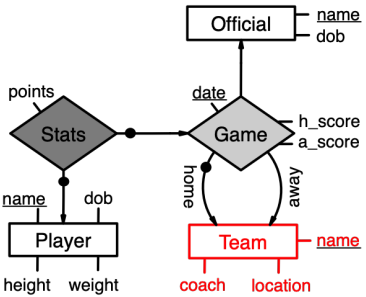

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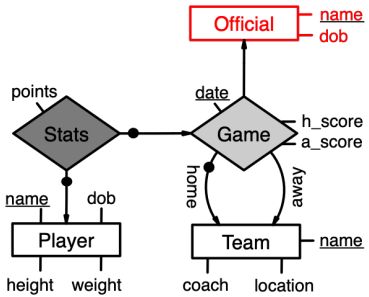

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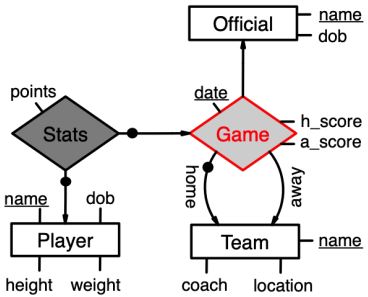

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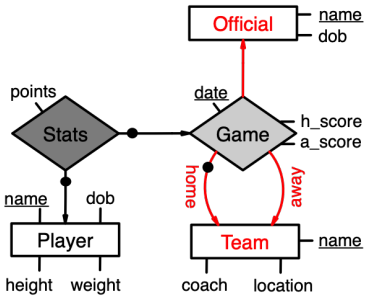

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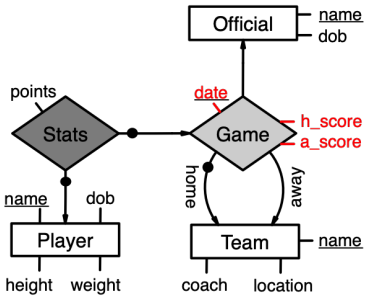

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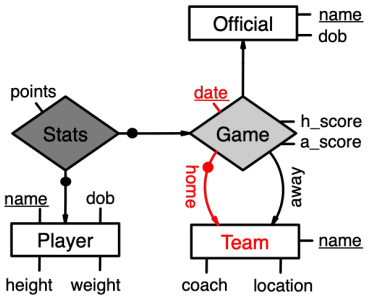

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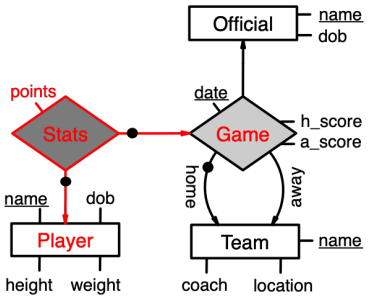

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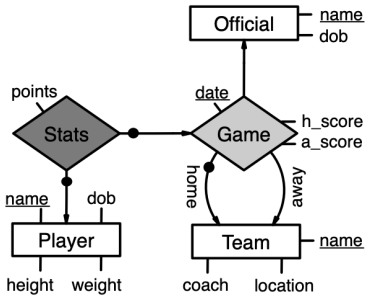

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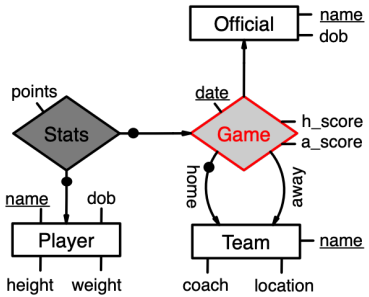

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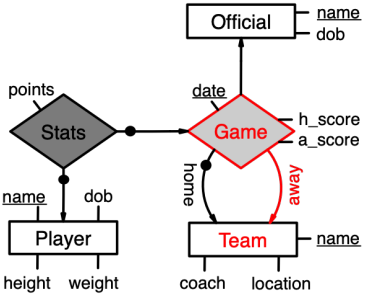

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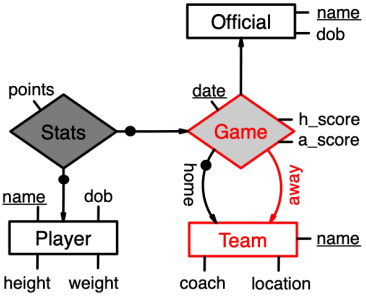

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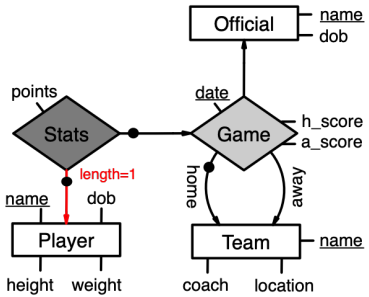

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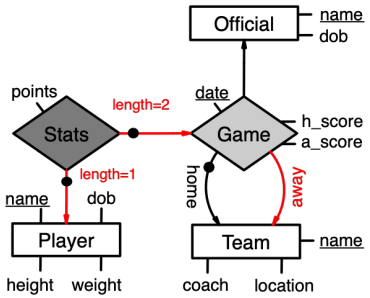

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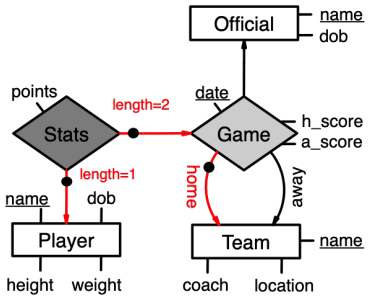

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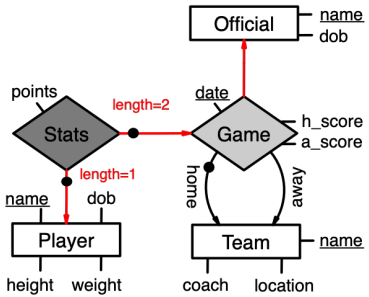

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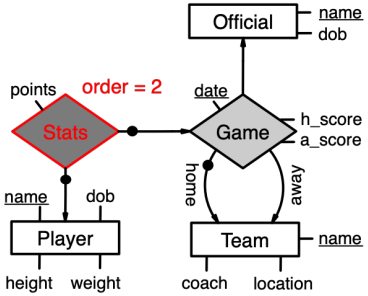

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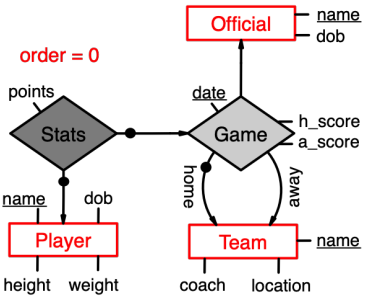

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Malone	24/07/1963	2.06m	117kg

Team

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Game

home:Team	away:TEAM	**date**	*h_score*	*a_score*
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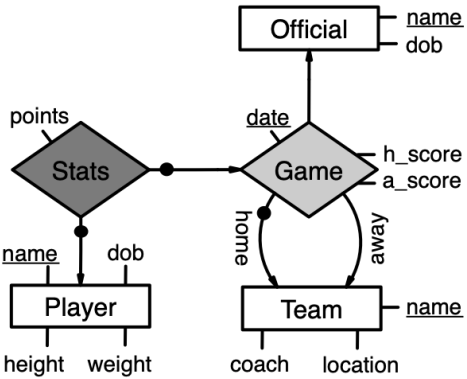

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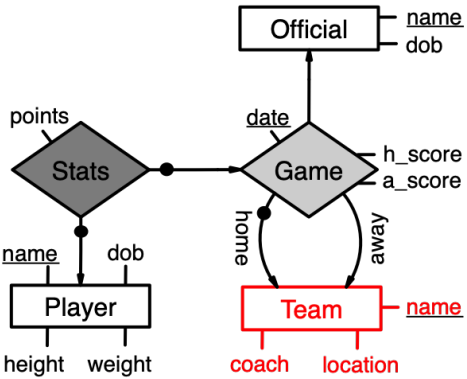

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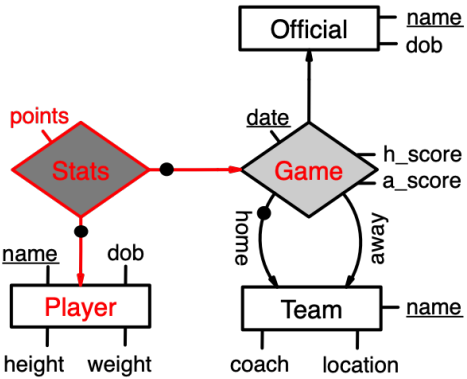

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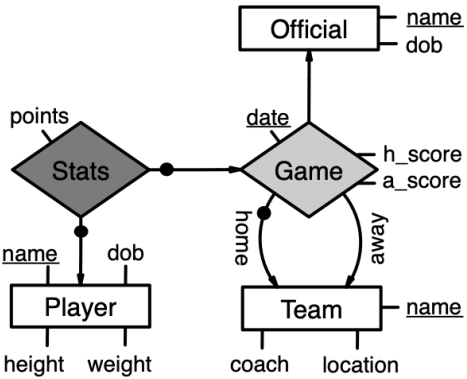

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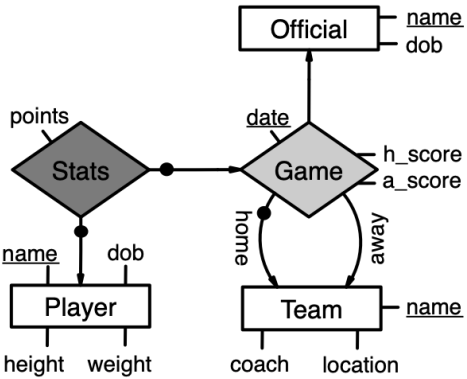

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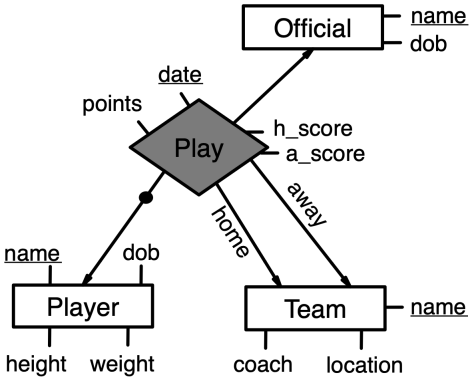
- STATS={*pname*, *hname*, *date*, *points*} with foreign keys
  - [*pname*] $\subseteq$ PLAYER[*name*]
  - [*hname*,*date*] $\subseteq$ GAME[*hname*,*date*]

Attribute Redundancy: Referential Integrity by Duplicating Keys as Foreign Keys

Object Types Not in BCNF Cause Data Value Redundancy



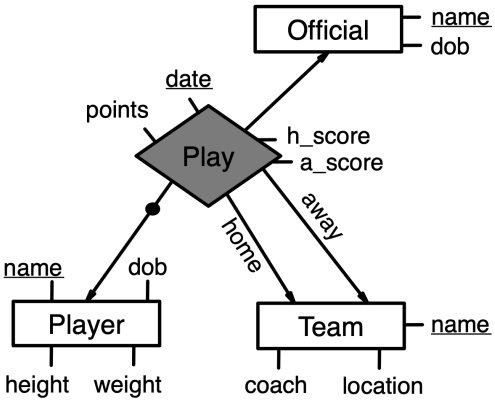
Data Value Redundancy

- Object types overloaded
- Inefficient for updates
- Source of inconsistency

Functional Dependency Causes Data Value Redundancy

PLAY : *date*, *home*:TEAM → *away*:TEAM, OFFICIAL, *h_score*, *a_score*

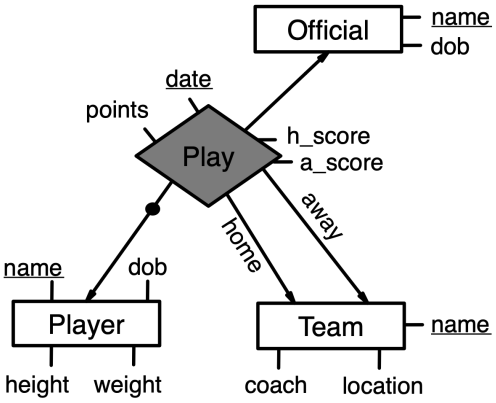
E/R Diagrams: No Data Redundancy or Update Inefficiency

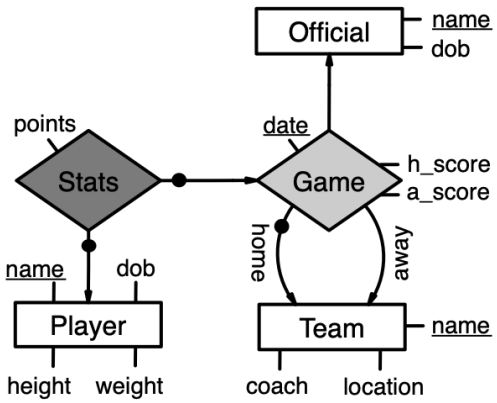


From Non-key Functional Dependency to Key

From: $\textsc{Play} : \textit{date}, \textit{home}{:}\textsc{Team} \rightarrow \textit{away}{:}\textsc{Team}, \textsc{Official}, \textit{h\_score}, \textit{a\_score}$

E/R Diagrams: No Data Redundancy or Update Inefficiency





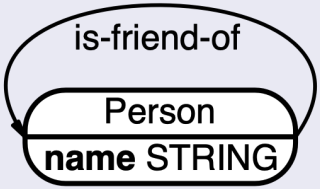
From Non-key Functional Dependency to Key

From: $\textsc{Play} : \mathit{date}, \mathit{home}{:}\textsc{Team} \rightarrow \mathit{away}{:}\textsc{Team}, \textsc{Official}, \mathit{h\_score}, \mathit{a\_score}$

To: $\mathit{id}(\textsc{Game}) = \{\mathit{date}, \mathit{home}{:}\textsc{Team}\}$

Directed Cycles in PG-Schema: Semantic Issues and Update Inefficiency

PG-Schema Definition



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Invalid Instance of PG-Schema

:Person
name = “Tik”

Directed Cycles in PG-Schema: Semantic Issues and Update Inefficiency

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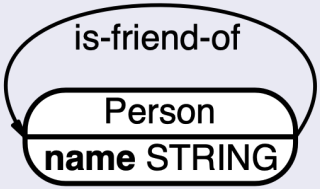

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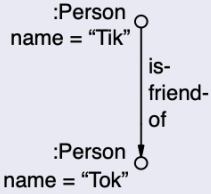

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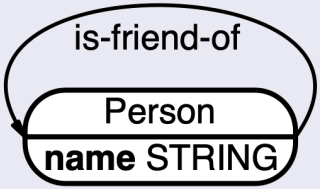

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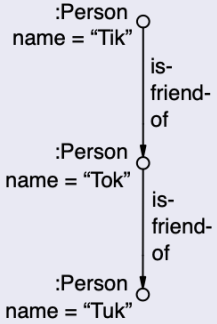

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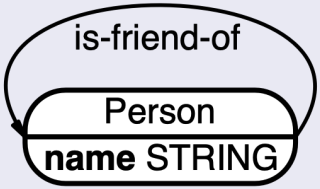

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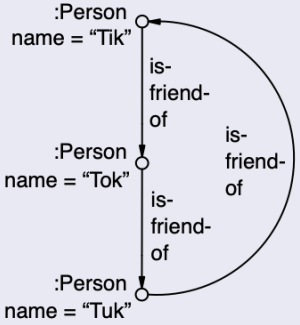

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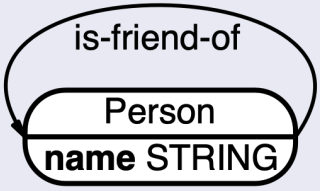

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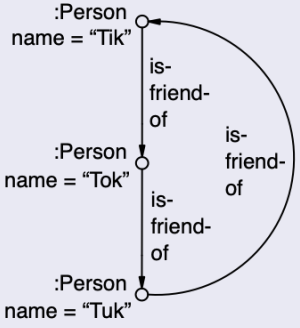
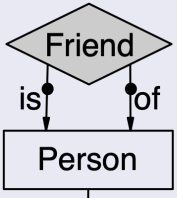


Potentially Unintended

This PG-Schema always requires some circle of friends

E/R Diagrams Promote Acyclicity

E/R Diagram



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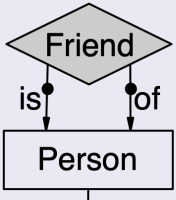
(x) —[is]—> (y:pType), (x) —[of]—> (z:pType)}

Instance of E/R Diagram

:Person
name = “Tik”

E/R Diagrams Promote Acyclicity

E/R Diagram



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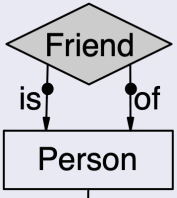
:Person
name = “Tik”

:Person
name = “Tok”

E/R Diagrams Promote Acyclicity

E/R Diagram



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Instance of E/R Diagram

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:Person
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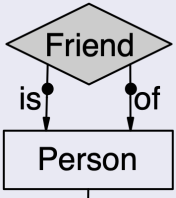
E/R Diagrams Promote Acyclicity

E/R Diagram



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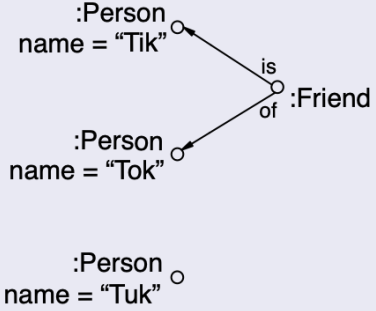
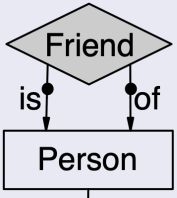


E/R Diagrams Promote Acyclicity

E/R Diagram



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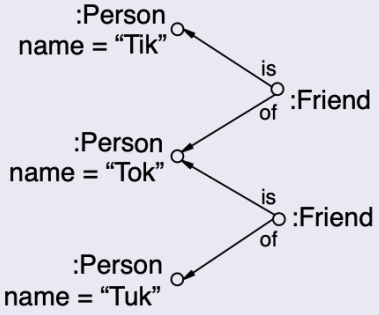

E/R Diagrams Promote Acyclicity

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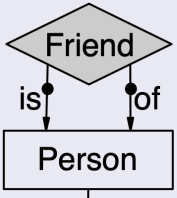

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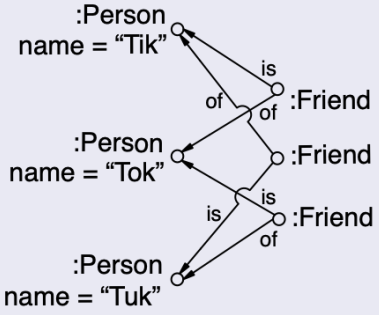

E/R Diagrams Promote Acyclicity

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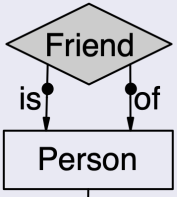

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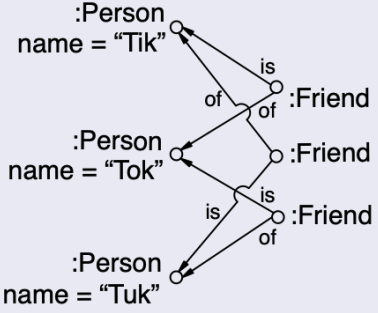
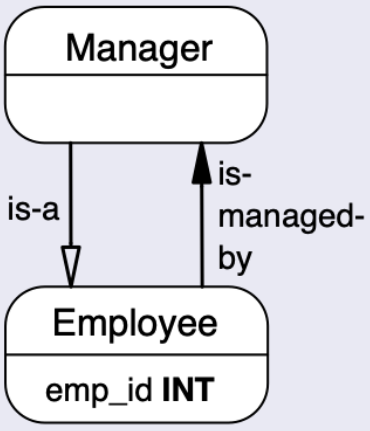


Circles of friends are included as one class of valid instances, but even their representation is still acyclic

E/R Diagrams: Breaking Directed Cycles

PG-Schema with Directed Cycle



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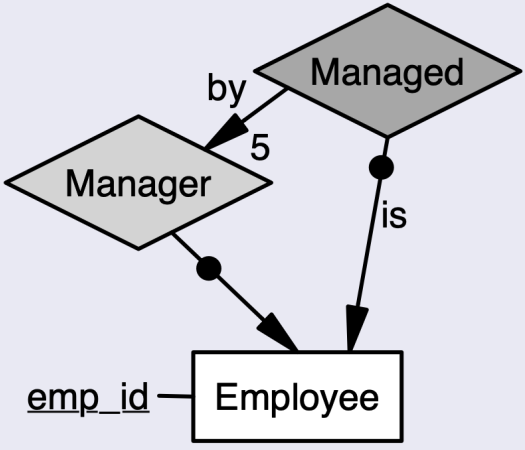
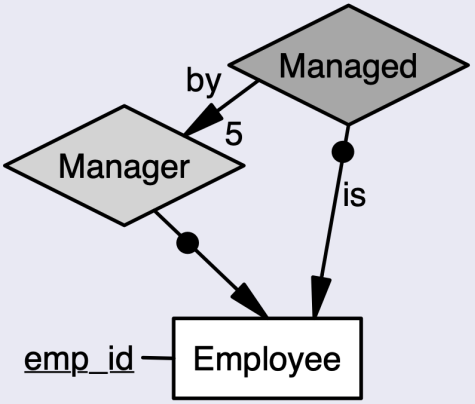


E/R Diagrams: Other Concepts

E/R Diagram



Some other E/R Concepts

- **Specialization** is unary relationship type
  $R = (\{C\}, attr(C), \{C\})$
  $\textsc{Manager} = (\{\textsc{Employee}\}, \emptyset, \{\textsc{Employee}\})$
- **Weak Entity Type**: objects cannot be identified without other object types
  $\textsc{Manager}$ requires $\textsc{Employee}$
- **Cardinality constraint**: Restrict number of objects in which objects of a component participate
  $card(\textsc{Managed}, by : \textsc{Manager}) \leq 5$ means every manager manages up to 5 employees
  $card(\textsc{Managed}, is : \textsc{Manager}) \leq 1$ expresses the key $id(\textsc{Managed})$=$\{is$:$\textsc{Employee}\}$

Background: PG-Key

Renzo Angles, Angela Bonifati, Stefania Dumbrava, George Fletcher, Keith W. Hare, Jan Hidders, Victor E. Lee, Bei Li, Leonid Libkin, Wim Martens, Filip Murlak, Josh Perryman, Ognjen Savkovic, Michael Schmidt, Juan F. Sequeda, Slawek Staworko, Dominik Tomaszuk: **PG-Keys: Keys for Property Graphs**. *SIGMOD Conference 2021*: 2423-2436

PG-Key: Basic goals

- Flexible and powerful framework for defining key constraints
- Designed by the Linked Data Benchmark Council's Property Graph Schema Working Group
  - with members from industry, academia, and the ISO Graph Query Language standards group
- PG-Key combines basic restrictions to identify, reference and constrain objects, including nodes, edges, and properties

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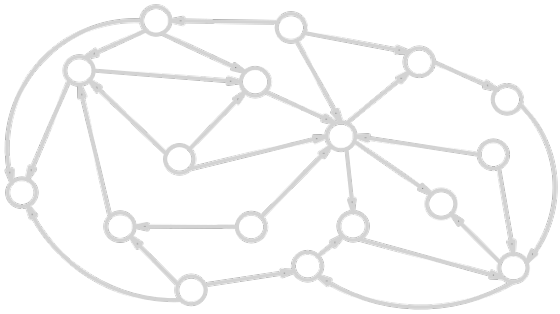
"PG-Keys aims to guide the evolution of the standardization efforts towards making systems more useful, powerful, and expressive".

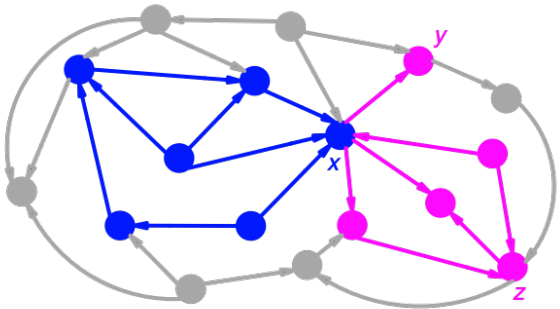
PG-Key: Informal definition

- Declared on property graph



PG-Key: Informal definition

- Declared on property graph
- Key scope
- Descriptor of values



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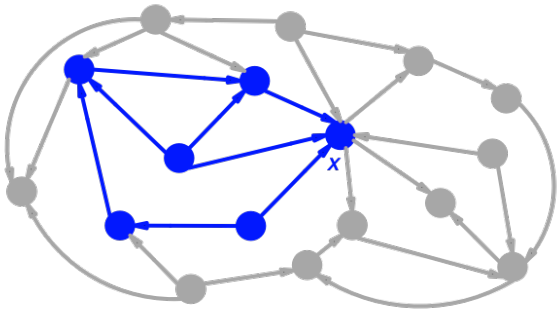
Key scope

`FOR x WITHIN`



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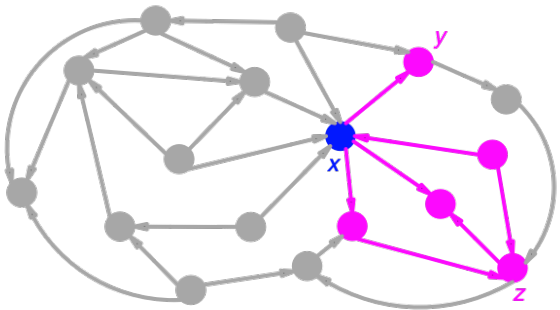
Key scope

`FOR` x `WITHIN`

Descriptor of key values

`IDENTIFIER` y, z `WITHIN`



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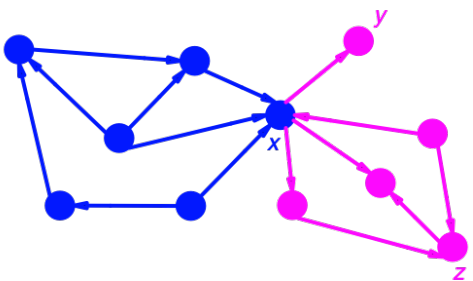

PG-Key: Formal definition

FOR $p(x)$ qualifier $q(x, \overline{y})$

- with scope query $p(x)$: all possible target objects that need to be identified
- and descriptor query $q(x, \overline{y})$: selecting the key values that identify target objects

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Objects comprise nodes, relationships, or properties

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Objects comprise nodes, relationships, or properties

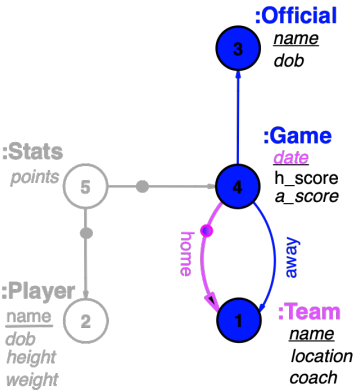
qualifier combines keywords

- `EXCLUSIVE`: where no two targets can share the same key value
- `MANDATORY`: where each target has at least one key value
- `SINGLETON`: where each target as at most one key value
- `IDENTIFIER`: shorthand for `EXCLUSIVE`, `MANDATORY` and `SINGLETON`.

PG-Key: Example

```
FOR g WITHIN
    (g:GAME)-[home]->(t:TEAM),
    (g:GAME)-[away]->(:TEAM),
    (g:GAME)-[ ]->(:OFFICIAL)
IDENTIFIER g.date, t
```

Each game between a home and away team refereed by some official **is identified by the date and home team**



Background: PG-Schema

Renzo Angles, Angela Bonifati, Stefania Dumbrava, George Fletcher, Alastair Green, Jan Hidders, Bei Li, Leonid Libkin, Victor Marsault, Wim Martens, Filip Murlak, Stefan Plantikow, Ognjen Savkovic, Michael Schmidt, Juan Sequeda, Slawek Staworko, Dominik Tomaszuk, Hannes Voigt, Domagoj Vrgoc, Mingxi Wu, Dusan Zivkovic: **PG-Schema: Schemas for Property Graphs**. *Proc. ACM Manag. Data* 1(2): 198:1-198:25 (2023)

PG-Schema: Basic goals

- Community-based effort of academics and practitioners to help with standardization efforts for a graph query language, in particular schema support
- PG-Schema aims at providing a simple yet powerful formalism for specifying property graph schemata, including flexible type definitions supporting multi-inheritance, and expressive constraints
- The formal syntax and semantics of PG-Schema is aimed at meeting principled design requirements of contemporary property graph management

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PG-Schema "deliberately target minimal data modeling capabilities and as a reference point ... take the most basic variant of Entity-Relationship (ER) diagrams ... as the ultimate lower bound in the expressiveness of conceptual modelling languages".

PG-Schema: Main features

Domain, Entity, Referential Integrity & Beyond

- Node types, Edge types, Constraints

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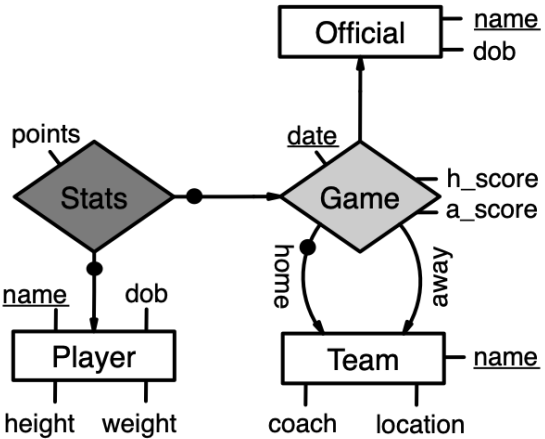
  - closed types allow only explicitly mentioned or inherited labels and properties
  - open types allow arbitrary additional labels and properties

E/R Diagrams are Property Graphs, PG Models, and PG-Schemata

Visual: E/R Diagrams as Property Graphs



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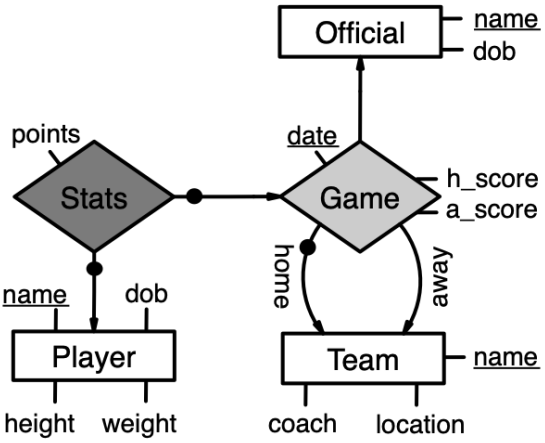
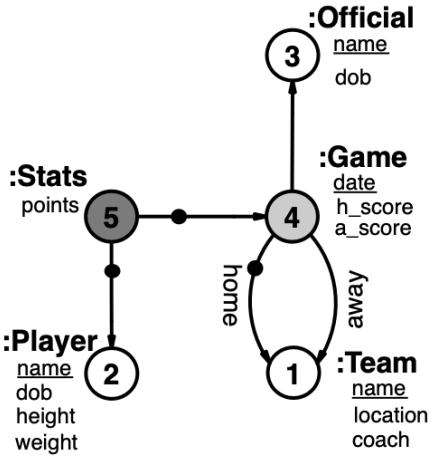
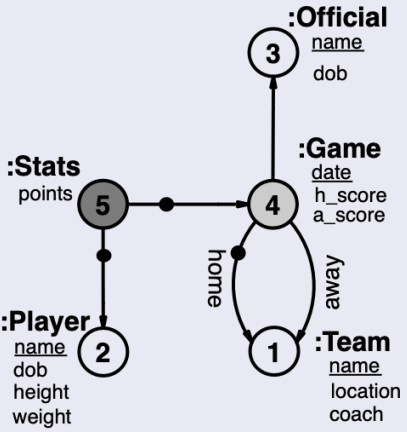




Formal definition: E/R Diagrams as Property Graphs

E/R Diagram D



Formal Property Graph $\mathcal{G}_D = (V_D, E_D, \eta_D, \lambda_D, \nu_D)$ for D

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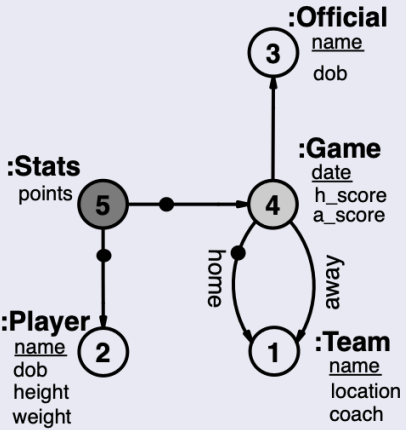

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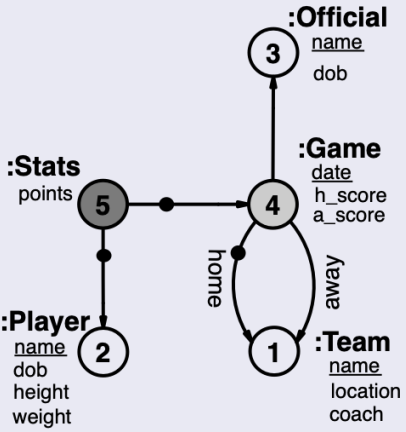

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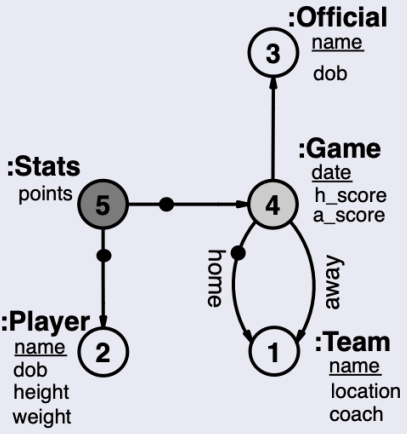

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- $\lambda_D : V_D \cup E_D \to \mathcal{P}(\mathcal{L})$:
  $i_1 \mapsto \{\text{Team}\}$, $i_2 \mapsto \{\text{Player}\}$, $i_3 \mapsto \{\text{Official}\}$,
  $i_4 \mapsto \{\text{Game}\}$, $i_5 \mapsto \{\text{Stats}\}$, $i_{(4,1,\text{home})} \mapsto \{\bullet, \text{home}\}$,
  $i_{(4,1,\text{away})} \mapsto \{\text{away}\}$, $i_{(5,2)} \mapsto \{\bullet\}$, $i_{(5,4)} \mapsto \{\bullet\}$, and

Formal definition: E/R Diagrams as Property Graphs

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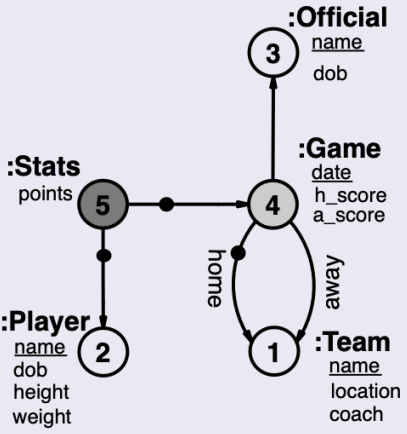

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- $\nu_D : (V_D \cup E_D) \times \mathcal{K}_D \to \mathcal{N}_D$:
  $(i_1, \text{name}) \mapsto \texttt{String}$, $(i_1, \text{location}) \mapsto \texttt{String}_\perp$, $(i_1, \text{coach}) \mapsto \texttt{String}_\perp$,
  $(i_2, \text{name}) \mapsto \texttt{String}$, $(i_2, \text{dob}) \mapsto \texttt{Date}_\perp$, $(i_2, \text{height}) \mapsto \texttt{String}_\perp$,
  $(i_2, \text{weight}) \mapsto \texttt{String}_\perp$,
  $(i_3, \text{name}) \mapsto \texttt{String}$, $(i_3, \text{dob}) \mapsto \texttt{Date}_\perp$,
  $(i_4, \text{date}) \mapsto \texttt{Date}$, $(i_4, \text{h\_score}) \mapsto \texttt{Int}_\perp$, $(i_4, \text{a\_score}) \mapsto \texttt{Int}_\perp$,
  $(i_5, \text{points}) \mapsto \texttt{Int}$.

Formal Translation: Base Sets

E/R Diagram $D = (V, E)$ for E/R Schema $\mathcal{S}$

- $\mathcal{O}_D = O_V \cup O_E$: set of object identifiers for D composed of the sets
  - O_V with unique identifiers i_o for each vertex in V,
  - O_E with unique identifiers $i_{(o,o',l)}$ for each edge in E,

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  - edge labels in D (resulting from labels l in $l : O' \in comp(O)$), and
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- $\mathcal{K}_D$: set of properties for D containing the attribute names for all object types in $\mathcal{S}$
- $\mathcal{N}_D$: the set of values for D with all domains $dom(A)$ for any attribute A of any object type in $\mathcal{S}$, but we distinguish between
  - $dom(A)$ which does not permit null marker occurrences
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Principle of entity integrity: as for primary keys, values for key properties are mandatory (`NOT NULL`)

Formal Translation: Definition of $\mathcal{G}_D = (V_D, E_D, \eta_D, \lambda_D, \nu_D)$

Given $\mathcal{O}_D$, $\mathcal{L}_D$, $\mathcal{K}_D$, and $\mathcal{N}_D$, the *E/R graph model* $\mathcal{G}_D = (V_D, E_D, \eta_D, \lambda_D, \nu_D)$ of an E/R diagram $D = (V, E)$ is defined as follows:

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4. $\lambda_D : V_D \cup E_D \to \mathcal{P}(\mathcal{L})$ is defined by $\lambda_D(i_o) = \{O\}$ for $O \in V$ with unique identifier $i_o \in O_V$, $l \in \lambda_D(i_{(o,o',l)})$ for $((O, O'), l) \in E$ with edge label l and unique identifier $i_{(o,o',l)} \in O_E$, and $\bullet \in \lambda_D(i_{(o,o',l)})$ for $((O, O'), l) \in E$ with key component $l : O' \in \mathit{comp}(O) \cap \mathit{id}(O)$ and unique identifier $i_{(o,o',l)} \in O_E$, and

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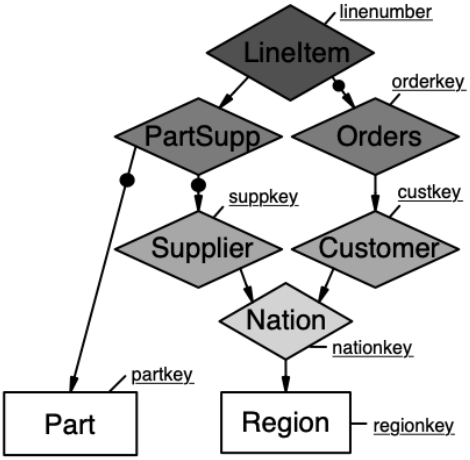
5. $\nu_D : (V_D \cup E_D) \times \mathcal{K}_D \to \mathcal{N}_D$ is defined by $\nu(i_o, A) = \mathit{dom}_\perp(A)$ for all attributes $A \in \mathit{attr}(O) - \mathit{id}(O)$ and all $O \in V$, and $\nu(i_o, A) = \mathit{dom}(A)$ for all key attributes $A \in \mathit{attr}(O) \cap \mathit{id}(O)$ and all $O \in V$.

The TPC-H Data Model as Property Graph



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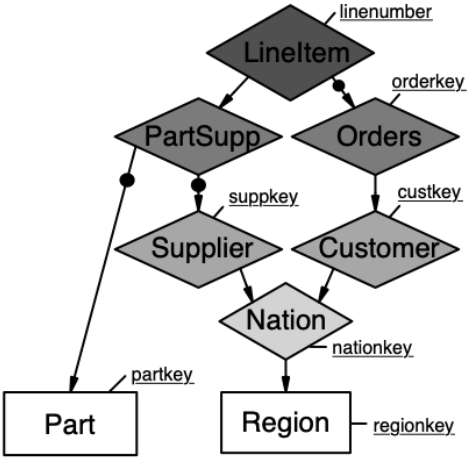

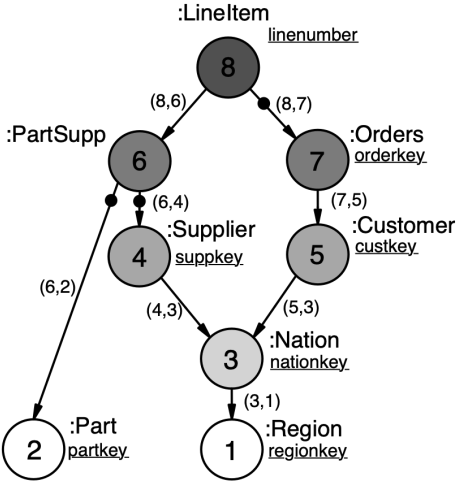

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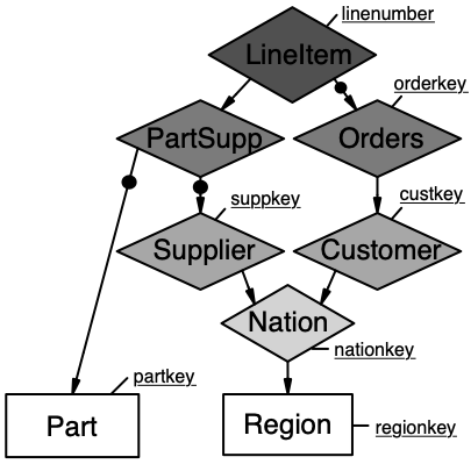
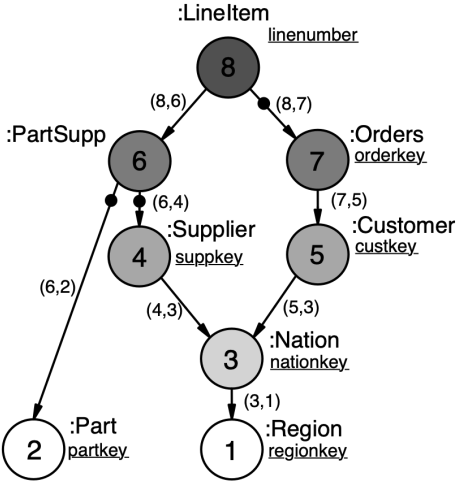




E/R instances (diagrams) are directed acyclic property graphs (models)

E/R Diagrams from a Sub-class of PG-Schema

Input: Object Type $O = (comp(O), attr(O), id(O))$

- $comp(O) = \{\ell_1{:}O_1, \ldots, \ell_k{:}O_k\}$
- $attr(O) = \{A_1, \ldots, A_l\}$, and
- $id(O) = \{\ell_{i_1}{:}O_{i_1}, \ldots, \ell_{i_m}{:}O_{i_m}, A_{j_1}, \ldots, A_{j_n}\}$

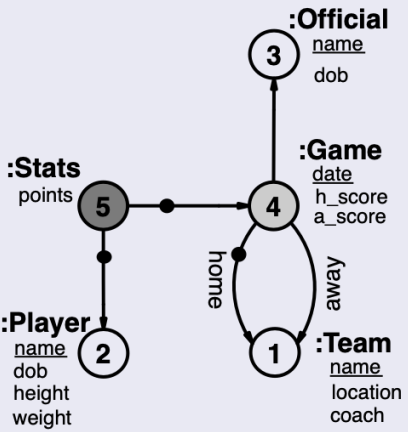
Output

- Node type $(O\text{-}Type : O\{A_1\ dom(A_1), \ldots, A_l\ dom(A_l)\})$, f
- For $i = 1, \ldots, k$, we add the
  - E/R link as edge type $(:O\text{-}Type) - [:\ell_i] \rightarrow (:O_i\text{-}Type)$
- PG-key:
  `FOR` $(x : O\text{-}Type)$ `ID` $y_{i_1}, \ldots, y_{i_m}, x.A_{j_1}, \ldots, x.A_{j_n}$ `WITHIN`
  $(x) - [:\ell_{i_1}] \rightarrow (y_{i_1}{:}O_{i_1}\text{-}Type), \ldots, (x) - [:\ell_{i_m}] \rightarrow (y_{i_m}{:}O_{i_m}\text{-}Type)$.

The translation of entity types is the special case where $comp(O) = \emptyset$

Example: E/R Diagram and Matching PG-Schema

E/R Diagram PLAY



PG-Schema (neglect domain definitions)

```
CREATE GRAPH TYPE PLAY STRICT {
```

- (pType:PLAYER{name, dob, height, weight}),
  `FOR x:pType IDENTIFIER x.name,`
- (tType:TEAM{name, coach, location}),
  `FOR x:tType IDENTIFIER x.name,`
- (oType:OFFICIAL{name, license}),
  `FOR x:oType IDENTIFIER x.name,`
- (gType:GAME{date,home_score,away_score}),
  (x)-[home]→(y:tType), (x)-[away]→(v:tType),
  (x)-[ ]→(w:oType),
  `FOR x:gType IDENTIFIER x.date,y WITHIN`
  (x),(x-[home]→y:tType)
- (sType:STATS{points}),
  (x)-[ ]→(y:gType), (x)-[ ]→(z:pType)
  `FOR x:sType IDENTIFIER y,z WITHIN`
  (x-[ ]→y:gType), (x-[ ]→z:pType)}

Pitstop 1

Contributions

- E/R Schema $\equiv$ E/R Diagrams $\equiv$ E/R Graph Models
- E/R Diagrams are directed acyclic property graphs and well-designed PG-Schema
- Compare:
  - Every XML Schema is an XML document
  - Every E/R Graph Model is a Property graph

Pitstop 1

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E/R Modeling is a Methodology for Designing Property Graphs Well

Pitstop 1

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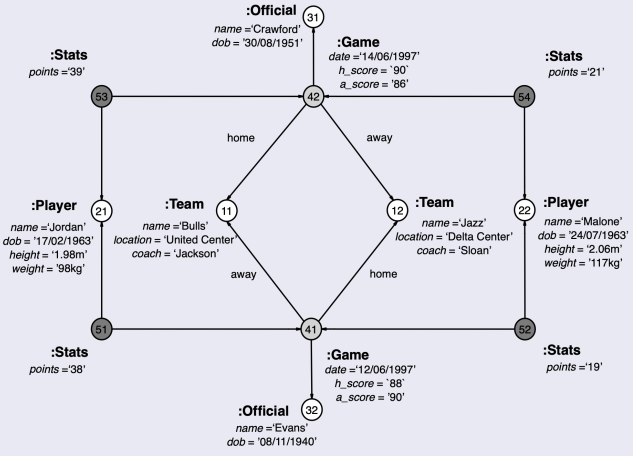
E/R Modeling is a Methodology for Designing Property Graphs Well

Next step: We want a graph semantics for PG modeling with E/R graph model

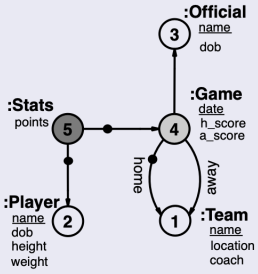
E/R Graphs as Instances of E/R Graph Models

Graph Semantics for E/R Models

E/R Graph

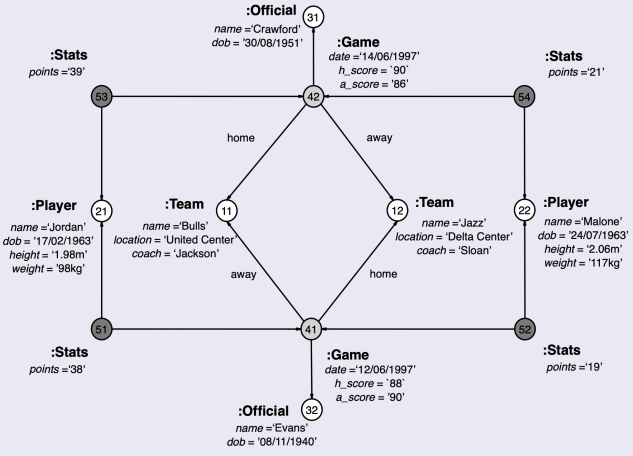


E/R Graph model

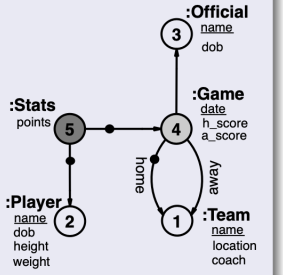


Graph Semantics for E/R Models

E/R Graph

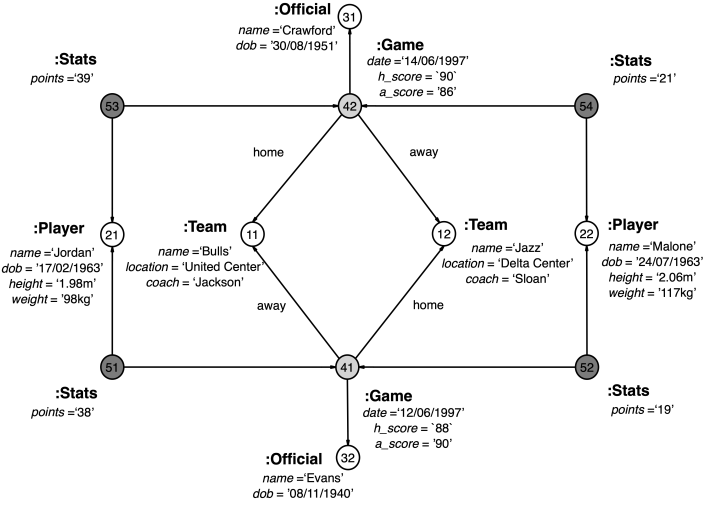


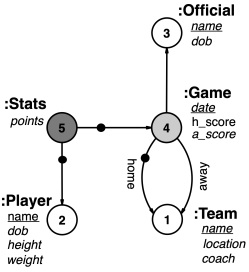
E/R Graph model



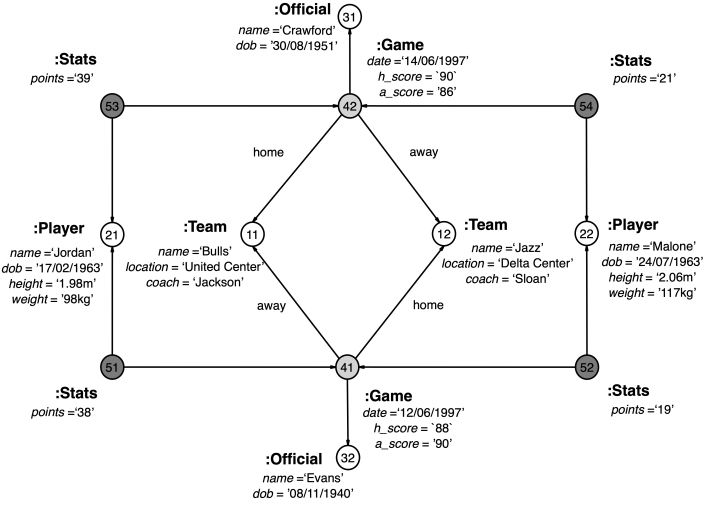
E/R graphs are instances of E/R graph models

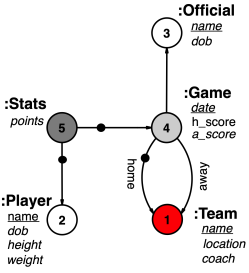
Illustrating how E/R graphs comply with E/R model graphs



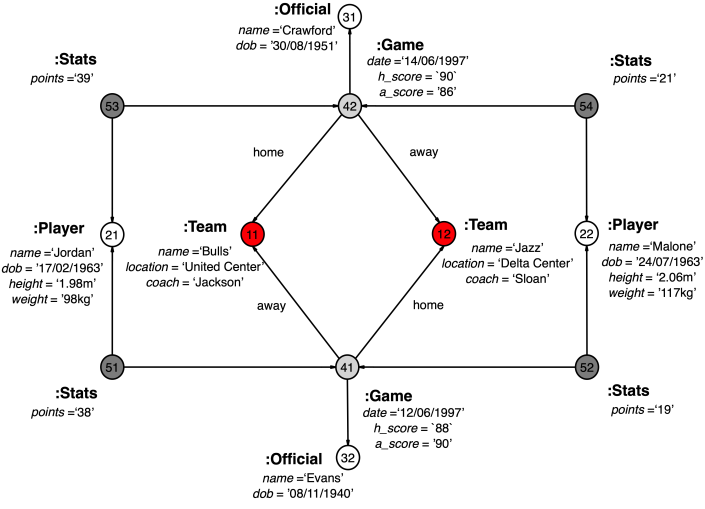


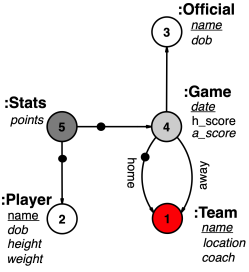
Homomorphism: Example of node image



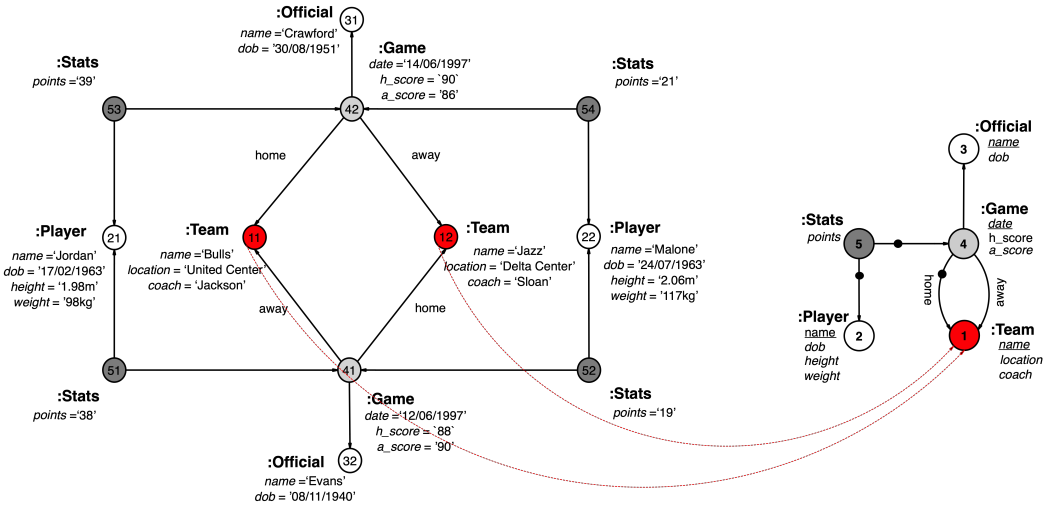


Homomorphism: Example of graph nodes mapped to node image

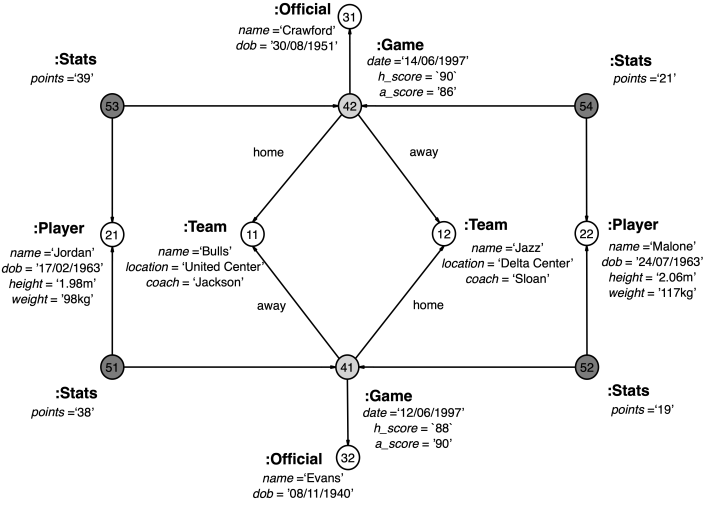


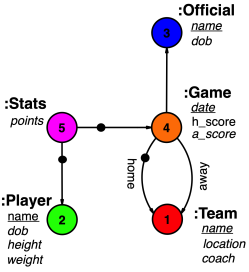


Homomorphism: Example of mapping graph nodes to node image

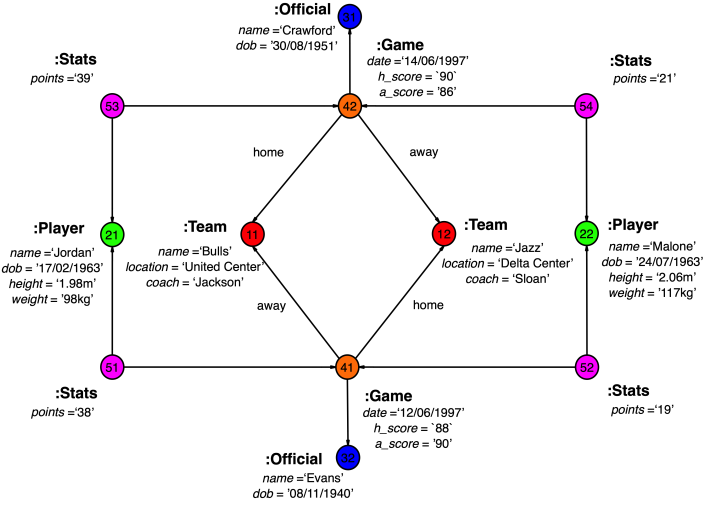


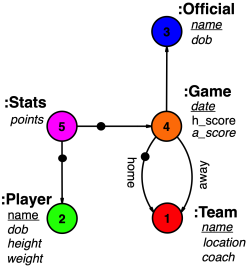
Homomorphism:Image for Node Mapping





Homomorphism: Mapping of graph nodes to graph model nodes





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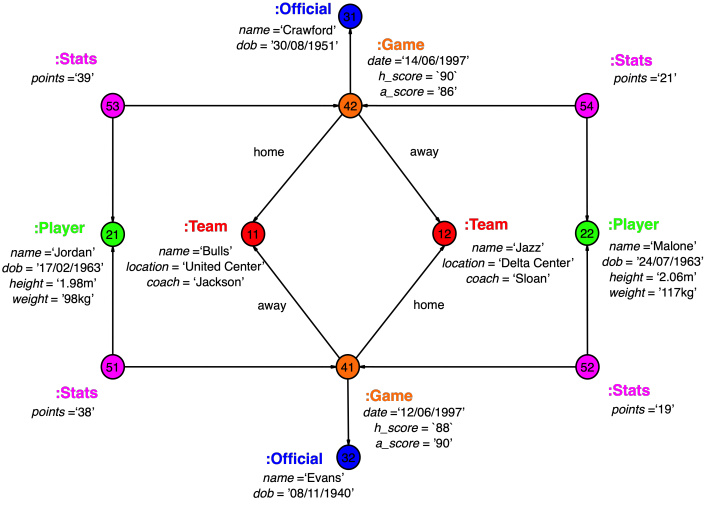
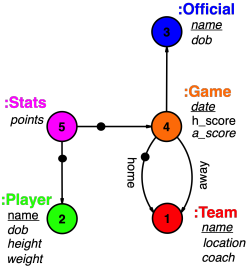
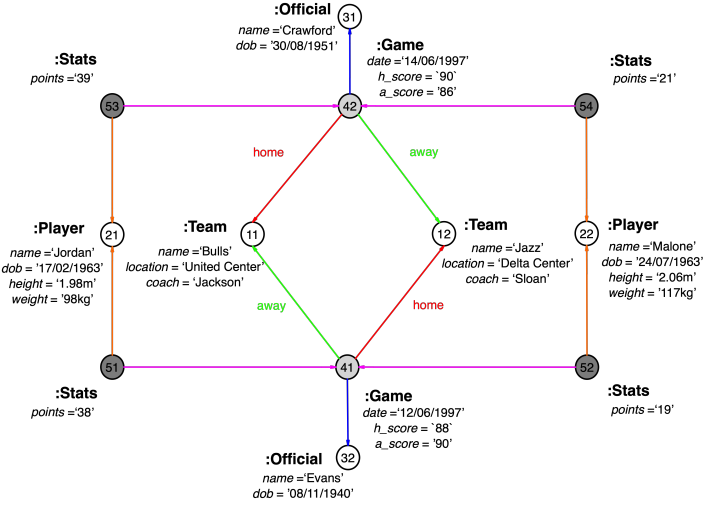
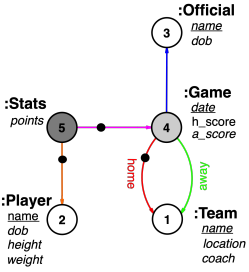


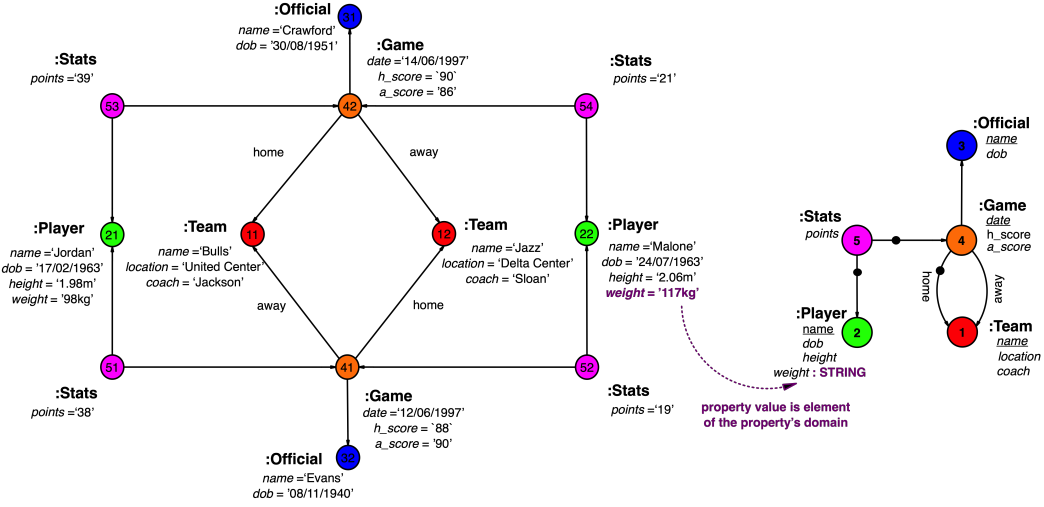


Homomorphism: Extension to edges

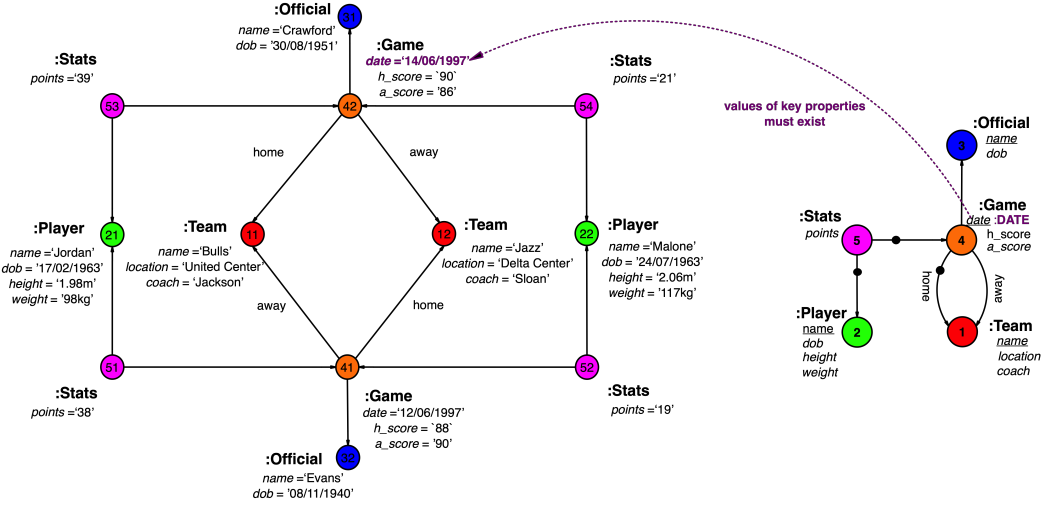




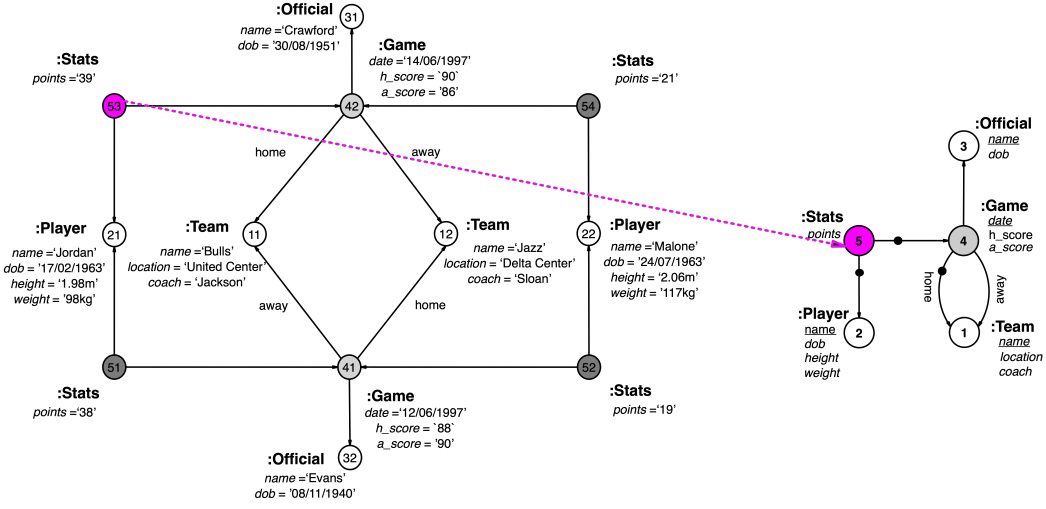
Homomorphism: Domain integrity



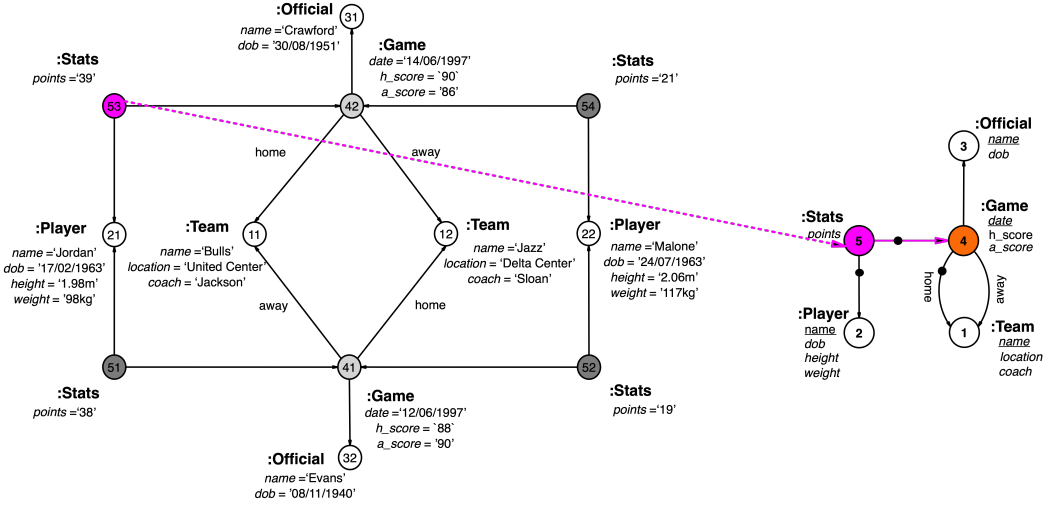
Homomorphism: Values of key properties must exist



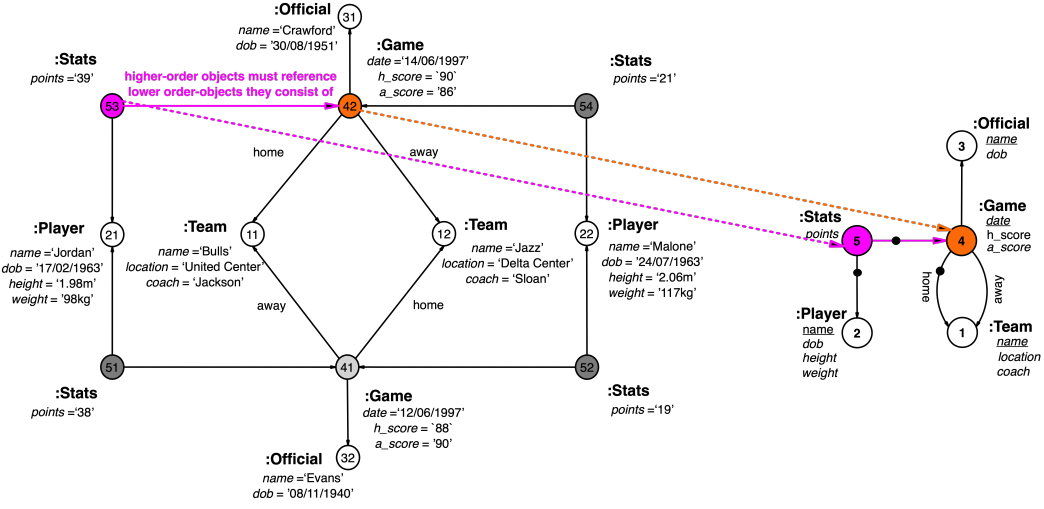
Homomorphism: Referential integrity for object



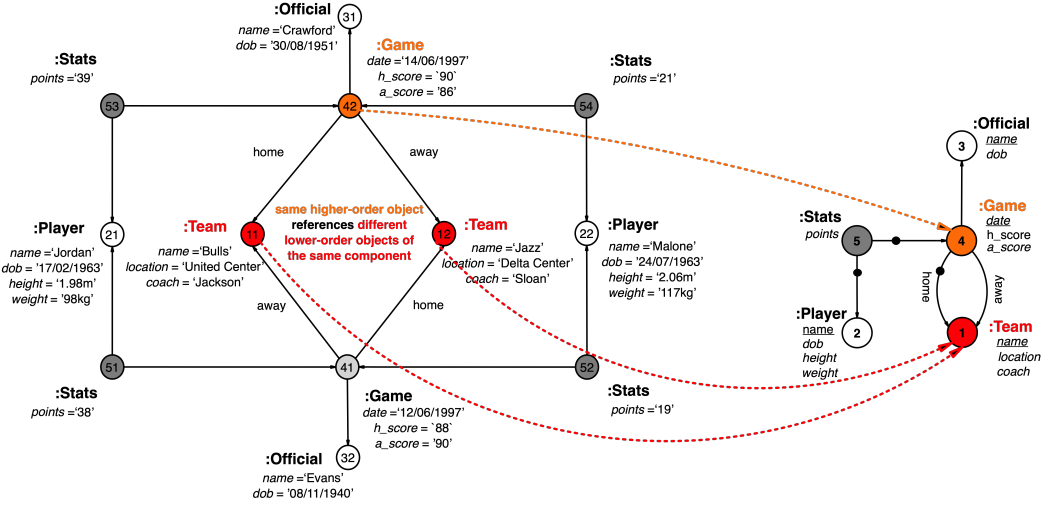
Homomorphism: Referential integrity for object of higher-order type



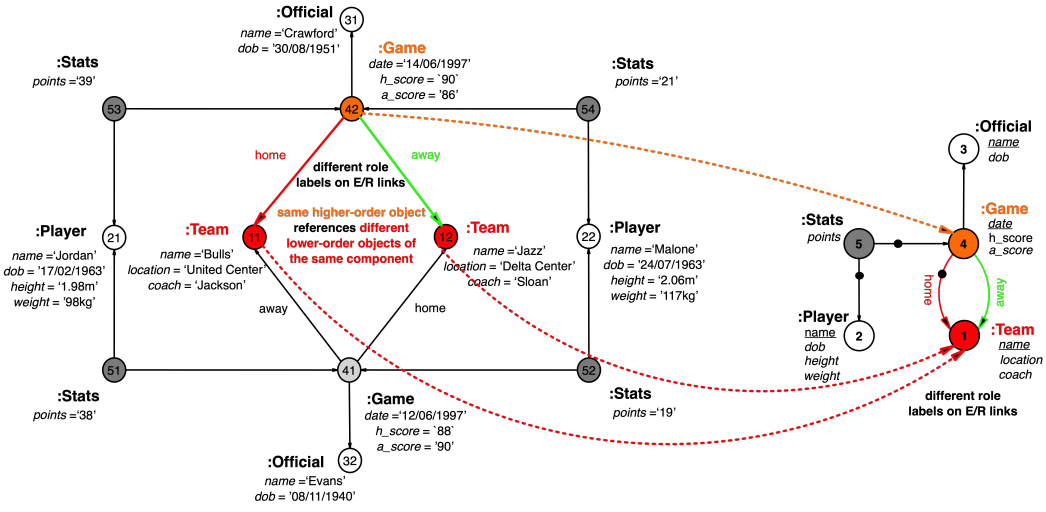
Homomorphism: Referential integrity for higher-order object with reference



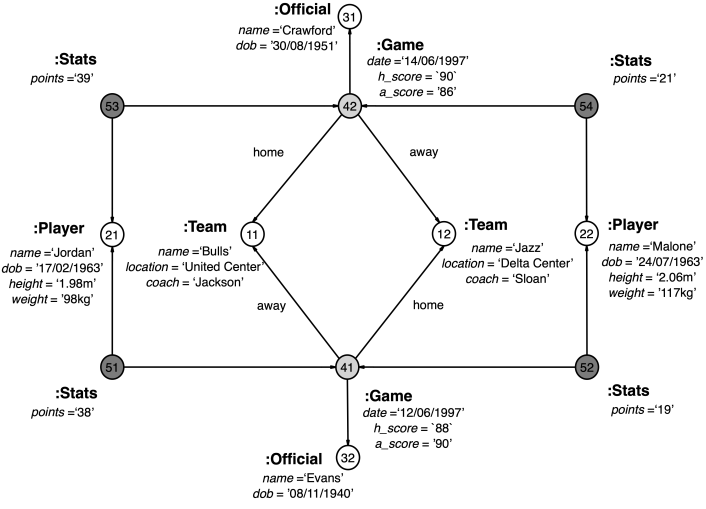
Homomorphism: Single-valued component

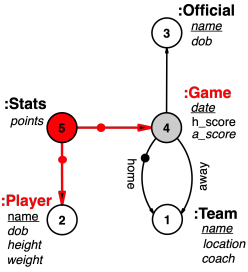


Homomorphism: Single-valued component distinguished by role labels

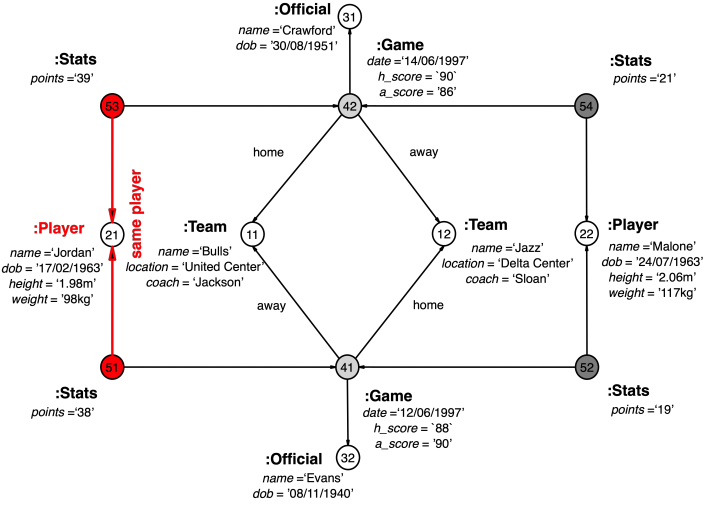


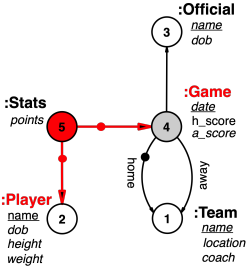
Homomorphism: Entity integrity on key STATS({PLAYER, GAME}, ∅)



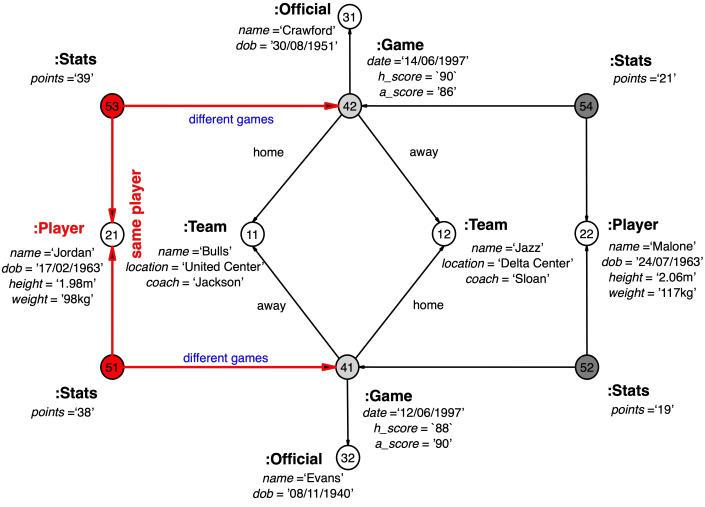


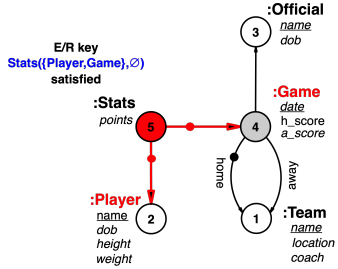
Homomorphism: Entity integrity on key $\textsc{Stats}(\{\textsc{Player}, \textsc{Game}\}, \emptyset)$



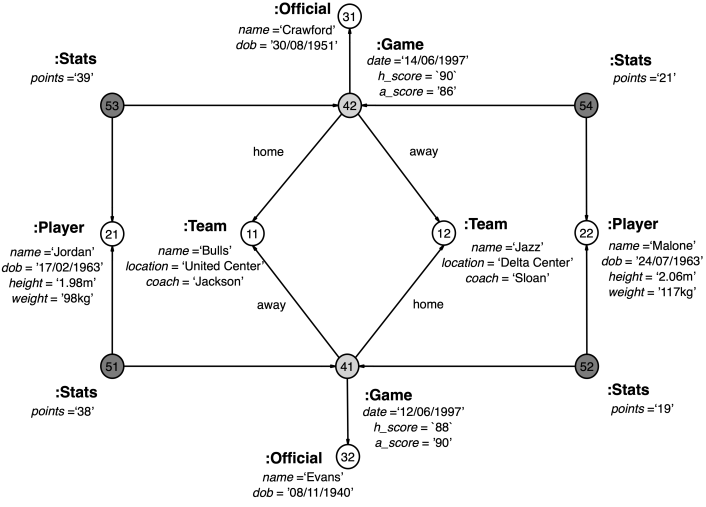


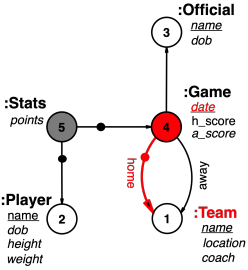
Homomorphism: Entity integrity on key STATS({PLAYER, GAME}, ∅)



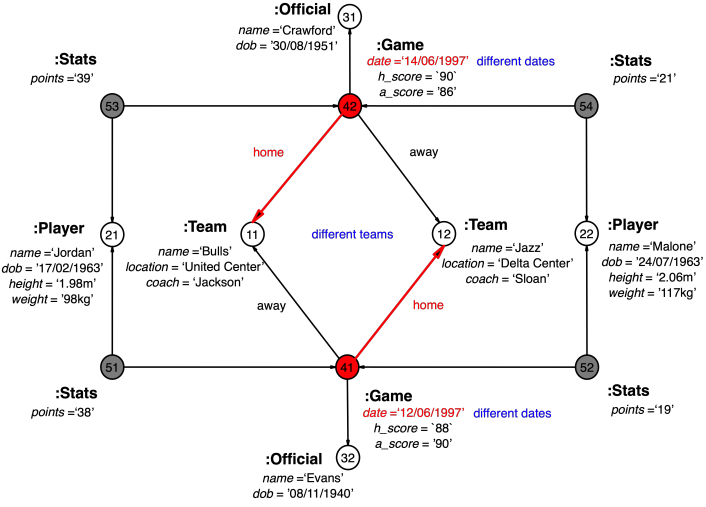


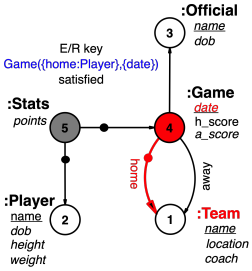
Homomorphism: Entity integrity on key GAME({*home*:TEAM}, {*date*})





Homomorphism: Entity integrity on key GAME({*home*:TEAM}, {*date*})





E/R graphs: Formal definition of E/R model graph compliance

E/R diagram $\mathcal{D} = (V, E)$ for E/R schema $\mathcal{S}$

- $\mathcal{O}_G$ as set of object identifiers,
- $\mathcal{L}_G \subseteq \mathcal{L}_D - \{\bullet\}$ as set of labels,
- $\mathcal{K}_G \subseteq \mathcal{K}_D$ as set of properties,
- $\mathcal{N}_G \subseteq \bigcup_{A \in attr(O), O \in \mathcal{S}} dom(A)$ as set of values

E/R graphs: Formal definition of E/R model graph compliance

A property graph $G = (V_G, E_G, \eta_G, \lambda_G, \nu_G)$ over $\mathcal{O}_G$, $\mathcal{L}_G$, $\mathcal{K}_G$, and $\mathcal{N}_G$ is called an *E/R graph* for $\mathcal{G}_D$ iff there is some function $h : \mathcal{O}_G \to \mathcal{O}_D$ such that:

E/R graphs: Formal definition of E/R model graph compliance

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1. $h(V_G) \subseteq V_D$ and $h(E_G) \subseteq E_D$,

E/R graphs: Formal definition of E/R model graph compliance

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1. $h(V_G) \subseteq V_D$ and $h(E_G) \subseteq E_D$,
2. $\forall o \in E_G \ (\eta_G(o) = (o_1, o_2) \Rightarrow h(\eta_G(o)) = (h(o_1), h(o_2)) = \eta_D(h(o)))$ (edge-preserving),

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3. $\forall o \in V_G, \ \lambda_G(o) = \lambda_D(h(o))$ (node label-preserving),

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3. $\forall o \in V_G, \ \lambda_G(o) = \lambda_D(h(o))$ (node label-preserving),
4. $\forall o \in E_G, l \neq \bullet, \ \lambda_G(o) = \{l\}$ if and only if $l \in \lambda_D(h(o))$ (role label-preserving),

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4. $\forall o \in E_G, l \neq \bullet, \ \lambda_G(o) = \{l\}$ if and only if $l \in \lambda_D(h(o))$ (role label-preserving),
5. $\forall v \in V_G, p \in \mathcal{K}_G \ (\nu_G(v, p) = val \Rightarrow val \in \nu_D(h(v), p))$ (type-preserving),

E/R graphs: Formal definition of E/R model graph compliance

A property graph $G = (V_G, E_G, \eta_G, \lambda_G, \nu_G)$ over $\mathcal{O}_G$, $\mathcal{L}_G$, $\mathcal{K}_G$, and $\mathcal{N}_G$ is called an *E/R graph* for $\mathcal{G}_D$ iff there is some function $h : \mathcal{O}_G \to \mathcal{O}_D$ such that:

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2. $\forall o \in E_G \; (\eta_G(o) = (o_1, o_2) \Rightarrow h(\eta_G(o)) = (h(o_1), h(o_2)) = \eta_D(h(o)))$ (edge-preserving),
3. $\forall o \in V_G, \; \lambda_G(o) = \lambda_D(h(o))$ (node label-preserving),
4. $\forall o \in E_G, l \neq \bullet, \; \lambda_G(o) = \{l\}$ if and only if $l \in \lambda_D(h(o))$ (role label-preserving),
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   (key properties must exist),

E/R graphs: Formal definition of E/R model graph compliance

A property graph $G = (V_G, E_G, \eta_G, \lambda_G, \nu_G)$ over $\mathcal{O}_G$, $\mathcal{L}_G$, $\mathcal{K}_G$, and $\mathcal{N}_G$ is called an *E/R graph* for $\mathcal{G}_D$ iff there is some function $h : \mathcal{O}_G \to \mathcal{O}_D$ such that:

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9. $\forall O \in V_D$ such that $id(O) = C \cup K$ with $C = \{O_1, \ldots, O_n\}$ and $K = \{K_1, \ldots, K_m\}$, G satisfies the E/R key $O(C, K)$ (entity integrity).

E/R keys

E/R keys for an E/R graph model $\mathcal{G}_D$ of an E/R diagram $\mathcal{D}$

- For every object type $O \in V_D$, it is an expression $O(C, K)$ with
  - $C = \{O_1, \ldots, O_n\} \subseteq comp(O)$ [key components]
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- An *E/R graph* G for $\mathcal{G}_D$ *satisfies* $O(C, K)$ iff for all $o, o' \in V_G$ such that
  - $(o, o_i), (o', o_i) \in E_G$ for $i = 1, \ldots, n$,
  - $\downarrow = \nu_G(o, K_j) = \nu_G(o', K_j) = \downarrow$ for $j = 1, \ldots, m$,
  - $\lambda_G(o) = O = \lambda_G(o')$, and $\lambda_G(o_i) = O_i$ for $i = 1, \ldots, n$,

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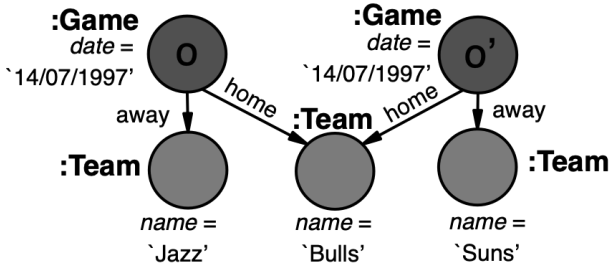
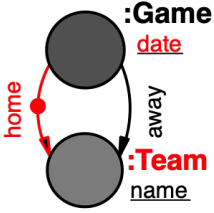
- E/R keys $E(C, K)$ where $C = \emptyset$ are called *property keys*
- E/R keys do not stipulate object uniqueness if any key property is undefined
- Principle of entity integrity requires some E/R key for each object type
  - ensures objects can be identified uniquely and accessed efficiently
- We recommend specifying all E/R keys that express integrity rules

E/R Key Example $\textsc{Game}(\{home{:}\textsc{Team}\}, \{date\})$

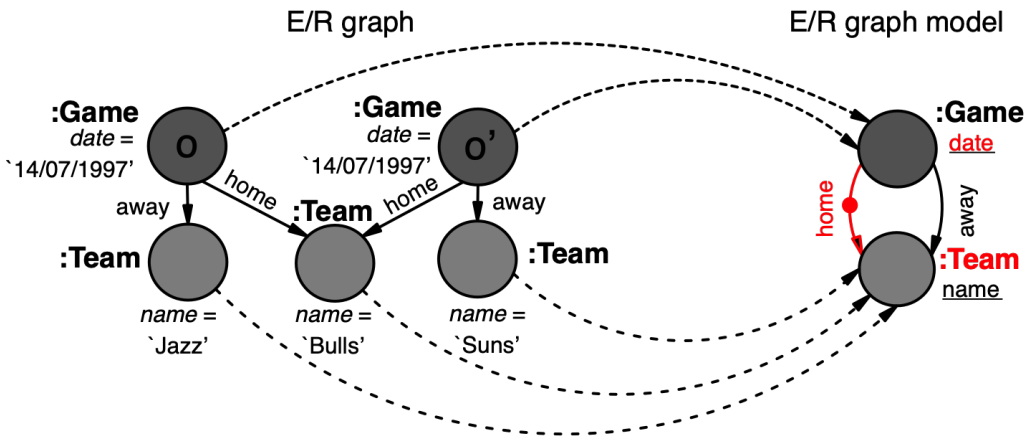
E/R graph



E/R graph model



E/R Key Example GAME({*home*:TEAM}, {*date*})



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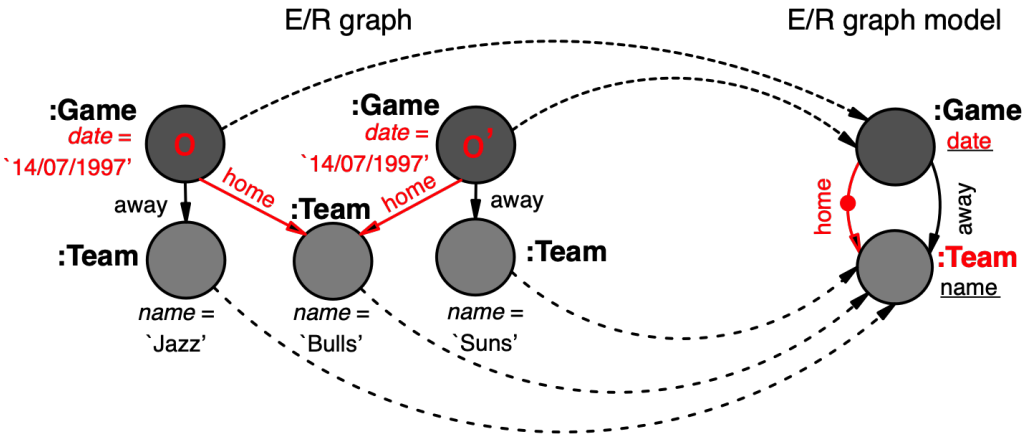

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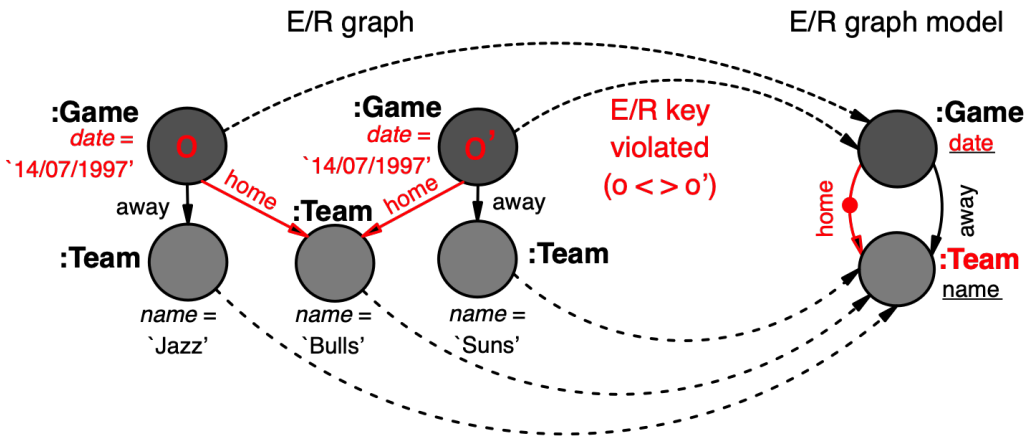
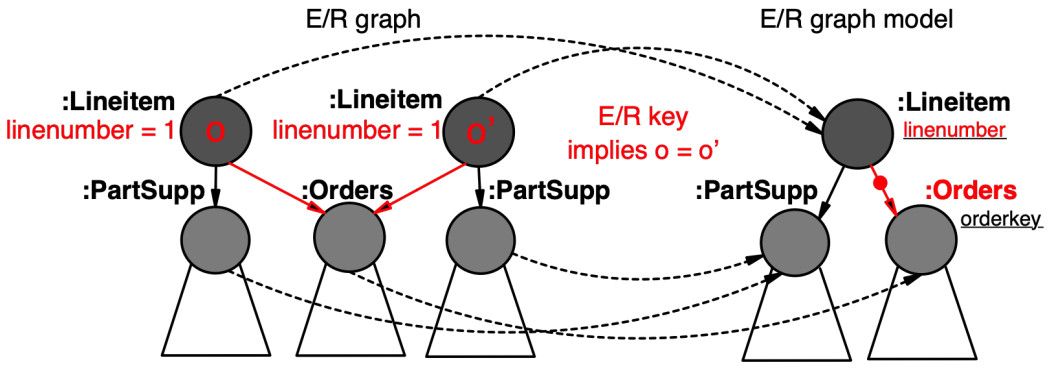


Another E/R Key Example LINEITEM({ORDERS}, {*linenumber*})



Pitstop 2

Contributions

- E/R Graphs are natural instances of E/R graph models, defining homomorphisms
- E/R Graphs provide a graph semantics for E/R schemata and diagrams

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E/R Graphs: A Natural Semantics for Well-designed Databases

Entity and Referential Integrity

PG-Key

When specifying some set of PG-keys, what other PG-keys do you specify implicitly?

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Relational translation of relational database into property graph

- (S, Σ_S): finite set of keys and foreign keys over relational database schema S
- Translate into $\mathcal{G}_S$ with a set $\Sigma_{\mathcal{G}_S} = \{\sigma_{\mathcal{G}_S} \mid \sigma \in \Sigma_S\}$ of PG-keys by mapping
  - every relation schema $R \in \mathcal{S}$ to a vertex v_R with label $:R$,

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    `FOR` x`:`R `IDENTIFIER` $x.A_1, \ldots, x.A_n$
  - every foreign key $\sigma = R[A_1, \ldots, A_n] \subseteq S[B_1, \ldots, B_n]$ to a PG-key $\sigma_{\mathcal{G}_S}$ over v_R:
    `FOR` x`:`R `MANDATORY` y `WITHIN` $(x), (y{:}S)$ `WHERE` $x.A_1 = y.B_1, \ldots, x.A_n = y.B_n$.

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- Extend mapping to relational databases over (S, Σ_S) by translating every record r over schema R to a node v_r with label $:R$ and the property-value pairs $A = r(A)$ for every $A \in R$

Reasoning about relational PG-Keys is infeasible

The *implication problem for relational PG-keys*

Decide whether for every given

$$(\mathcal{S}, \Sigma_{\mathcal{S}} \cup \{\varphi\}),$$

every instance $\mathcal{I}(\mathcal{G}_{\mathcal{S}})$ over $\mathcal{G}_{\mathcal{S}}$ that satisfies all elements of $\Sigma_{\mathcal{G}_{\mathcal{S}}}$ will also satisfy $\varphi_{\mathcal{G}_{\mathcal{S}}}$

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Theorem

The implication problem of relational PG-keys is undecidable.

E/R Keys and their Implication Problem

E/R Keys as PG-Keys

E/R key $O(\{O_1, \ldots, O_n\}, \{K_1, \ldots, K_m\})$ for $\mathcal{G}_D$ is satisfied by an E/R graph G for $\mathcal{G}_D$ if and only if G satisfies the following PG-key

$$\texttt{FOR } (x{:}O) \texttt{ IDENTIFIER } x.K_1, \ldots, x.K_m, y_1, \ldots, y_n \texttt{ WITHIN}$$
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Implication problem for E/R keys

Decide whether for every E/R graph model $\mathcal{G}_D$ and every set $\Sigma \cup \{\varphi\}$ of E/R keys for $\mathcal{G}_D$, every E/R graph that satisfies all E/R keys in Σ also satisfies φ.

Efficient Reasoning about E/R Keys

Theorem (Axiomatic Characterization)

The implication of E/R keys is finitely axiomatized by the extension rule:

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Corollary (Algorithmic Characterization)

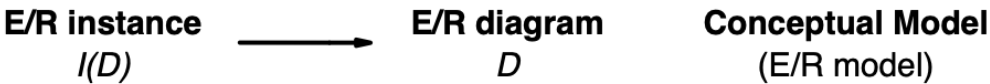
The implication of E/R keys is decidable in time linear in the input.

E/R keys

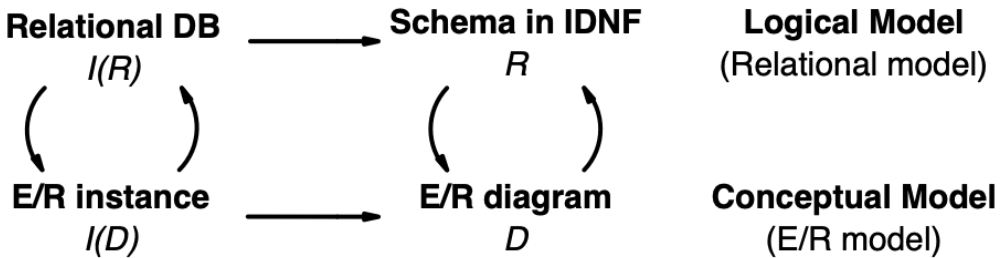
form an efficient fragment of PG-Key for managing entity and referential integrity of well-designed property graphs

E/R graphs and their models
unify
conceptual, logical and graph modeling

Well-designed Databases: Conceptual Perspective



Well-designed Databases: Conceptual, Logical Perspective



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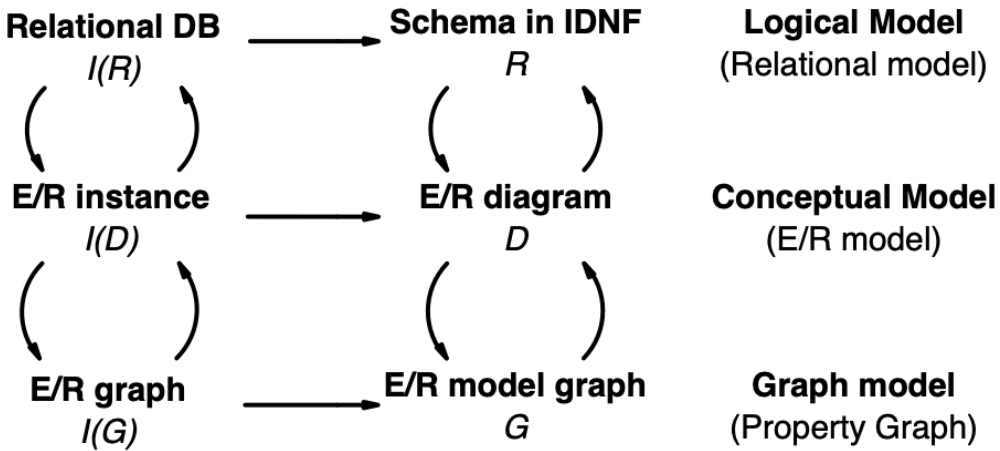
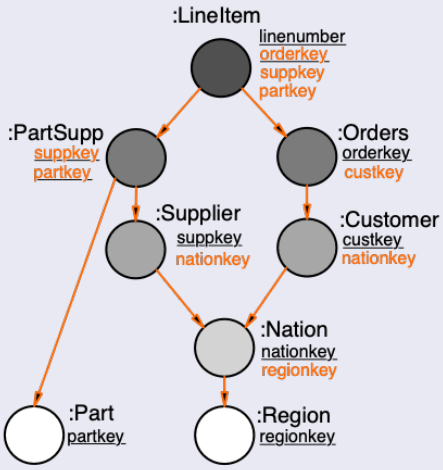


Taking the Management of Entity and Referential Integrity Management to the Next Level

Integrity for Property Graphs: Spoilt for Choice

E/R graph model



E/R Keys

- Each E/R link provides a choice:
  - duplicate key properties of target node on source node, or
  - keep the E/R link
- for the first choice:
  - E/R links are redundant
  - E/R keys reduce to property keys
- for the second choice:
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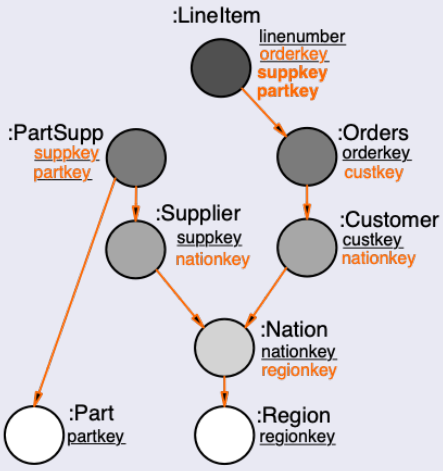

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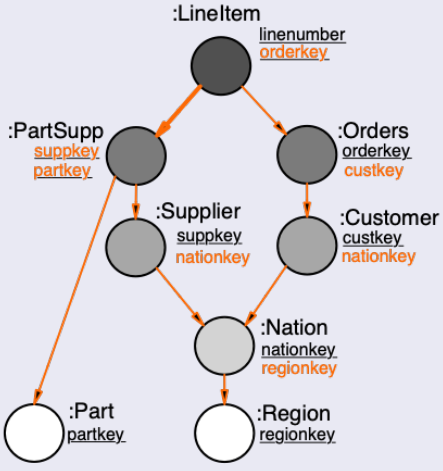

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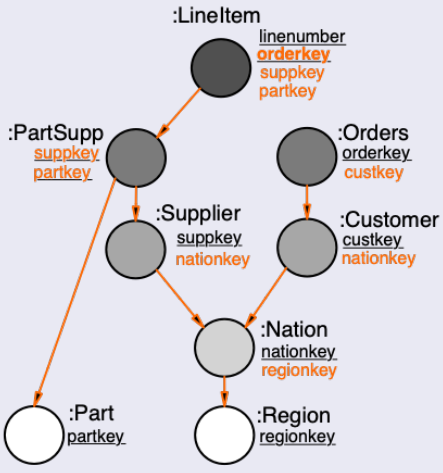

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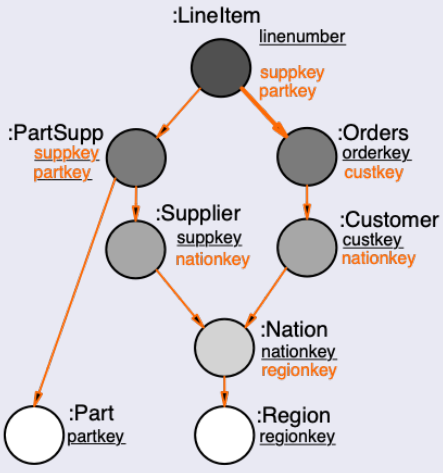
Integrity for Property Graphs: Spoilt for Choice

E/R graph model

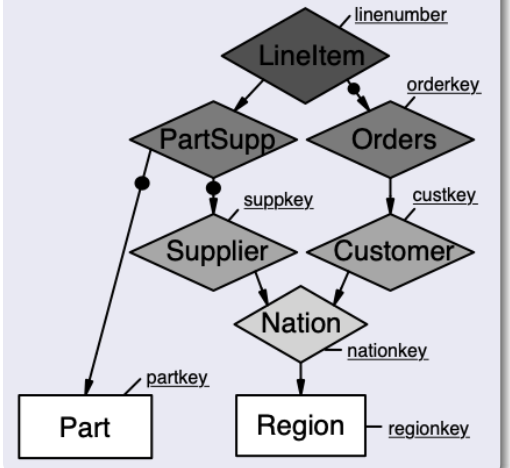


E/R Keys

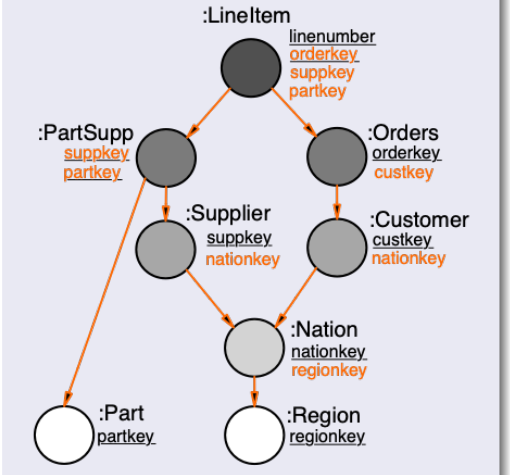
- Each E/R link provides a choice:
  - duplicate key properties of target node on source node, or
  - keep the E/R link
- for the first choice:
  - E/R links are redundant
  - E/R keys reduce to property keys
- for the second choice:
  - no property redundancy
  - E/R keys may require key components

Relational semantics: duplicate key properties and use property keys

E/R graph model

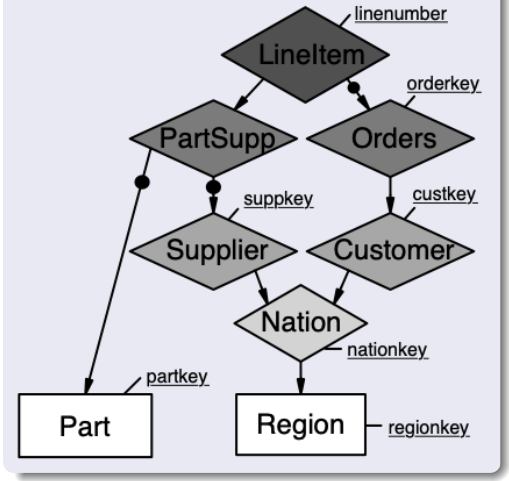


Relational semantics

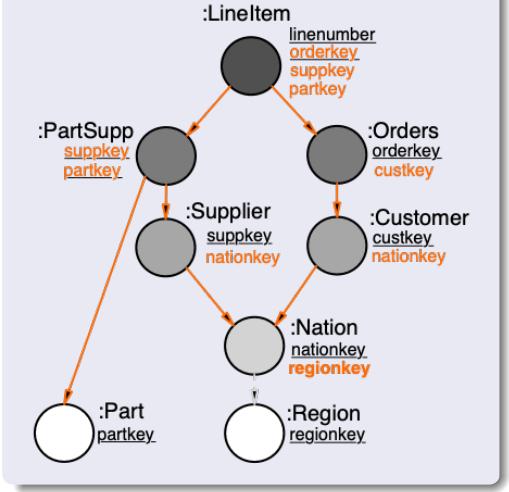


Relational semantics: duplicate key properties and use property keys

E/R graph model

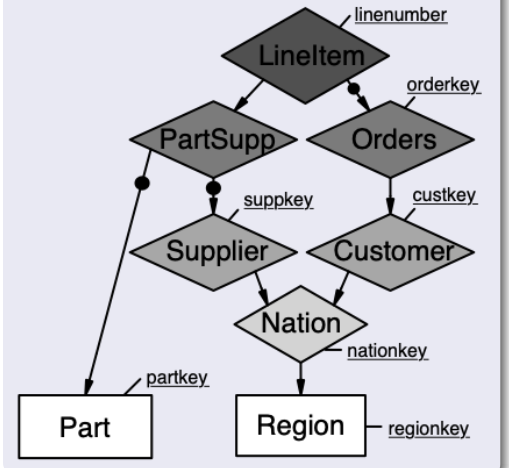


Relational semantics

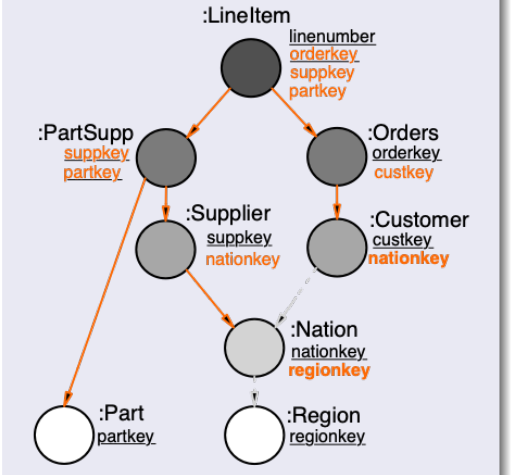


Relational semantics: duplicate key properties and use property keys

E/R graph model

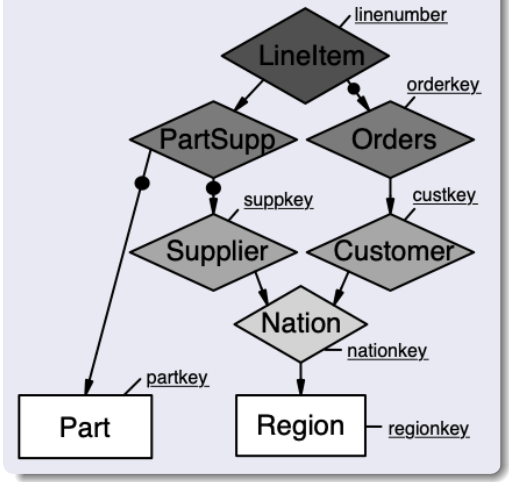


Relational semantics

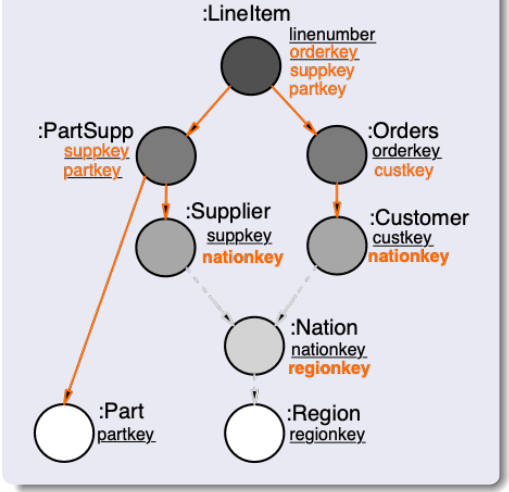


Relational semantics: duplicate key properties and use property keys

E/R graph model

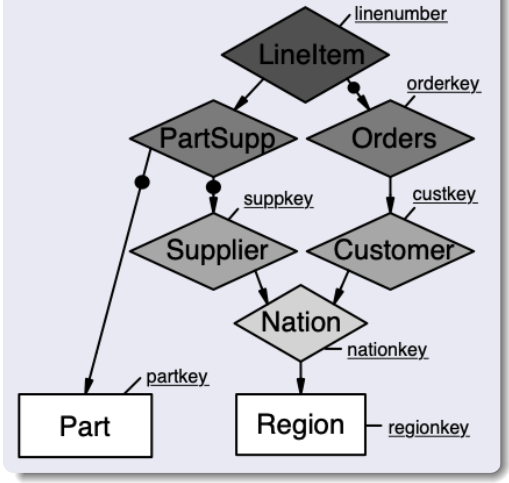


Relational semantics

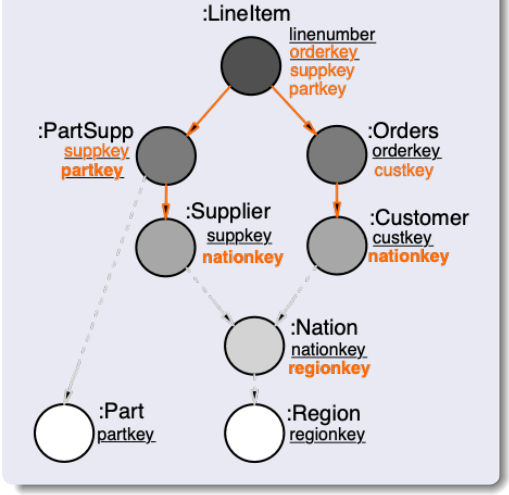


Relational semantics: duplicate key properties and use property keys

E/R graph model

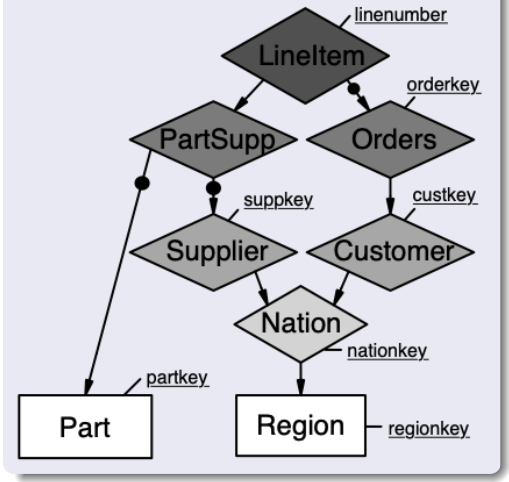


Relational semantics

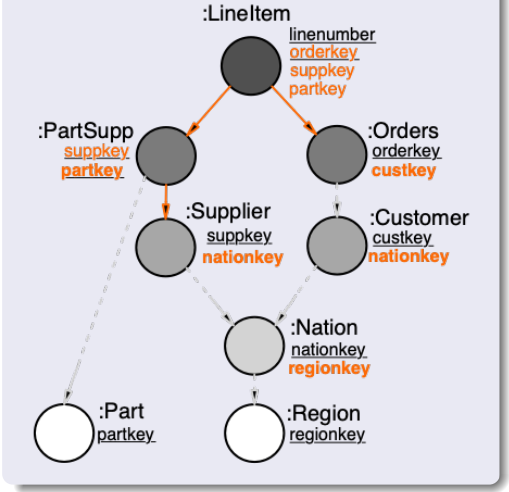


Relational semantics: duplicate key properties and use property keys

E/R graph model

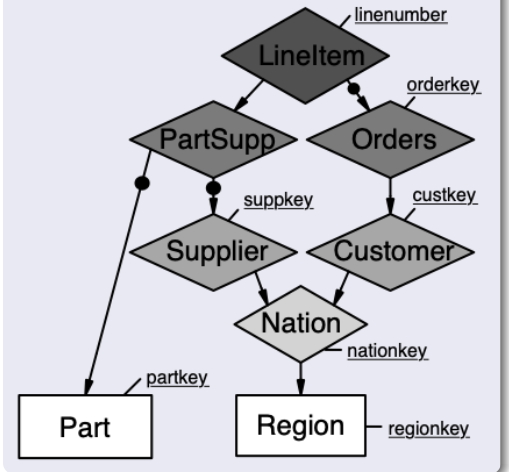


Relational semantics

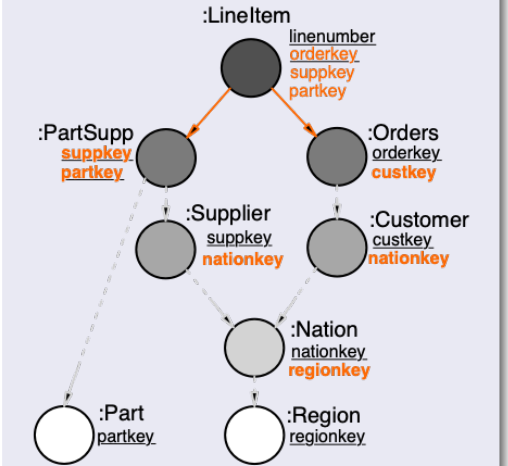


Relational semantics: duplicate key properties and use property keys

E/R graph model

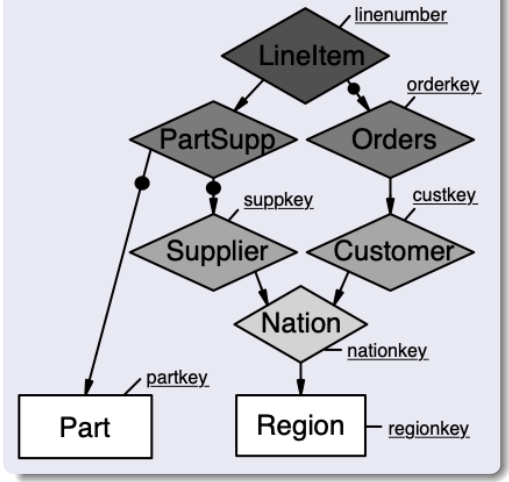


Relational semantics

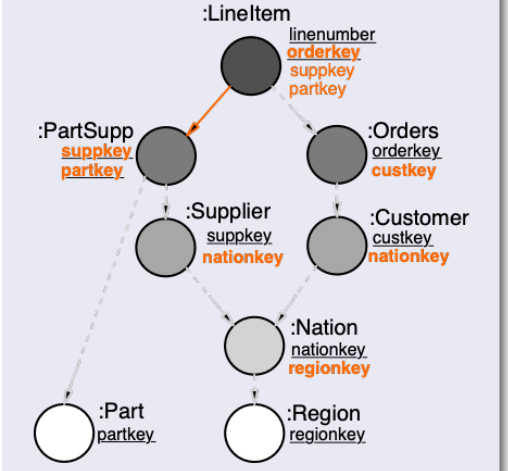


Relational semantics: duplicate key properties and use property keys

E/R graph model

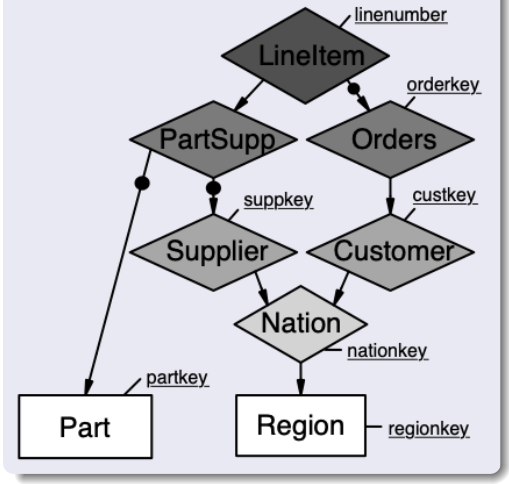


Relational semantics

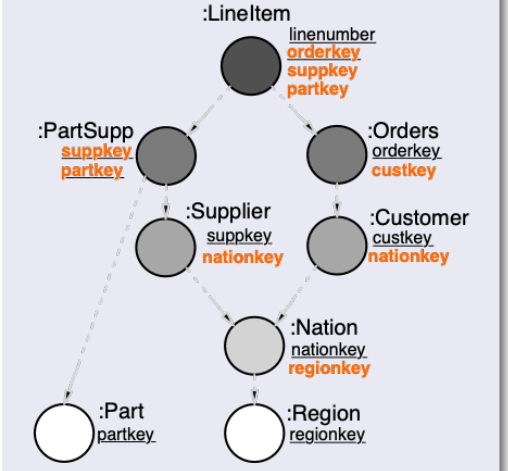


Relational semantics: duplicate key properties and use property keys

E/R graph model

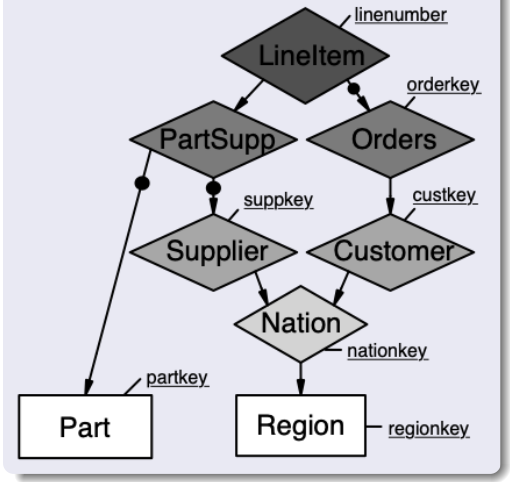


Relational semantics

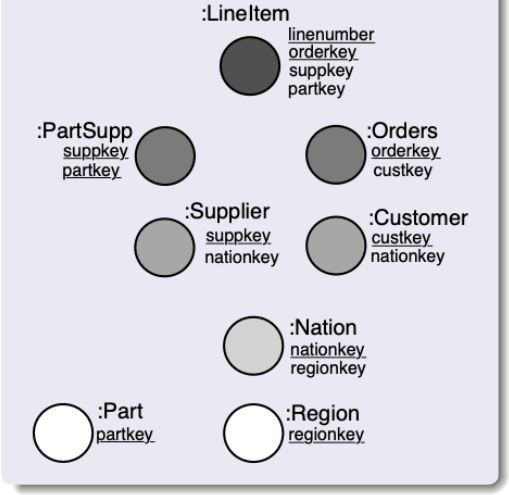


Relational semantics: duplicate key properties and use property keys

E/R graph model

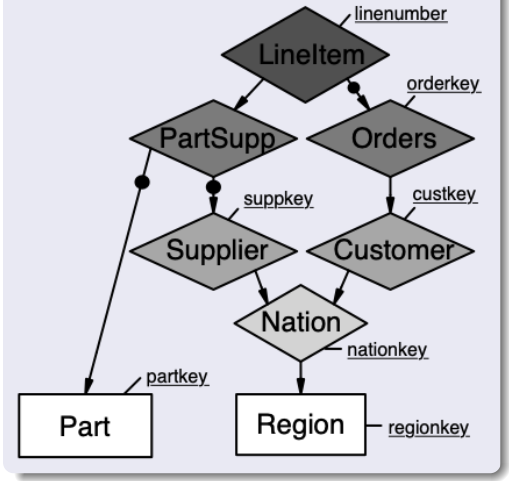


Relational semantics

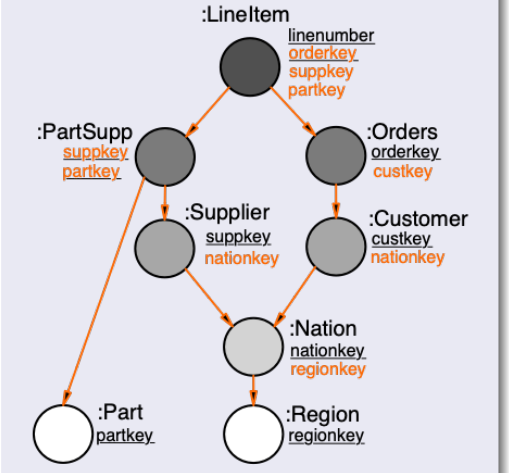


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

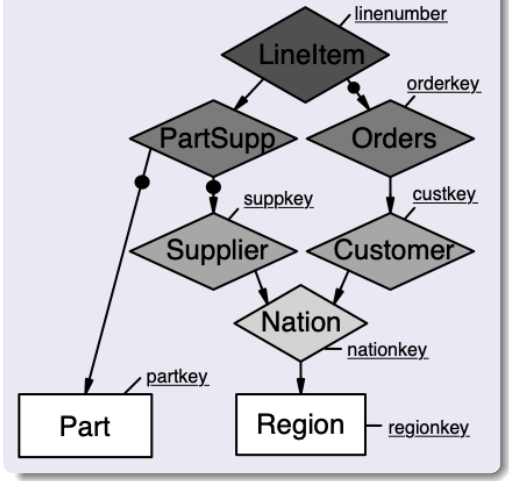


Graph semantics

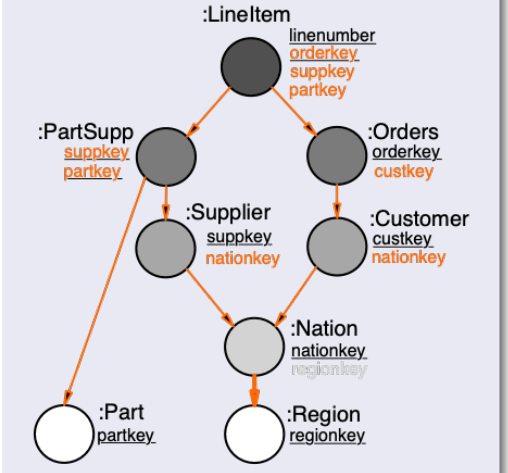


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

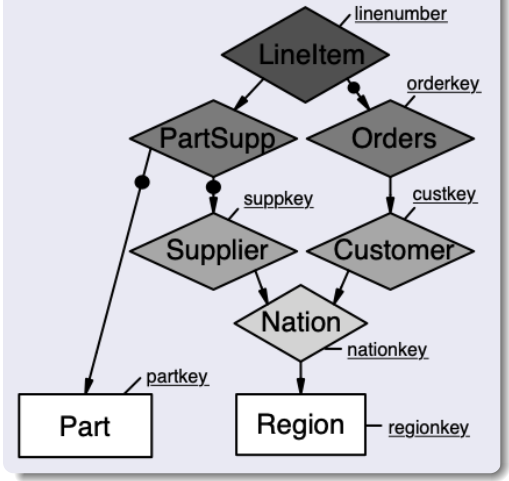


Graph semantics

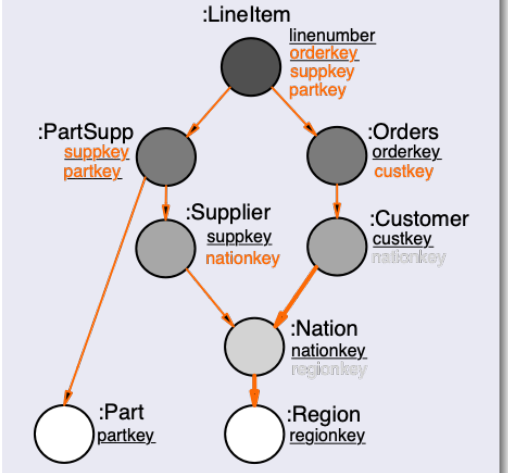


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

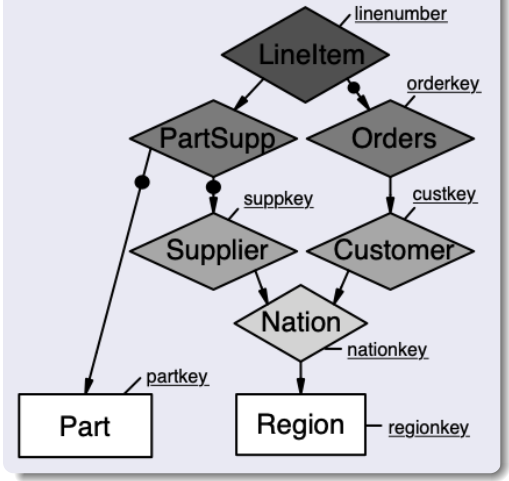


Graph semantics

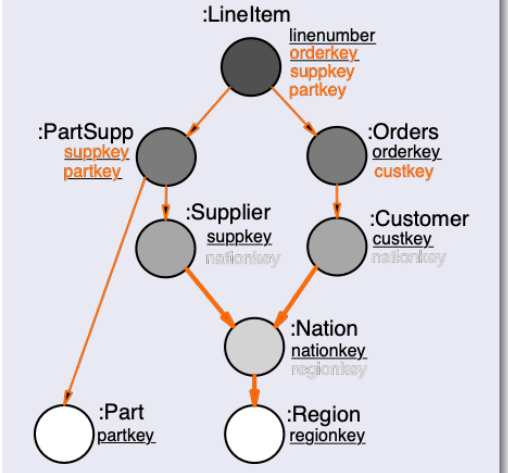


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

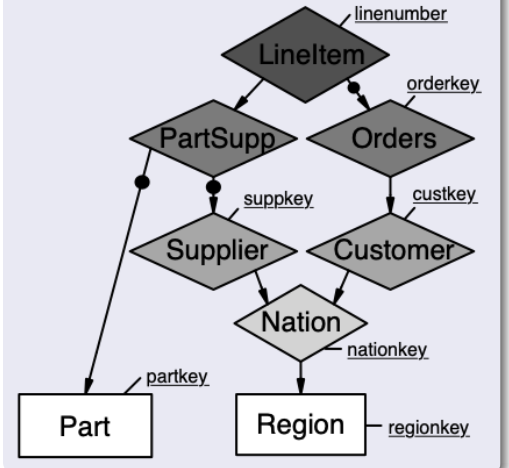


Graph semantics

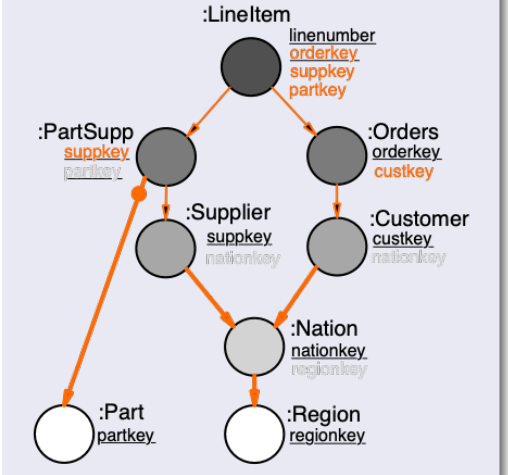


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

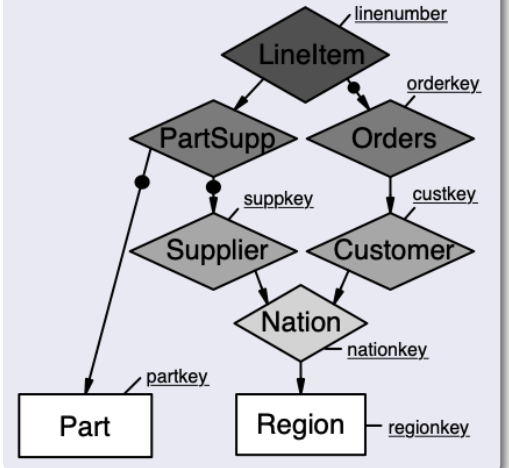


Graph semantics

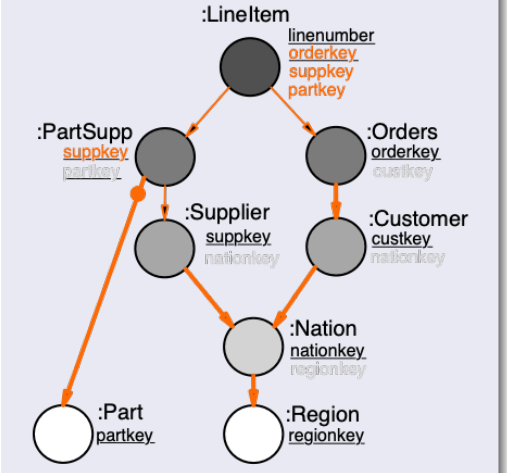


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

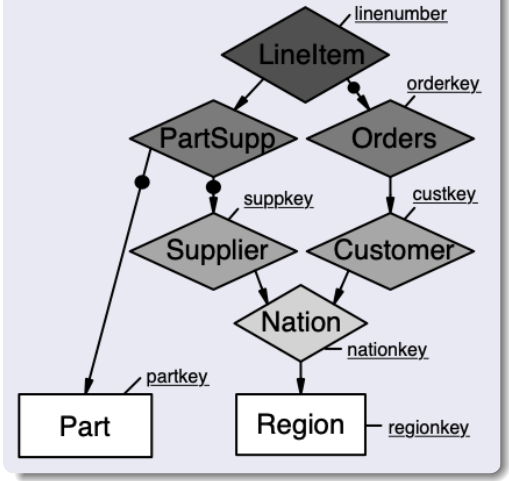


Graph semantics

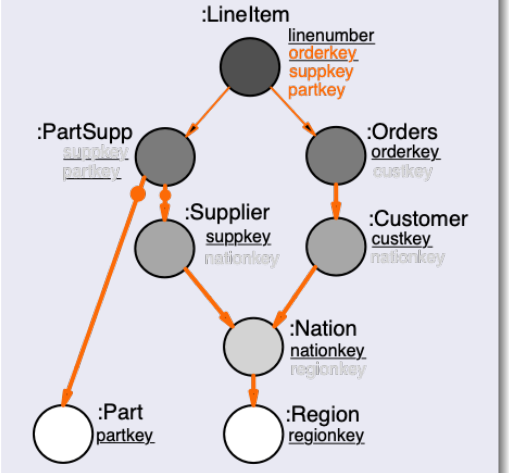


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

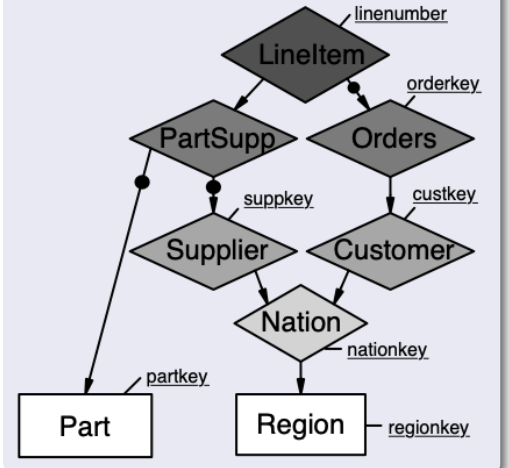


Graph semantics

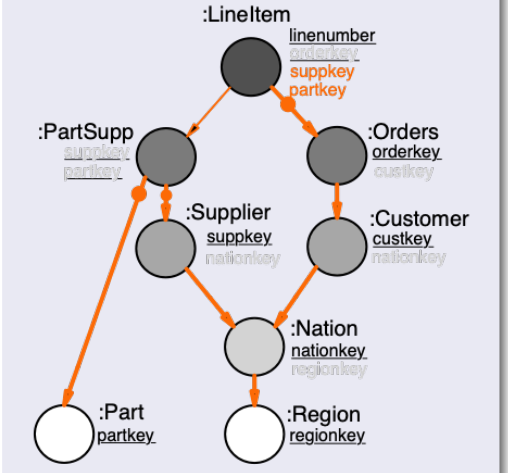


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

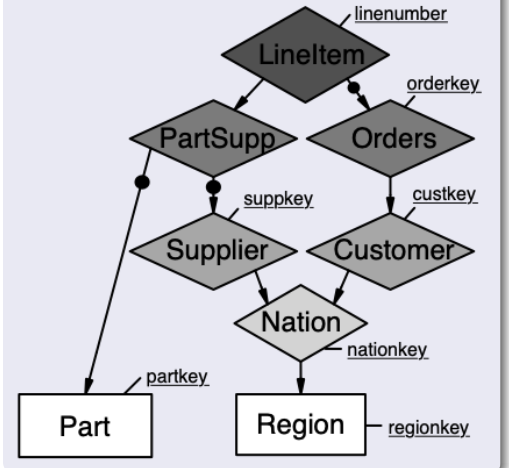


Graph semantics

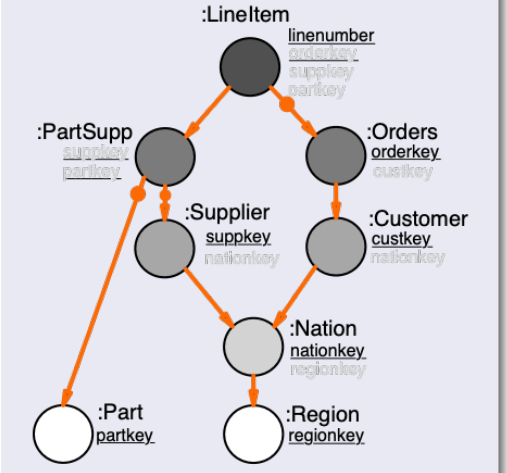


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

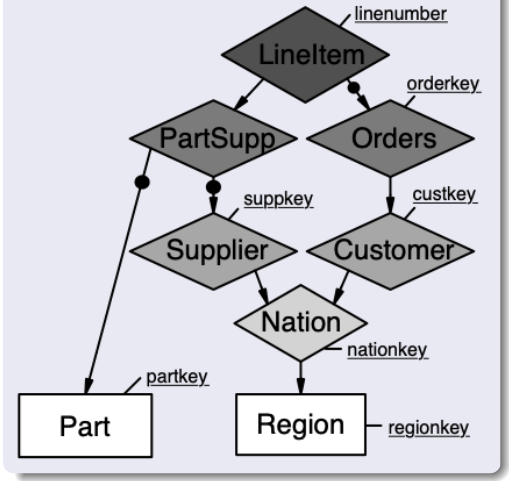


Graph semantics

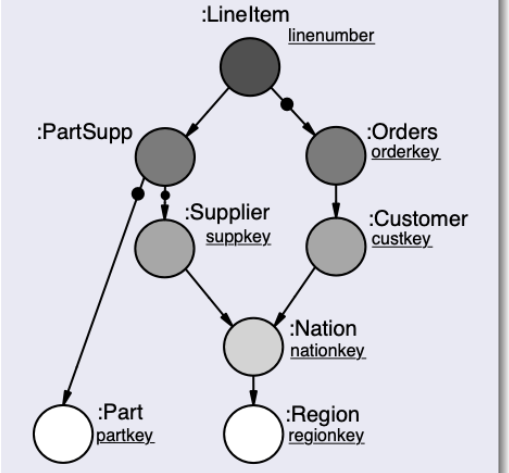


Graph semantics: no duplication of properties but need E/R keys

E/R graph model

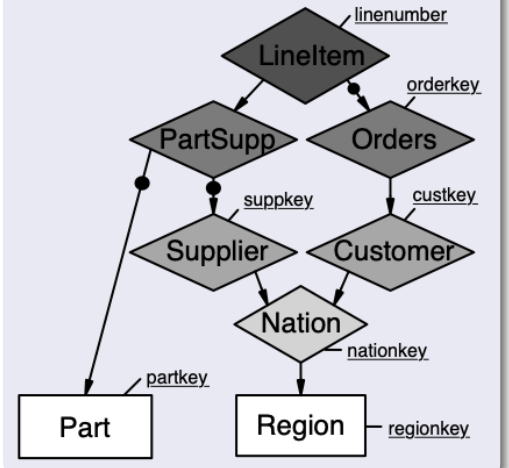


Graph semantics

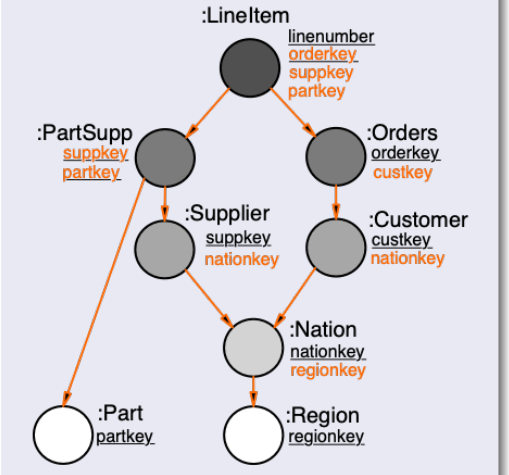


Mixed semantics: only keep E/R links to non-key components

E/R graph model

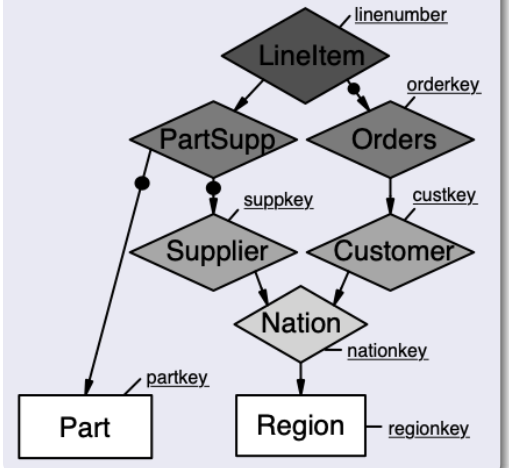


Mixed semantics

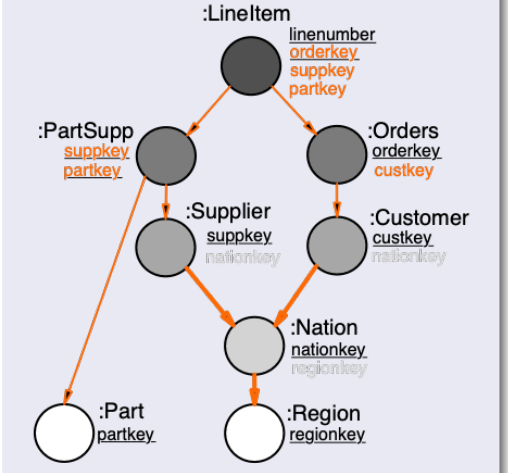


Mixed semantics: only keep E/R links to non-key components

E/R graph model

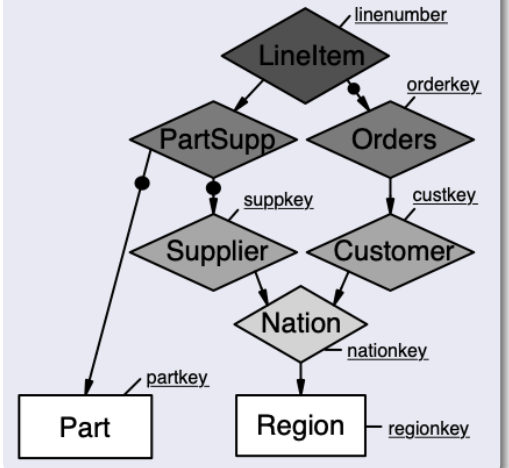


Mixed semantics

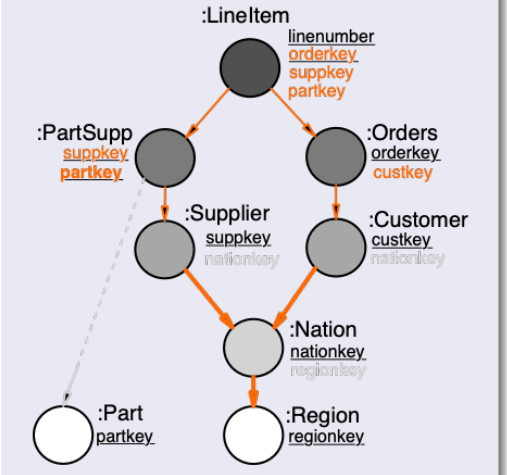


Mixed semantics: only keep E/R links to non-key components

E/R graph model

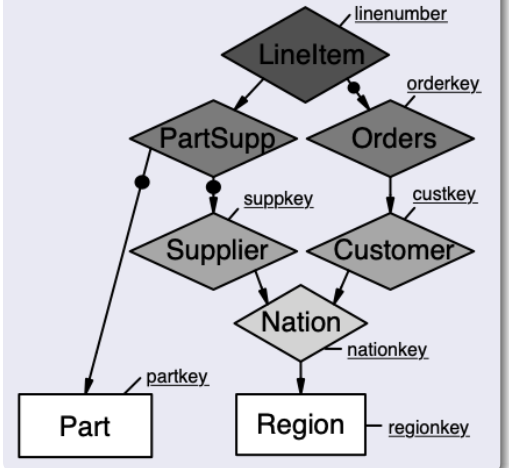


Mixed semantics

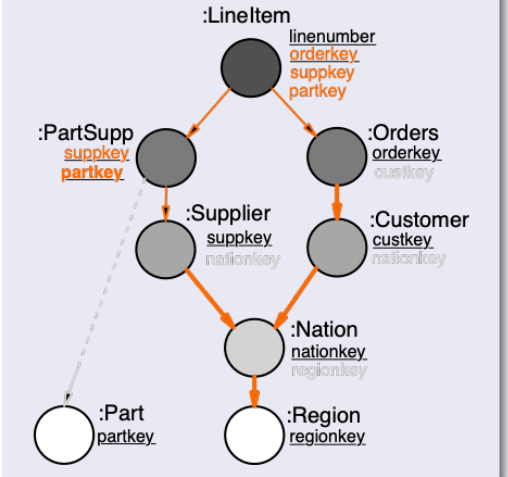


Mixed semantics: only keep E/R links to non-key components

E/R graph model

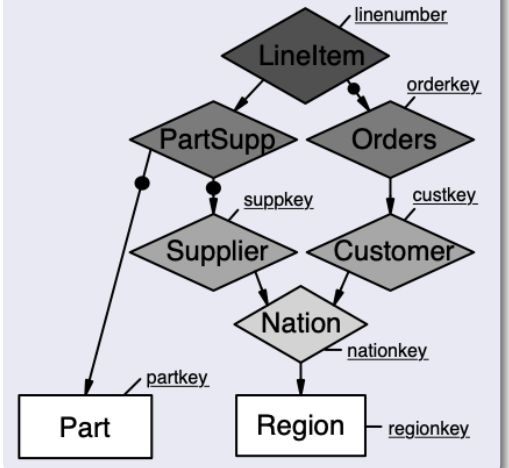


Mixed semantics

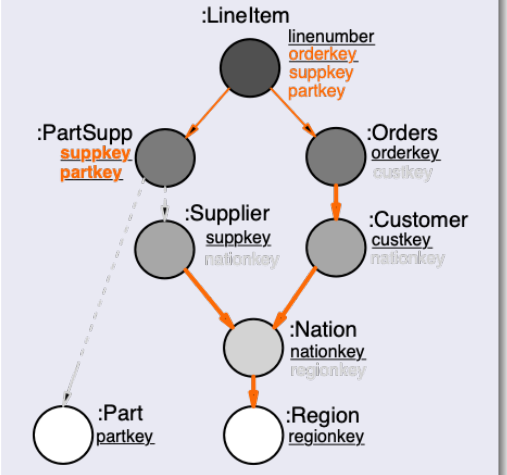


Mixed semantics: only keep E/R links to non-key components

E/R graph model



Mixed semantics



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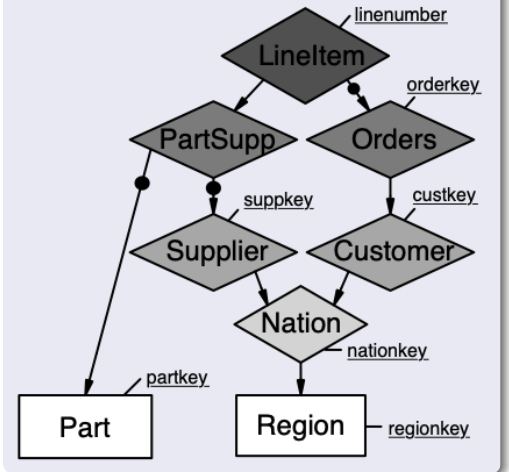

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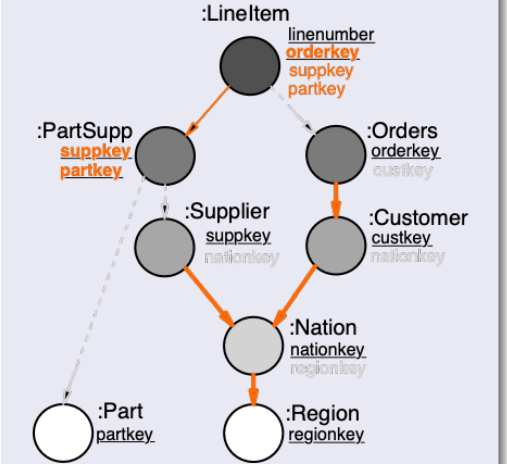

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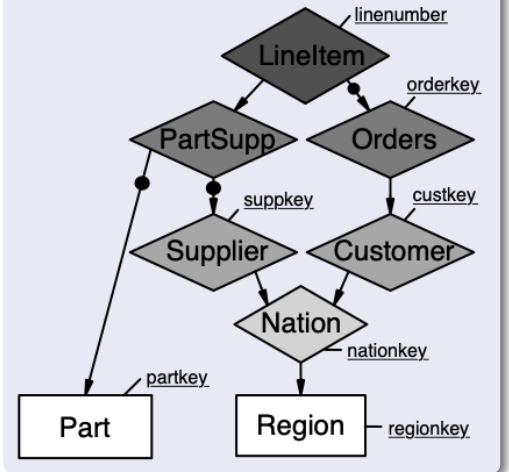

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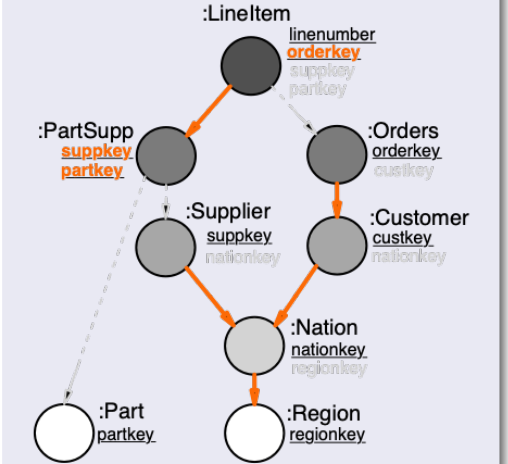

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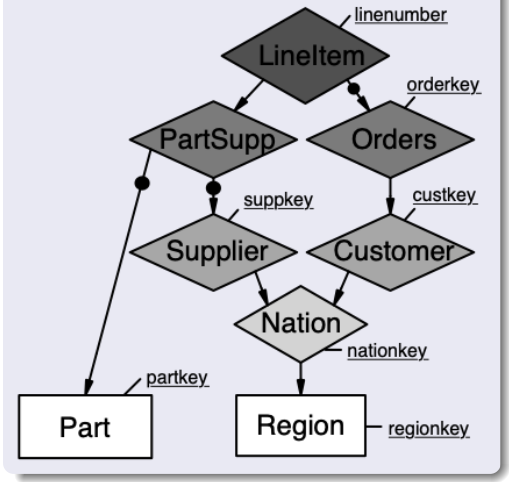

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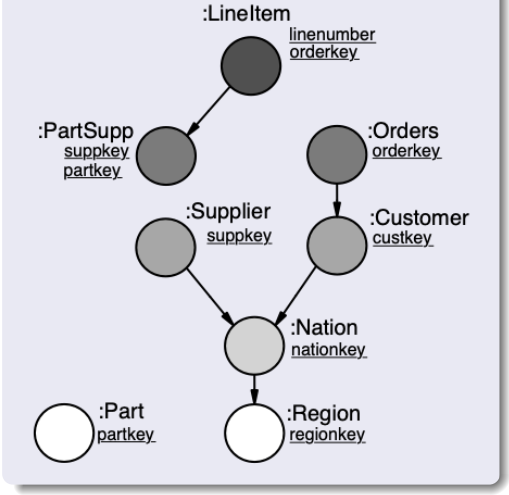

Formal Translations for Well-designed Relational Databases

Input

- Relational database schema $(\mathcal{S}, \Sigma_{\mathcal{S}})$ that is in
  Inclusion Dependency Normal Form (acyclic, key-based, foreign-key based)
- Relational database instance $\mathcal{I}(\mathcal{S})$ over $\mathcal{S}$ that satisfies $\Sigma_{\mathcal{S}}$

Order of relation schema $R \in \mathcal{S}$

The length of a longest key/foreign key chain from R to another relation schema $S \in \mathcal{S}$ on which no foreign key is defined

Relational Semantics

Firstly, the E/R graph model is obtained by mapping every relation schema $R \in \mathcal{S}$ to

- a vertex v_R with label R, and
- for every attribute $A \in R$ with $dom(A)$ we have a property-value pair $P_A = dom(A)$ on v_R,

Relational Semantics

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- for every key $\{A_1, \ldots, A_n\}$ of R we have a property key $R(\emptyset, \{P_{A_1}, \ldots, P_{A_n}\})$ implemented as PG-key:

  `FOR` $(x{:}R)$ `IDENTIFIER` $x.P_{A_1}, \ldots, x.P_{A_n}$

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  `FOR` $(x{:}R)$ `IDENTIFIER` $x.P_{A_1}, \ldots, x.P_{A_n}$
- for every foreign key $R[A_1, \ldots, A_m] \subseteq S[B_1, \ldots, B_m]$ of R we have a PG-key:

  `FOR` $(x{:}R)$ `MANDATORY` y `WITHIN` (x),$(y{:}S)$ `WHERE` $x.P_{A_1} = y.P_{B_1}, \ldots, x.P_{A_m} = y.P_{B_m}$.

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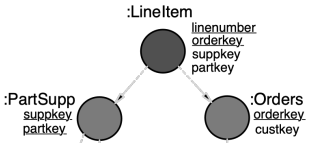
- tuple $t \in \mathcal{I}(R)$ for every $R \in \mathcal{S}$ to a node v_t with label R such that, for all $A \in R$, $\nu(v_t, P_A) := t(A)$.

Example: Relational Semantics

Relation schema LINEITEM

- attributes *suppkey*, *partkey*, *orderkey*, *linenumber*, ..., *comment*
- key {*linenumber*,*orderkey*} and foreign keys
  - [*suppkey*, *partkey*] $\subseteq$ PARTSUPP[*suppkey*, *partkey*] and
  - [*orderkey*] $\subseteq$ ORDERS[*orderkey*]



Vertex v_{LINEITEM} with label LINEITEM

- properties $P_{suppkey}$, $P_{partkey}$, $P_{orderkey}$, $P_{linenumber}$ ..., $P_{comment}$,
- E/R-key LINEITEM($\emptyset$, {*linenumber*, *orderkey*}) implemented as PG-Key:

  `FOR` (x:LINEITEM) `IDENTIFIER` $x.P_{linenumber}, x.P_{orderkey}$
- Further PG-keys:
  - `FOR` (x:LINEITEM) `MANDATORY` y `WITHIN` (x), (y:PARTSUPP) `WHERE` $x.P_{suppkey} = y.P_{suppkey}, x.P_{partkey} = y.P_{partkey}$
  - `FOR` (x:LINEITEM) `MANDATORY` y `WITHIN` (x), (y:ORDERS) `WHERE` $x.P_{orderkey} = y.P_{orderkey}$.

Graph Semantics

- R_{g}: Attributes of R that do not occur in any foreign key of R (others to be covered by E/R links)
- $C_R = \{S \in \mathcal{S} \mid \exists R[X] \subseteq S[Y] \in \Sigma_{\mathcal{S}}\}$ (the components of relationship type R)

Firstly, the E/R graph model is obtained by mapping every relation schema $R \in \mathcal{S}$ to

- a vertex v_R with label R, and
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- for distinguished key K of R in Σ, we have an E/R key $R(C_K, P_K)$ where
  $C_K := \{S \in C_R \mid K \cap S \neq \emptyset\} = \{S_1, \ldots, S_m\}$ and
  $P_K := \{P_A \mid A \in K\} - \{P_A \mid A \in S, S \in C_K\} = \{P_{A_1}, \ldots, P_{A_n}\}$, implemented as:
  - directed edges $(v_R, v_{S_1}), \ldots, (v_R, v_{S_m})$ with label $\bullet$ each,
  - `FOR` $(x{:}R)$ `IDENTIFIER` $x.P_{A_1}, \ldots, x.P_{A_n}, y_1, \ldots, y_m$ `WITHIN` $(x){\rightarrow}(y_1{:}S_1), \ldots, (x){\rightarrow}(y_m{:}S_m)$

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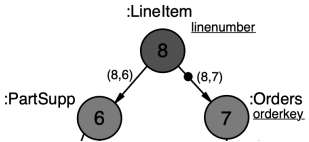
Example: Graph Semantics

Relation schema LINEITEM

- attributes *suppkey*, *partkey*, *orderkey*, *linenumber*, ..., *comment*
- key {*linenumber*,*orderkey*} and foreign keys
  - [*suppkey*, *partkey*] ⊆ PARTSUPP[*suppkey*, *partkey*] and
  - [*orderkey*] ⊆ ORDERS[*orderkey*]



Vertex v_{LINEITEM} with label LINEITEM

- properties $P_{linenumber}, \ldots, P_{comment}$ ($P_{suppkey}$, $P_{partkey}$, $P_{orderkey}$ not required)
- an E/R-key LINEITEM({ORDERS}, {*linenumber*}) implemented as
  - `FOR` (*x*:LINEITEM) `IDENTIFIER` $x.P_{linenumber}$, *y* `WITHIN` (*x*) → (*y*:ORDERS)
- further PG-key:
  - `FOR` (*x*:LINEITEM) `MANDATORY` *y* `WITHIN` (*x*) → (*y*:PARTSUPP).

Note that relation schema ORDERS has the key {*orderkey*}, and relation schema PARTSUPP has the key {*partkey*,*suppkey*}.

Mixed Semantics

- $R_{\text{mix}} := R - (F_R - K_R)$: remove attributes that belong to some foreign key but not to the key
- No foreign key for which some of its attributes belong to the key and some others do not

Firstly, the E/R graph model is obtained by mapping every relation schema $R \in \mathcal{S}$ to

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Example: Mixed Semantics

Relation schema LINEITEM

- attributes *suppkey*, *partkey*, *orderkey*, *linenumber*, ..., *comment*
- key {*linenumber*,*orderkey*} and foreign keys
  - [*suppkey*, *partkey*] $\subseteq$ PARTSUPP[*suppkey*, *partkey*] and
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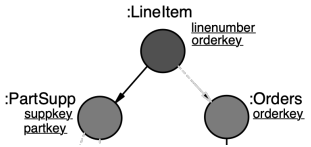


Vertex v_{LINEITEM} with label LINEITEM

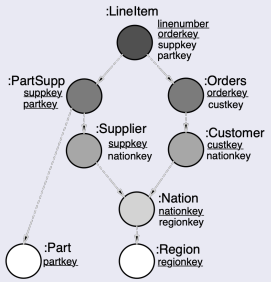
- $P_{orderkey}, P_{linenumber}, \ldots, P_{comment}$ ($P_{suppkey}$ and $P_{partkey}$ are not required),
- a property key LINEITEM($\emptyset$, {*linenumber*, *orderkey*}) implemented as
  - For (x:LINEITEM) `IDENTIFIER` $x.P_{linenumber}, x.P_{orderkey}$
- and further PG-keys:
  - `FOR` (x:LINEITEM) `MANDATORY` y `WITHIN` (x) $\rightarrow$ (y:PARTSUPP), and
  - `FOR` (x:LINEITEM) `MANDATORY` y `WITHIN` (x), (y:ORDERS) `WHERE` $x.P_{orderkey} = y.P_{orderkey}$.

Note that relation schema PARTSUPP has the key {*partkey*,*suppkey*}.

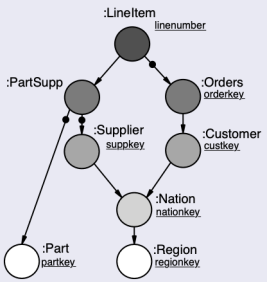
Three Principled Choices for Managing Entity and Referential (E/R) Integrity

- Relational: Use property keys uniformly, duplicate key properties as foreign keys
- Graphs: Use E/R keys and E/R links uniformly
- Mixed: Use property keys uniformly, and maximize use of E/R links

Relational semantics



Graph semantics



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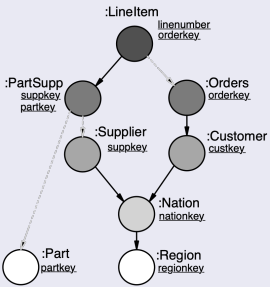

Experiments

Executive Summary

Question

How well do database workloads perform under relational, mixed, and graph semantics?

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Main findings

- Graph semantics saves orders of magnitude in maintaining entity and referential integrity
- Mixed semantics most efficient in current limits that only support property keys
- SQL benchmark queries perform remarkably well in graph databases compared to SQL
- Note: E/R keys do not enjoy native support in graph database systems yet

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TPC-H in IDNF, quantifies (dis-)advantages of the semantics due to (1) key/foreign key chain of different lengths, (2) E/R and property keys, and (3) different data volumes

Set Up

For each of the three semantics, we translated the TPC-H database into Neo4j using scaling factors small (0.01), medium (0.1) and large (1).

Semantics	*sf=0.01*		*sf=0.1*		*sf=1*	
	$\|V\|$	$\|E\|$	$\|V\|$	$\|E\|$	$\|V\|$	$\|E\|$
Relational	86,805	0	866,602	0	8,661,245	0
Mixed	86,805	76,800	866,602	766,597	8,661,245	7,661,240
Graph	86,805	152,975	866,602	1,527,169	8,661,245	15,262,455

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Specifications

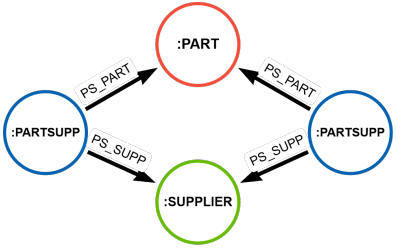
- Neo4j with its query language Cypher in conjunction with Python 3.9.13
- 64-bit operating system with an Intel Core i7 Processor and 16GB RAM

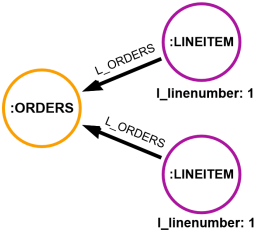
`https://github.com/graphdbexperiments/er_graph_experiments`

Experiments: Entity Integrity Checking

Entity Integrity

- Perform Cypher queries that validate E/R keys to check entity integrity
  - $\mathrm{PS}_k = \mathrm{PS}(\{\textsc{Part}, \textsc{Supp}\}, \emptyset)$ on PARTSUPP
  - $\mathrm{L}_k = \mathrm{L}(\{\textsc{Orders}\}, \{\textit{linenumber}\})$ on LINEITEM





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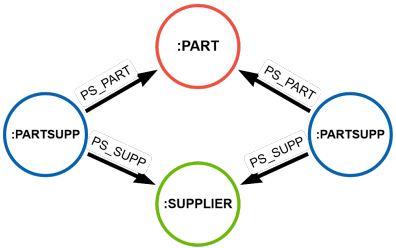

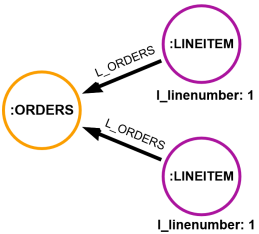

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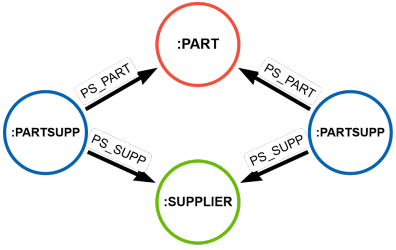

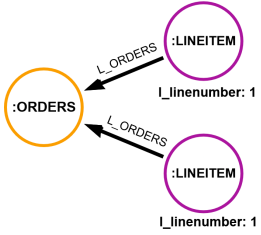

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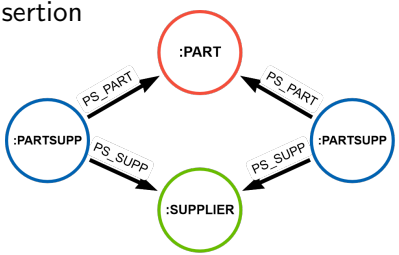
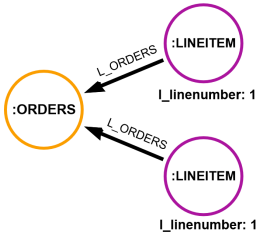
- Under graph semantics (g), the E/R keys are not supported by Neo4j (or any other graph system), so we cannot benefit from any index structure either
- Under all semantics, we check with a Cypher query whether inserting a new node in the graph will maintain entity integrity, that is, still satisfy the E/R key after insertion

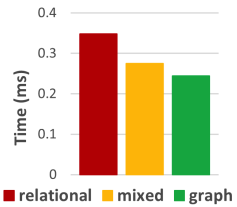




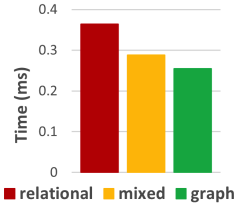
E/R keys used for entity integrity checking

Sem	E/R key	PG-Key	Cypher Query	Index
r and m	$\mathrm{PS}_k(r) = \mathrm{PS}_k(m)$: $\mathrm{PS}(\emptyset, \{partkey, suppkey\})$	`FOR (ps:PS) ID` ps.*partkey*, ps.*suppkey*	`MATCH (ps1:PS), (ps2:PS) WHERE` id(ps1) < id(ps2) `AND` ps1.*partkey* = <*partkey*> `AND` ps2.*partkey* = <*partkey*> `AND` ps1.*suppkey* = <*suppkey*> `AND` ps2.*suppkey* = <*suppkey*> `RETURN` ps1, ps2	PS(*partkey*, *suppkey*)
g	$\mathrm{PS}_k(g)$: $\mathrm{PS}(\{\mathrm{P}, \mathrm{S}\}, \emptyset)$	`FOR (ps:PS) ID` p, s `WITHIN` (s:S) ← (ps) → (p:P)	`MATCH` q1 = (s:S)<-[]-(ps1:PS)-[]->(p:P), q2=(s:S)<-[]-(ps2:PS)-[]->(p:P) `WHERE` id(ps1) < id(ps2) `AND` p.*partkey* = <*partkey*> `AND` s.*suppkey* = <*suppkey*> `RETURN` q1, q2	no index
r and m	$\mathrm{L}_k(r) = \mathrm{L}_k(m)$: $\mathrm{L}(\emptyset, \{orderkey, linenumber\})$	`FOR (l:L) ID` l.*orderkey*, l.*linenumber*	`MATCH` (l1:L), (l2:L) `WHERE` id(l1) < id(l2) `AND` l1.*orderkey* = <*orderkey*> `AND` l2.*orderkey* = <*orderkey*> `AND` l1.*linenumber* = <*linenumber*> `AND` l2.*linenumber* = <*linenumber*> `RETURN` l1, l2	L(*orderkey*, *linenumber*)
g	$\mathrm{L}_k(g)$: $\mathrm{L}(\{\mathrm{O}\}, linenumber)$	`FOR (l:L) ID` l.*linenumber*, o `WITHIN` (l) → (o:O)	`MATCH` q = (l1:L)-[]->(o:O)<-[]-(l2:L) `WHERE` id(l1) < id(l2) `AND` o.*orderkey* = <*orderkey*> `AND` l1.*linenumber* = <*linenumber*> `AND` l2.*linenumber* = <*linenumber*> `RETURN` q	no index

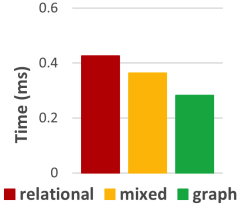
How does entity integrity checking compare across the different semantics?



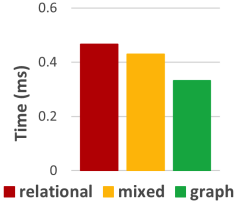
(a) PS_k: sf=0.01



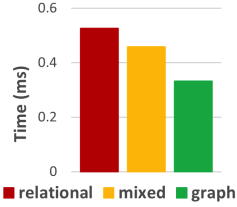
(b) PS_k: sf=0.1



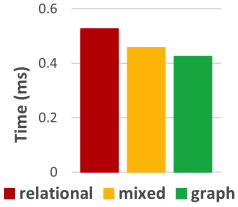
(c) PS_k: sf=1



(d) L_k: sf=0.01



(e) L_k: sf=0.1



(f) L_k: sf=1

Measuring the efficiency of entity integrity checking

E/R key	Sem	*sf=0.01*		*sf=0.1*		*sf=1*	
		index	no index	index	no index	index	no index
PS_k	r	0.348	5.617	0.364	14.949	0.426	203.713
	m	0.275	4.042	0.288	13.694	0.364	142.031
	g	–	0.244	–	0.255	–	0.282
L_k	r	0.466	34.877	0.526	206.668	0.527	26721.735
	m	0.43	33.783	0.458	215.784	0.468	15260.388
	g	–	0.332	–	0.368	–	0.426

- Relational and mixed semantics utilize UNIQUE index from property key definition
- As E/R keys are not supported in Neo4j, no index support available
- Nevertheless, entity integrity checking still most efficient under graph semantics
- Due to focus of graph database systems on efficient processing of edges
- Huge potential for further improvements once indices available for E/R keys

Experiments: Referential Integrity

Referential Integrity

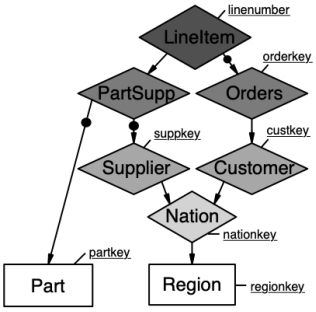
Key/foreign key chains

P_c: $\textsc{Part}(P) \subset \textsc{PartSupp}(PS) \subset \textsc{LineItem}(L)$

S_c: $\textsc{Supplier}(S) \subset \textsc{PartSupp}(PS) \subset \textsc{LineItem}(L)$

C_c: $\textsc{Customer}(C) \subset \textsc{Orders}(O)$

O_c: $\textsc{Orders}(O) \subset \textsc{LineItem}(L)$



Referential Integrity

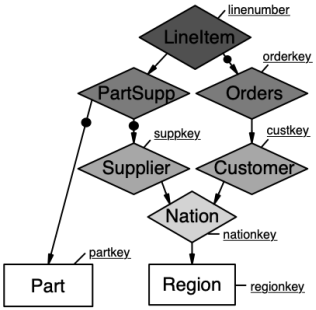
Key/foreign key chains

P_c: $\text{PART}(P) \subset \text{PARTSUPP}(PS) \subset \text{LINEITEM}(L)$

S_c: $\text{SUPPLIER}(S) \subset \text{PARTSUPP}(PS) \subset \text{LINEITEM}(L)$

C_c: $\text{CUSTOMER}(C) \subset \text{ORDERS}(O)$

O_c: $\text{ORDERS}(O) \subset \text{LINEITEM}(L)$



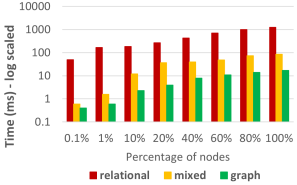
S_c : Update of *suppkey* value

- for different percentages of SUPPLIER-nodes (all semantics)
- requires updates for *suppkey* on PARTSUPP-nodes (relational and mixed)
- which requires updates for *suppkey* on LINEITEM-nodes (for relational only)

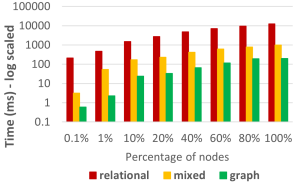
Update propagation for $S \subset PS \subset L$ by semantics

Semantics	Update propagation
relational	MATCH (s:S), (ps:PS), (l:L) WHERE ps.*suppkey* = s.*suppkey* AND l.*suppkey* = s.*suppkey* SET s.*suppkey* = RIGHT(('00000000' + toString(s.*suppkey*)), 8), ps.*suppkey* = RIGHT(('00000000' + toString(ps.*suppkey*)), 8), l.*suppkey* = RIGHT(('00000000'+toString(l.*suppkey*)), 8)
mixed	MATCH (s:S), (ps:PS) WHERE ps.*suppkey* = s.*suppkey* SET s.*suppkey* = RIGHT(('00000000' + toString(s.*suppkey*)), 8), ps.*suppkey* = RIGHT(('00000000' + toString(ps.*suppkey*)), 8)
graph	MATCH (s:S) SET s.*suppkey* = RIGHT(('00000000' + toString(s.*suppkey*)), 8)

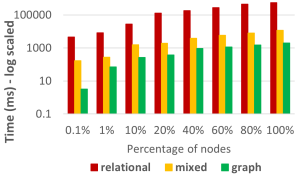
How well does update propagation work under different semantics? - 1/4



(a) *sf=0.01*: $P \subset PS \subset L$

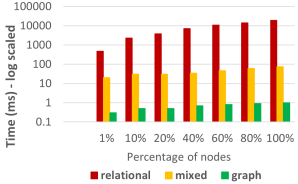


(b) *sf=0.1*: $P \subset PS \subset L$



(c) *sf=1*: $P \subset PS \subset L$

How well does update propagation work under different semantics? - 2/4



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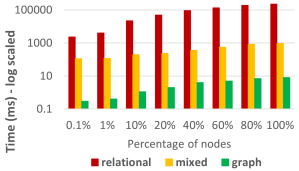

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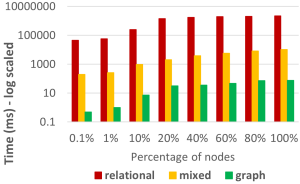
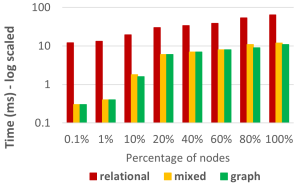
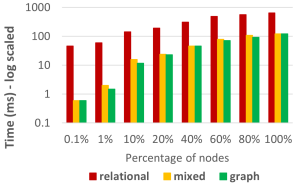


(c) *sf=1*: $S \subset PS \subset L$

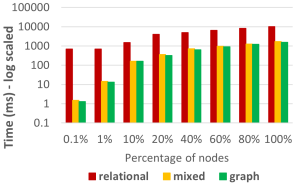
How well does update propagation work under different semantics? - 3/4



(a) *sf=0.01*: $C \subset O$

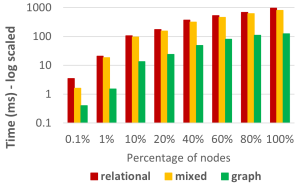


(b) *sf=0.1*: $C \subset O$



(c) *sf=1*: $C \subset O$

How well does update propagation work under different semantics? - 4/4



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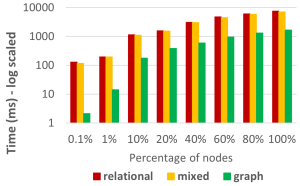
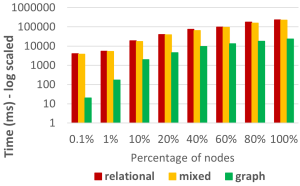


(b) *sf=0.1*: $O \subset L$



(c) *sf=1*: $O \subset L$

Quantifying the effort of update propagation in TPC-H (1%)

chain	semantics	*sf=0.01*		*sf=0.1*		*sf=1*	
		db hits	time (ms)	db hits	time (ms)	db hits	time (ms)
P_c	relational	311,642	168	1,816,084	469	8,243,302	8,418
	mixed	761	1.6	83,402	54	834,002	276
	graph	101	0.6	601	2.3	6,001	73
S_c	relational	884,973	470	5,958,206	4,101	47,587,284	57,795
	mixed	24,641	20	243,201	117	864,102	261
	graph	6	0.3	31	0.4	301	1.0
C_c	relational	75,131	13	456,155	60	4,621,519	709
	mixed	76	0.4	451	2.0	4,501	15
	graph	76	0.4	451	1.5	4,501	13
O_c	relational	65,161	21	626,036	198	6,256,283	5,447
	mixed	65,111	19	626,011	195	6,256,090	5,237
	graph	751	1.5	4,501	14	45,001	173

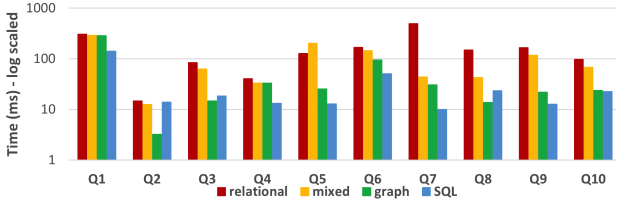
Quantifying the effort of update propagation in TPC-H (100%)

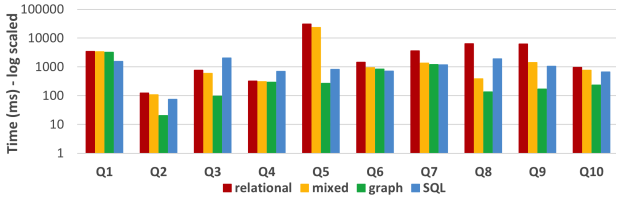
chain	semantics	*sf=0.01*		*sf=0.1*		*sf=1*	
		db hits	time (ms)	db hits	time (ms)	db hits	time (ms)
P_c	relational	1,514,378	1,251	16,315,447	12,579	151,030,378	559,684
	mixed	42,002	87	420,002	932	4,200,002	11,689
	graph	6,001	17	60,001	200	600,001	2,022
S_c	relational	29,072,628	18,996	290,157,279	231,131	2,904,674,611	2,216,487
	mixed	40,102	76	401,002	925	4,010,002	10,584
	graph	301	1	3,001	8	30,001	78
C_c	relational	106,502	64	1,065,002	648	10,650,002	10,146
	mixed	4,501	12	45,001	123	450,001	1,707
	graph	4,501	11	45,001	122	450,001	1,582
O_c	relational	315,877	964	3,152,862	7,606	31,522,479	231,556
	mixed	315,877	807	3,152,862	7,175	31,522,479	224,179
	graph	45,001	125	450,001	1,682	4,500,001	23,516

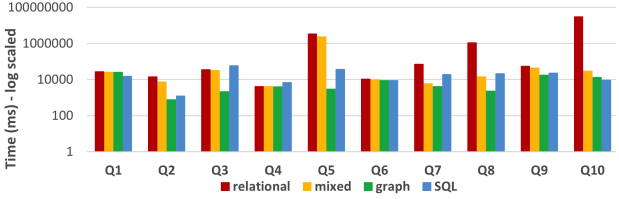
Eliminating property/attribute redundancy is a holy grail in database design

Experiments: Benchmark Queries

TPC-H Benchmark Queries: Small, Medium and Large Scale



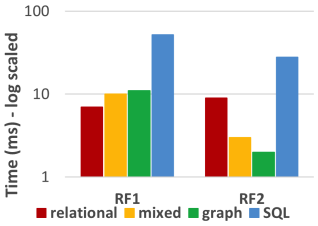


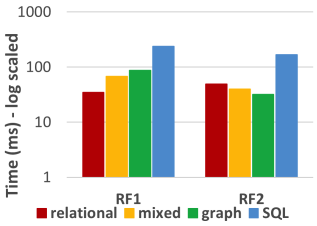


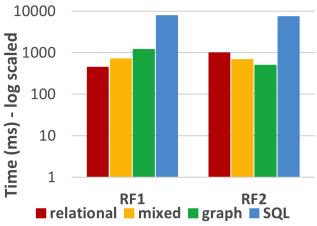
Observations

- Relational databases are fine-tuned for benchmarks
- Queries under graph semantics in graph database (Neo4j) very competitive with SQL queries (MySQL)
- Graphs outperform SQL on joins and scale
- SQL outperforms graphs on aggregations (normalize in graphs!)

TPC-H Benchmark: Refresh Operations (Small, Medium, Large Scale)







Slower insert from relational to mixed to graph semantics, reversed for deletions
SQL much slower

Conclusion

Contributions: “Lean and Mean”

Conceptual modeling

- E/R graphs as semantics of E/R models
- Implementation of E/R modeling within a native technology platform

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- Property graphs (E/R graph models) as a general data modeling tool

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Entity and referential integrity management

- E/R keys as efficient fragment of PG-Key for managing integrity of well-designed databases
- Relational, mixed, graph semantics as principled, complementary choices for managing integrity

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General

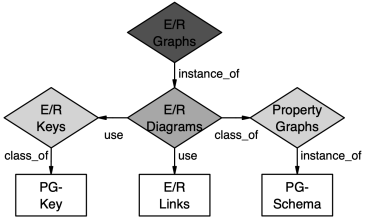
- Unifying conceptual, logical and graph data modeling
- Taking integrity management to the next level by eliminating property redundancy
- Relational benchmark queries perform very well under graph semantics, especially joins

Our work has turned a major inhibitor to the uptake of graph databases into a strong driver

Future Work

Fundamentals

- What are other sweet spots of PG-Schema?
- What are other sweet spots of PG-Key?
- What are classes of well-designed databases?
- How to optimize E/R graph models?
- How to infer models from property graphs?



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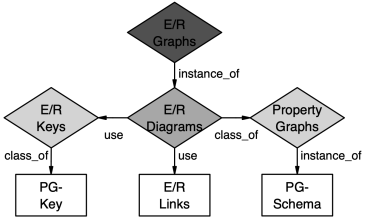

Practical

- What does index support for E/R keys and PG-Keys look like?
- What does support for other classes of graph data dependencies look like?

Summary: References

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