

Comment on Snizkho et *al.*, Phys. Rev. Research 4, 023179 (2021)

Graciana Puentes^{1,2}

1- *Departamento de Física, Facultad de Ciencias Exactas y Naturales, Pabellon 1, Ciudad Universitaria, 1428 Buenos Aires, Argentina*

2- *CONICET-Universidad de Buenos Aires. Instituto de Física de Buenos Aires (IFIBA). Buenos Aires, Argentina.*

In a recent paper published in Phys. Rev. Research by Snizkho et *al.* [1], the authors explore two timely concepts at the forefront of quantum mechanics. Namely, quantum weak measurements and topological phase transitions. In particular, they address the measurement-induced geometric phase which emerges following an infinite sequence of weak measurements. Moreover, they outline a general framework for treating measurement-induced phase factors, and apply it to a broad class of measurements. Nevertheless, we argue that the general framework proposed by the authors strongly relies on a continuous limit approximation which does not hold in the limiting cases of infinite measurement strength, and infinite measurement back-action, as explored by the authors.

More specific, for the continuous measurement limit, an analytic expression for the measurement-induced geometric phase is obtained by first considering the explicit form of the Kraus decomposition, given by:

$$\langle \psi_0 | M_{N-1}^{(0)} \dots M_1^{(0)} | \psi_0 \rangle = \langle \uparrow | \delta R (M^{(0)} \delta R)^{N-1} | \uparrow \rangle, \quad (1)$$

where $|\psi_0\rangle$ is the initial state, δR is a matrix independent of the measurement index $k = 1, \dots, N-1$, and $M^{(0)}$ is a 2×2 diagonal Kraus operator, whose diagonal elements are $(1, \text{Exp}(-2\frac{C+iA}{N}) + O(\frac{1}{N^2}))$ [1]. The calculation of the matrix power $(M^{(0)} \delta R)^{N-1}$ is performed by expressing the product $(M^{(0)} \delta R)$ in its diagonal basis, in the form $V D V^{-1}$, where $D = (I + \frac{\Lambda}{N})^N$, and V is the corresponding matrix for the change of basis.

For the continuous-measurement limit ($N \rightarrow \infty$), the authors invoke the approximation $(I + \frac{\Lambda}{N})^N \approx e^\Lambda$, which is of course only valid if Λ is finite, as it can be easily demonstrated following L'Hopitals rule [2]. Doing some basic matrix manipulation, it can be shown that for the specific Kraus operators considered in [1], the Λ matrix takes the form:

$$\Lambda = \begin{bmatrix} -C - i(\pi + A) - \tau & 0 \\ 0 & -C - i(\pi + A) + \tau \end{bmatrix}, \quad (2)$$

where C is the measurement strength, A is the non-Hermitian component of the measurement backaction [1], and τ is a function of C , A , and the polar angle θ , of the form $\tau = \sqrt{Z^2 - \pi^2 \sin^2 \theta}$, with $Z = (C + iA + i\pi \cos \theta)$. From these derivations, the authors obtain an analytic expression for the geometric phase, of the form:

$$\sqrt{P} e^{\chi_{\text{geom}}} = e^{i\pi(\cos \theta - 1) - Z} (\cosh \tau + Z \sinh \tau / \tau), \quad (3)$$

recovering the analytic expression reported in [3]. For ease of notation we consider a fixed measurement orientation $d = +$.

In what follows, the authors explore the limiting cases $A \rightarrow \infty$ and $C \rightarrow \infty$, by expanding the logarithmic of Eq. (3), at infinite A and infinite C [1].

For the case of $A \rightarrow \infty$, they obtain :

$$\begin{aligned} \chi_{\text{geom}} = \pi(\cos \theta - 1) + \frac{\pi^2 \sin^2 \theta}{2A} \\ - \frac{\pi^2 \sin^2 \theta}{4A^2} [e^{2C} \sin(2A + 2\pi \cos \theta) - 2\pi \cos \theta] + O(A^{-3}) \end{aligned} \quad (4)$$

while for the case of $C \rightarrow \infty$ they obtain:

$$\chi_{\text{geom}} = \pi(\cos \theta - 1) + \frac{\pi^2 \sin^2 \theta}{2C^2} [A + \pi \cos \theta] + O(C^{-3}). \quad (6)$$

Nevertheless, it should be noted that due to the linear dependence of Λ on C and A (Eq. (2)), it turns out that $\Lambda \rightarrow \infty$, when $C \rightarrow \infty$ or $A \rightarrow \infty$. Therefore, in such limiting cases where Λ is unbounded, the approximation $(I + \frac{\Lambda}{N})^N \approx e^\Lambda$ does not hold, thus Eq. (3) cannot be used.

REFERENCES

- [1] Kyrylo Snizkho, Nihal Rao, Parveen Kumar, and Yuval Gefen, *Weak-measurement-induced phases and dephasing: Broken symmetry of the geometric phase*, Phys. Rev. Research **3**, 043045 (2021).
- [2] Visit our GitHub repository for an annotated derivation of L'Hopitals rule <https://github.com/grapts/geometric-phases>
- [3] Valentin Gebhart, Kyrylo Snizkho, Thomas Wellens, Andreas Buchleitner, Alessandro Romito, and Yuval Gefen, *Topological transition in measurement-induced geometric phases*, PNAS **117**, 5706 (2020).