

```

1  include("hw4_helpers.jl");
2
3  const λ = 1.0;
4  const a = 1.0;
5
6  asoln(x, t) = sin(π * (x - t));
7
8  # debugging flag
9  const test_with_gauss_elim = false;
10
11 for h in [1/10; 1/20; 1/40]
12     println("Crank-Nicolson, h = $h");
13     const k = λ * h;
14     const aa = -a * λ / 4;
15     const bb = 1.0;
16     const cc = -aa;
17
18     xs = linspace(-1.0, 1.0, Int(round((2.0) / h)));
19     ts = linspace(0.0, 1.0, Int(round(1.0 / k)));
20     const M, K = length(xs), length(ts);
21
22     u = zeros(M, K);
23     u[:, 1] = map(x -> asoln(x, 0.0), xs);
24     # used for debugging
25     u_debug = (test_with_gauss_elim) ? zeros(M, K) : zeros(1, 1);
26     if test_with_gauss_elim
27         u_debug[:, 1] = map(x -> asoln(x, 0.0), xs);
28     end
29
30     @time for n in 1:K-1
31         # used for debugging
32         A_debug = (test_with_gauss_elim) ? zeros(M, M) : zeros(1, 1);
33         b_debug = (test_with_gauss_elim) ? zeros(M) : zeros(1);
34         if test_with_gauss_elim
35             A_debug[1, 1] = 1.0;
36             b_debug[1] = asoln(-1.0, ts[n+1]);
37             for m=2:M-1
38                 A_debug[m, m-1] = aa;
39                 A_debug[m, m] = bb;
40                 A_debug[m, m+1] = cc;
41                 b_debug[m] = u_debug[m, n] - cc * u_debug[m+1, n] - aa * u_debug[m-1, n];
42             end
43             A_debug[M, M-1] = -λ;
44             A_debug[M, M] = 1+λ;
45             b_debug[M] = u_debug[M, n];
46         end
47
48         # calculate pi and qi for Thomas' algorithm
49         p = zeros(M);
50         q = zeros(M);
51         p[2], q[2] = 0.0, asoln(-1.0, ts[n+1]);
52         for m=2:M-1
53             dd = u[m, n] - a * λ * (u[m+1, n] - u[m-1, n]) / 4;
54             denom = aa * p[m] + bb;
55             p[m+1] = -cc / denom;
56             q[m+1] = (dd - aa * q[m]) / denom;
57         end
58         u[M, n+1] = (u[M, n] + q[M]*λ) / (1 + λ - p[M]*λ);
59         for m=M-1:-1:1
60             u[m, n+1] = p[m+1] * u[m+1, n+1] + q[m+1];
61         end
62
63         if test_with_gauss_elim
64             u_debug[:, n+1] = A_debug \ b_debug;

```

```

65         @show norm(u[:, n+1] - u_debug[:, n+1], Inf);
66     end
67 end
68
69 plot_solution(xs, ts, u, asoln; t="Crank-Nicolson, \h = \h\%",
70             show_plot=false, fname="cn_thomas_M-$M.png");
71 if test_with_gauss_elim
72     plot_solution(xs, ts, u_debug, asoln; t="Crank-Nicolson, \h = \h\%",
73             show_plot=false, fname="cn_gauss_M-$M.png");
74 end
75 u_a = map(x -> asoln(x, 1.0), xs);
76 println("Relative L2 error: ", norm(u[:, end] - u_a, 2) / norm(u_a, 2));
77 println("Relative L $\infty$  error: ", norm(u[:, end] - u_a, Inf) / norm(u_a, Inf));
78 println();
79 end

```