Report on a comment by Graciana Puentes

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In this report I discuss the "Comment on Snizkho et al., Phys. Rev. Resarch 3, 043045 (2021)" by Graciana Puentes. In that comment, the author criticises Ref. [1] with respect to four aspects:

1. Taking the limit of strong measurement ($C \to \infty$) or quasiadiabatic evolution ($A \to \infty$) is claimed to be invalid.
2. The issue of non-Hermitian Kraus operators and their relation to the topological transitions in the measurement-induced phase is claimed to be unclear and contradictive to Ref. [2].
3. The separation of measurement-induced phases into dynamical and geometrical is criticized.
4. The possibility of an experimental apparatus implementing the measurements of the considered class is doubted.

Below, I will address the above points, fully refuting items 1, 3, and 4. As for the item 2, I acknowledge that there is a distinction of the definitions used in Refs. [2] and [1], which should have been highlighted, in order to help the readers transition between the two papers; yet I emphasize that the statements made in each paper separately (with the definitions used in each respective paper) are correct.

I. TAKING THE LIMIT OF STRONG MEASUREMENT ($C \to \infty$) OR QUASIADIABATIC EVOLUTION ($A \to \infty$)

Most of the comment space is dedicated to arguing that taking the limit $C \to \infty$ or $A \to \infty$ is not valid since the calculation involves a step $\left(I + \frac{\Lambda}{N}\right)^N \approx e^{\Lambda}$, which requires $\Lambda/N \ll 1$. I would like to point out that the relevant section announces the interest in the limit $N \to \infty$ before performing the calculation and before taking the $C \to \infty$ or $A \to \infty$ limits (see the text between Eqs. (34) and (35) in Ref. [1]). The latter limits are analysed after the $N \to \infty$ calculation has been performed (see Eqs. (48) and (50) in Ref. [1]). The above means that a specific order of limits is implied. For example, the $C \to \infty$ limit should be understood as $1 \ll C \ll N$. With such a hierarchy of variables, all the steps in the calculation are valid.

As further evidence, I present Fig. 1. Here, the measurement-induced phase for $N = 500$ with $C = 100 \gg 1$ and $A = 0$ is calculated numerically. Note that this agrees well with the analytically found result in this limit, $\chi = \pi(\cos\theta - 1)$ (see Eq. (50) in Ref. [1] and Eq. (6) in the comment under discussion). Further, I present a numerical curve obtained for the same protocol using projective measurements; that is, using

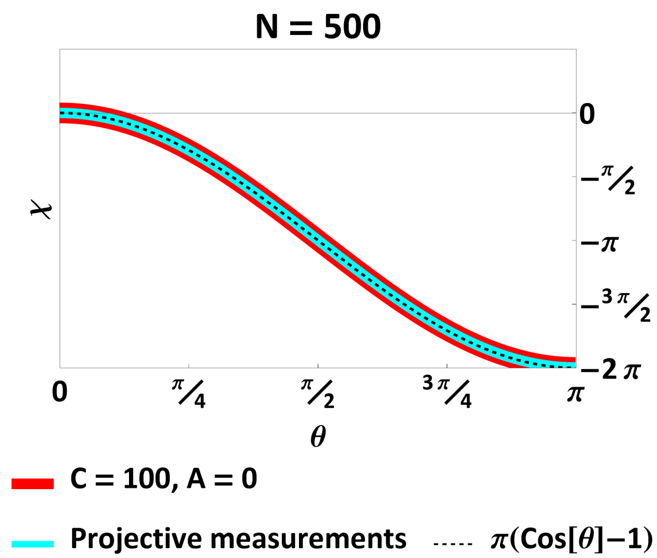

FIG. 1. Comparison of the numerically calculated phase in the limit $1 \ll C \ll N$ — $C = 100$, $A = 0$, $N = 500$ (red) — with the analytically derived answer $\pi(\cos\theta - 1)$ (black dashed), as well as with the phase that would be induced by projective measurements (cyan). The three curves lie on top of each other.

$$P^{(0)} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \tag{1}$$

instead of $M^{(0)}$ in Eq. (35) of Ref. [1]. The curve coincides with the above two, implying that even though strict projective measurements are not within the scope of the analytic calculation, our calculation in the limit $1 \ll C \ll N$ adequately represents also the behaviour of phases induced by projective measurements. With this report, I attach a Mathematica code that will allow the reviewers and the comment author to perform the same comparison for any values of C, A, and N. In particular, one can verify the behaviour in the limit $A \to \infty$, i.e., $1 \ll A \ll N$.

II. THE ISSUE OF NON-HERMITIAN KRAUS OPERATORS AND THEIR RELATION TO THE TOPOLOGICAL TRANSITIONS

In addition to the quote given in the comment, I would like to highlight another quote from Ref. [2] on the same issue: "*Note that the existence of a sharp transition at* $\theta = \pi/2$ *is protected by the fact that* $M_\eta(e_z, r)$ *is real (Eq. 17), which guarantees that the trajectory always remains on the equator and thus* $\chi_{geom}(\pi/2) \in \{0, -\pi\}$." This highlights the source and the resolution of the apparent contradiction with the results of Ref. [1]. Indeed, Ref. [2] claims that the topological transition in measurement-induced phases is protected by the Hermiticity of Kraus operators, while Ref. [1] finds transitions also for non-Hermitian operators. The difference lies in the definition of the topological transition.

In the process of research after publishing Ref. [2], we have realized that considering $\chi(\theta = \pi) - \chi(\theta = 0)$ (used in Ref. [1], cf. Eq. (59)) provides a better definition for the topological transition than $\chi(\theta = \pi/2) - \chi(\theta = 0)$ (used in Ref. [2]). Namely, for Hermitian Kraus operators, there is additional symmetry, which forces the transition to $\theta = \pi/2$; yet even without this symmetry there are topological transitions captured by $\chi(\theta = \pi) - \chi(\theta = 0)$, but not by $\chi(\theta = \pi/2) - \chi(\theta = 0)$. In this respect, Ref. [1] generalizes Ref. [2] in terms of definitions and results (the results of Ref. [1] for $A = 0$ are consistent with those of Ref. [2]).

I acknowledge that we should have alerted the readers to this change of definitions in order to facilitate a smooth transition of the readers between the papers. At the same time, I emphasize that the results and claims of both references are factually correct with the definitions used in the respective papers.

III. THE SEPARATION OF MEASUREMENT-INDUCED PHASES INTO DYNAMICAL AND GEOMETRICAL

In section II.C of Ref. [1] ("Classification of measurement-induced phases"), we compare the measurement-induced phase with the Pancharatnam phase acquires by a state following the same trajectory in the Hilbert space. We show that unless the Kraus operators are Hermitian, the two do not coincide. Naturally calling the Pancharatnam phase the geometric phase, we conclude that the measurement-induced phase is not fully geometric. Lacking a better name and in analogy with the Hamiltonian physics embodied in the non-Hermiticity parameter A (cf. Eq. (37) in Ref. [1]), we call the difference a dynamic phase.

This realization that measurement-induced phase is not necessarily geometric is reflected in the title difference between Refs. [2] and [1]: "measurement-induced geometric phases" and "weak-measurement-induced phases" respectively.

Note that the Pancharatnam phase is completely describable in terms of the Berry connection (the Panchatatnam phase for a discrete trajectory is exactly the Berry phase on a continuous trajectory composed of the shortest geodesics connecting the discrete points, cf. Ref. [3, section 5.3.4]). In this respect, we have actually fulfiled the request of the comment's author: "*in order to properly distinguish geometric from dynamical components in the acquired phase, the authors should have also analyzed the Berry connection and the Berry curvature*".

The comment author also claims that the only possible dynamical phase is the common phase factor $e^{i\gamma}$. In a system with multiple levels involved, evidently, each level can acquire its own dynamical phase. Therefore, there can be multiple dynamical phases at play.

IV. THE POSSIBILITY OF AN EXPERIMENTAL APPARATUS IMPLEMENTING THE MEASUREMENTS OF THE CONSIDERED CLASS

The comment author writes that we do not mention explicitly an experimental device that can lead to the measurement back action of the form we consider. Let me first point out that in section III.A of Ref. [1] ("Measurement model") we introduce a model of system-detector interaction, which results in the Kraus operators we use throughout the rest of the article. Such an interaction may not be natural, yet it may be engineerable in modern well-controlled quantum systems.

Further, there are a few simple examples that result in the back action of the form of Eqs. (29) or (35) in Ref. [1]. One example is an imperfect polarizer: if the polarizer transmits two polarizations, they may acquire different phases while propagating through the polarizer (e.g., because of the difference in the refraction indices for the ordinary and the extraordinary components in a birefringent crystal). In fact, a phrase from Ref. [2] quoted in the comment, "*The polarizers must be carefully designed such that no additional phase difference between the two polarizations is accumulated by the light passing through a polarizer*", refers exactly to the need to eliminate this physics if one wants to have Hermitian Kraus operators. Without special effort, this phase is naturally present; it may not be controllable easily, so that investigating the dependence on the parameter A may require physically swapping different polarizers or adding extra wave plates.

Another example which demonstrates essentially the same physics is an imperfect Stern-Gerlach device. It separates the two spin species spatially by a magnetic field gradient. Yet there is Zeeman splitting, inducing dynamical phases to each of the spin states. If the spatial separation of the beams is not sufficient, spatial filtering of the beams effectively results in the measurement back action of the form of Eqs. (29) or (35) in Ref. [1] on top of the common phase $\sim e^{i\gamma}$. Note that the phase in the matrix $M^{(0)}$ is clearly of dynamical origin in this example, making a connection to the issue of dynamical and geometric components in the measurement-induced phases discussed in the previous section.

Finally, in the time that has passed since the publication of Ref. [1], two experimental works observing the transition have appeared, see Refs. [4] (for $N = 6$ measurements) and [5] ($N = 3$ measurements). In particular, Ref. [5] implemented measurement specifically via the above polarizer model and put special effort to ensure $A = 0$ (cancel A via additional wave plates).

Therefore, the measurements discussed in Ref. [1] can be implemented in practice. Further, Ref. [1] has discussed an implementation of the measurements considered. I do not think that the paper had to discuss various implementations of the measurement procedure or do it in detail.

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[4] Y. Wang, K. Snizhko, A. Romito, Y. Gefen, and K. Murch, Observing a topological transition in weak-measurement-induced geometric phases, Phys. Rev. Res. **4**, 023179 (2022), arXiv:2102.05660.

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