```
dfilename(h, t) = @sprintf("h-%04d_t-%04d.csv", Int(round(h*100)), Int(round(t*100)));
dfilename(h) = @sprintf("h-%04d_end.csv", Int(round(h*100)));

asoln(x, y, t) = exp(0.75 * t) * sin(2 * x - y) * cosh(1.5 * (x + y));

errs = [];
hs = [1/10; 1/20; 1/40];

for h in hs
  k = h^2;
  output_step = Int(round(1 / k * 0.2));

  μ = k / (h*h);
  println("k = $k, h = $h");

  const aax = -μ;
  const bbx = (2 * μ + 1);
  const ccx = aax;

  const aay = -2 * μ;
  const bby = (4 * μ + 1);
  const ccy = aay;

  xs = linspace(0.0, 1.0, Int(round(1.0 / h)));
  ys = copy(xs);
  ts = linspace(0.0, 1.0, Int(round(1.0 / k)));
  const M, L, K = length(xs), length(ys), length(ts);

  u = zeros(M, L, K);
  for (m, x) in zip(1:M, xs), (l, y) in zip(1:L, ys)
    u[m, l, 1] = asoln(x, y, 0);
  end

  for n in 1:K-1
    u_temp = zeros(M, L);
    thalf = (ts[n]+ts[n+1]) / 2;
    # calculate boundary terms
    for l in 1:L
      u_temp[1, l] = asoln(0.0, ys[l], thalf);
      u_temp[M, l] = asoln(1.0, ys[l], thalf);
      u[1, l, n+1] = asoln(0.0, ys[l], ts[n+1]);
      u[M, l, n+1] = asoln(1.0, ys[l], ts[n+1]);
    end
    for m in 2:M-1
      u_temp[m, 1] = asoln(xs[m], 0.0, thalf);
      u_temp[m, L] = asoln(xs[m], 1.0, thalf);
      u[m, 1, n+1] = asoln(xs[m], 0.0, ts[n+1]);
      u[m, L, n+1] = asoln(xs[m], 1.0, ts[n+1]);
    end

    for l in 2:L-1
      # calculate pi and qi for Thomas' algorithm
      p = zeros(L);
      q = zeros(L);
      p[2], q[2] = 0.0, asoln(0.0, ys[l], thalf);
      for m=2:M-1
        dd = u[m, l, n] + (2 * μ *
                           (u[m, l+1, n] - 2 * u[m, l, n] + u[m, l-1, n]));
        denom = aax * p[m] + bbx;
        p[m+1] =  -ccx / denom;
        q[m+1] = (dd - aax * q[m]) / denom;
      end
      u_temp[M, l] = asoln(1.0, ys[l], thalf);
```

```
64        for m=M-1:-1:2
65          u_temp[m, l] = p[m+1] * u_temp[m+1, l] + q[m+1];
66        end
67        u_temp[1, l] = asoln(0.0, ys[l], thalf);
68      end
69
70      for m in 2:M-1
71        # calculate pi and qi for Thomas' algorithm
72        p = zeros(M);
73        q = zeros(M);
74        p[2], q[2] = 0.0, asoln(xs[m], 0.0, ts[n+1]);
75        for l=2:L-1
76          dd = u_temp[m, l] - (2 * μ *
77                               (u[m, l+1, n] - 2 * u[m, l, n] + u[m, l-1, n]));
78          denom = aay * p[l] + bby;
79          p[l+1] =  -ccy / denom;
80          q[l+1] = (dd - aay * q[l]) / denom;
81        end
82        u[m, L, n+1] = asoln(xs[m], 1.0, ts[n+1]);
83        for l=L-1:-1:2
84          u[m, l, n+1] = p[l+1] * u[m, l+1, n+1] + q[l+1];
85        end
86        u[m, 1, n+1] = asoln(xs[m], 0.0, ts[n+1]);
87      end
88
89      u_exact = zeros(M, L);
90      for m in 1:M, l in 1:L
91        u_exact[m, l] = asoln(xs[m], ys[l], ts[n+1]);
92      end
93
94      if n % output_step == 0
95        println("t=$(ts[n+1]), relative L∞ error: ", norm(u[:, :, n+1] - u_exact,
    Inf) / norm(u_exact, Inf));
96        println("t=$(ts[n+1]), relative L2 error: ", norm(u[:, :, n+1] - u_exact,
    2) / norm(u_exact, 2));
97        open(w -> begin
98          for m in 1:M, l in 1:L
99            write(w, "$(xs[m]),$(ys[l]),$(u[m, l, n+1]),$(asoln(xs[m], ys[l], ts[n
    +1]))\n");
100         end
101         end, dfilename(h, ts[n+1]), "w");
102     end
103   end
104
105   u_exact = zeros(M, L);
106   for m in 1:M, l in 1:L
107     u_exact[m, l] = asoln(xs[m], ys[l], ts[K]);
108   end
109   push!(errs, maximum(map(x -> abs(x), u[:, :, K] - u_exact)));
110   println("t=1.0, relative L∞ error: ", norm(u[:, :, K] - u_exact, Inf) / norm
    (u_exact, Inf));
111   println("t=1.0, relative L2 error: ", norm(u[:, :, K] - u_exact, 2) / norm
    (u_exact, 2));
112   println();
113
114   open(w -> begin
115     for m in 1:M, l in 1:L
116       write(w, "$(xs[m]),$(ys[l]),$(u[m, l, K]),$(asoln(xs[m], ys[l], ts[K]))
    \n");
117     end
118     end, dfilename(h), "w");
119 end
120
121 println(@sprintf("%10s %10s %10s", "h", "max(|e|)", "ratio"));
```

```
122   println(@sprintf("%10.4lf %10.4lf %10s", hs[1], errs[1], "N/A"));
123   println(@sprintf("%10.4lf %10.4lf %10lf", hs[2], errs[2], errs[1]/errs[2]));
124   println(@sprintf("%10.4lf %10.4lf %10lf", hs[3], errs[3], errs[2]/errs[3]));
```