IS Übung

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January 15, 2019

1 3.

3.1. College Life $p(f_m) = .25, p(f_c) = .15, p(f_c, f_m) = .1.$

3.1. a) $p(f_m|f_c) = \frac{p(f_m, f_c)}{p(f_c)} = .67$

3.1. b) $p(f_c|f_m) = \frac{p(f_m, f_c)}{p(f_m)} = .25$

3.1. c) $p(f_c) + p(f_m) = .4$

3.2 Tossing Dice $p(1) = ... = p(6) = \frac{1}{6}$

3.2. a) Mögliche Paare: $(d_1,\ d_2)$: (1,5),(2,4),(3,3),(4,2),(5,1). $p(A) = \frac{5}{36}$.

3.2. b) $p(A) = \frac{5}{36}$.

3.2. c) Mögliche Paare: $(d_1,\ d_2)$: (1,1),(1,2),(1,3),(2,1),(2,2),(3,1). $p(A) = \frac{1}{6}$.

3.3. College Life Again

3.3. a) bgbgbgb: $p(A) = \frac{4*3*3*2*2*1*1}{7!} = .0286$

3.3. b) gbgbgb or bgbgbg: $p(A) = 2 * \frac{3*3*2*2*1*1}{6!} = .1$

3.4 Classical Birthday Problem

$$\frac{\frac{356!}{(356-n)!}}{356^n} \geq .5 \rightarrow n = 23 \tag{1}$$

$$\frac{\frac{356!}{(356-n)!}}{356^n} \geq .99 \rightarrow n = 57 \tag{2}$$

3.5 Family Life

3.5. a) $p(B_1, B_2) = \frac{1}{2^2-2} = .5$

3.5. b) $p(B_1, B_2) = \frac{1}{2^2-1} = .33$

3.6 Manufacturing Error $p(A) = .6, p(B) = .3, p(C) = .1, p(d|A) = .02, p(d|B) = .03, p(d|C) = .04$

3.6 a) $p(d) = \sum_{X\in\{A,B,C\}} p(d|X)p(X) = .02 * .6 + .03 * .3 + .04 * .1 = .025.$

3.6 b) $p(C|d) = \frac{p(C,d)}{p(d)} = p(d|C)\frac{p(C)}{p(d)} = .04\frac{.1}{.025} = .16$

3.7 Drawing Marbles (red, white): $A(3,5), B(2,1), C(2,3)$. $p(A|\text{red}) = \frac{p(A,\text{red})}{p(\text{red})} = \frac{\frac{3}{16}}{\frac{7}{16}} = .43$

3.8 Target Practice $p(h_A) = .25, p(h_B) = .4$. $p(h) = 1 - (1 - p(h_A))(1 - p(h_B)) = .55$

3.9 Tossing a Coin X = 0 head on the first toss, 1 otherwise Y = Number of occurring heads.

3.9. a) $p(X = 1) = .5$, $p(X = 0) = .5$. $p(Y = 0) = \frac{1}{8}, p(Y = 1) = \frac{3}{8}, p(Y = 2) = \frac{3}{8}, p(Y = 3) = \frac{1}{8}$.

3.9 b)

X	Y	$p(X\|Y)$	$p(Y)$	$p(X,Y)$
0	0	0	$\frac{1}{8}$	0
1	0	1	$\frac{1}{8}$	$\frac{1}{8}$
0	1	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{8}$
1	1	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{2}{8}$
0	2	$\frac{2}{3}$	$\frac{3}{8}$	$\frac{2}{8}$
1	2	$\frac{1}{3}$	$\frac{3}{8}$	$\frac{1}{8}$
0	3	1	$\frac{1}{8}$	$\frac{1}{8}$
1	3	0	$\frac{1}{8}$	0

3.9. c)

$$E[XY] = \sum_{z \in \{0,1,2,3\}} p(XY = z)z \tag{3}$$

$$= p(X = 1, Y = 1) + 2p(X = 1, Y = 2) + 3p(X = 1, Y = 3) \tag{4}$$

$$= \frac{1}{8} + 2\frac{1}{8} = \frac{3}{8} \tag{5}$$

$$E[X]E[Y] = \frac{1}{2} * \frac{12}{8} = \frac{6}{8} \tag{6}$$

$$cov(X, Y) = E[XY] - E[X]E[Y] = \frac{3}{8} - \frac{6}{8} = -\frac{3}{8} \tag{7}$$

$$\tag{8}$$

They're not indepedent. A negative correlation exists, implying that lower values of X (i.e. 0) correspond to higher values of Y (i.e. greater than 0), which makes sense, as head in the first round means possibly more heads in all three rounds together than no head.

3.10. Classifying Cars - Naive Bayes Die Klassifikation ergibt "not stolen".