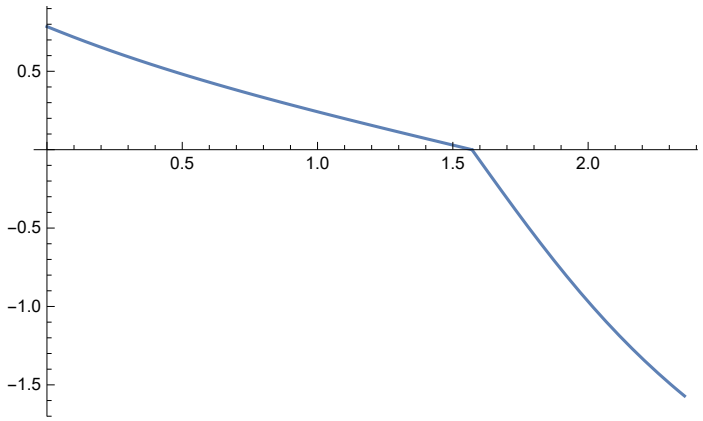
In[1]:=
```
$Assumptions = θ ∈ Reals && 0 ≤ θ ≤ π;
```

In[2]:=
```
ωA = ArcSin[(2 * Cos[θ] * (Sqrt[2] - Sin[θ])) / (3 + Cos[2 * θ])];
ωB = ArcSin[(2 * Cos[θ] * (Sqrt[2] + Sin[θ])) / (3 + Cos[2 * θ])];
```

In[4]:=
```
Plot[If[θ ≤ π / 2, ωA, ωB], {θ, 0, 3 * π / 4}]
```

Out[4]=



In[5]:=
```
ω[θArg_] := If[θArg ≤ π / 2, ωA /. {θ → θArg}, ωB /. {θ → θArg}]
```

In[6]:=
```
ω[0]
```

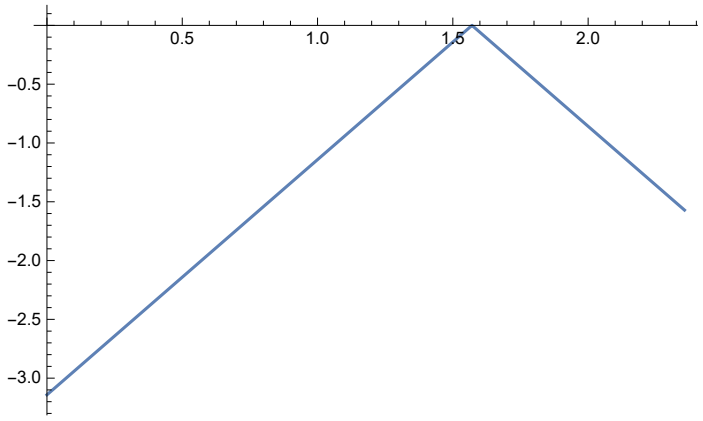
Out[6]=
$$\frac{\pi}{4}$$

In[7]:=
```
FullSimplify[ω[3 * π / 4]]
```

Out[7]=
$$-\frac{\pi}{2}$$

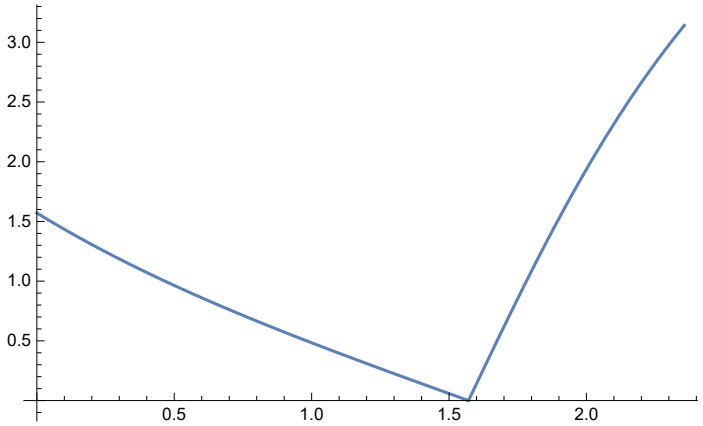
In[8]:=
```
γm[θ_] := -Abs[2 * θ - π];
Plot[γm[θ], {θ, 0, 3 * π / 4}]
```

Out[9]=



In[10]:=
```
γv[θ_] := If[θ ≤ π/2, 2 (ArcCsc[√(1 + Cos[θ]^2)] - ArcCsc[√(1 + Cos[θ]^2) Csc[θ]]),
  2 (ArcCos[Sin[θ]/√(1 + Cos[θ]^2)] + ArcSec[√(1 + Cos[θ]^2)])];
Plot[γv[θ], {θ, 0, 3 * π / 4}]
```

Out[11]=

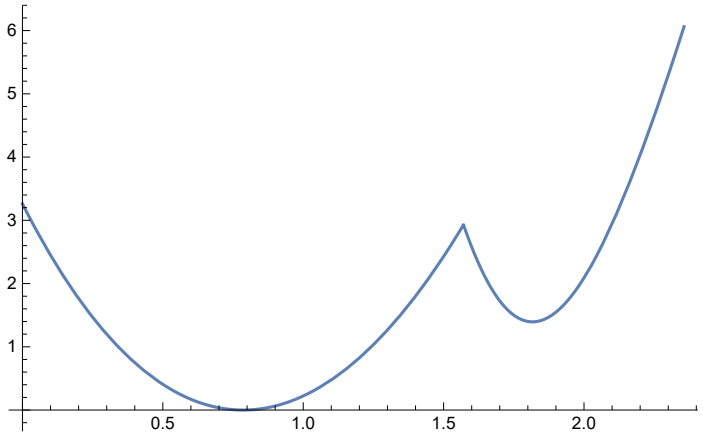


In[12]:=
```
reflIf[θ_, flag_] := If[flag, π - θ, θ]
```

In[13]:=
```
U[kmVec_, kvVec_, θ0mVec_, θ0vVec_, reflFlags_, θ_] :=
  Sum[kmVec[[i]]/2 * (γm[reflIf[θ, reflFlags[[i]]]] - γm[θ0mVec[[i]]])^2 + kvVec[[i]]/2 *
      (γv[reflIf[θ, reflFlags[[i]]]] - γv[θ0vVec[[i]]])^2, {i, 1, Length[kmVec]}]
```

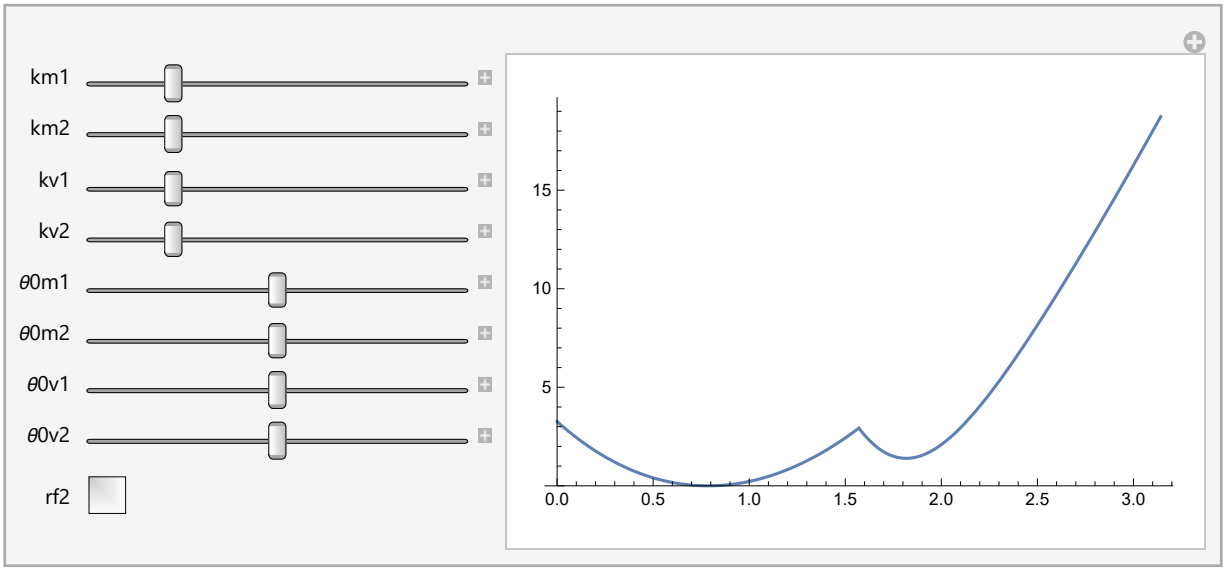
In[14]:=
```
Plot[U[{2}, {2}, {π / 4}, {π / 4}, {False}, θ], {θ, 0, 3 * π / 4}]
```

Out[14]=



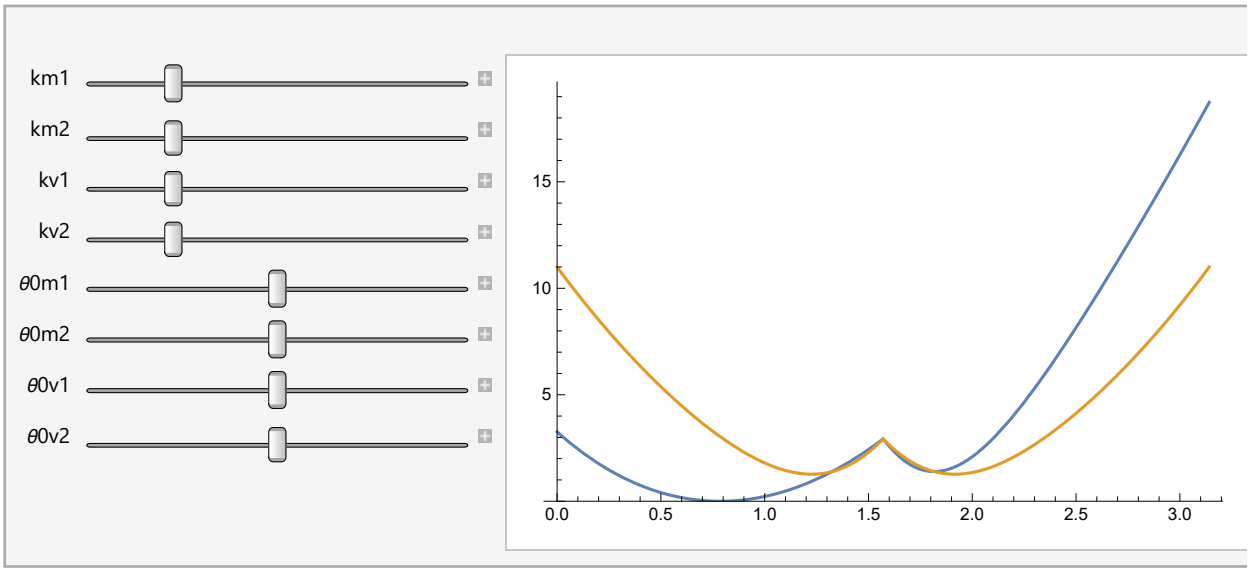
In[15]:=
```
Manipulate[Plot[U[{km1, km2}, {kv1, kv2}, {θ0m1, θ0m2}, {θ0v1, θ0v2}, {False, rf2}, θ],
  {θ, 0, π}, PlotRange → {0, Automatic}], {{km1, 1}, 0, 5, 0.5},
 {{km2, 1}, 0, 5, 0.5}, {{kv1, 1}, 0, 5, 0.5}, {{kv2, 1}, 0, 5, 0.5},
 {{θ0m1, π / 4}, 0, π / 2, π / 24}, {{θ0m2, π / 4}, 0, π / 2, π / 24},
 {{θ0v1, π / 4}, 0, π / 2, π / 24}, {{θ0v2, π / 4}, 0, π / 2, π / 24}, {rf2, {False, True}}]
```

Out[15]=



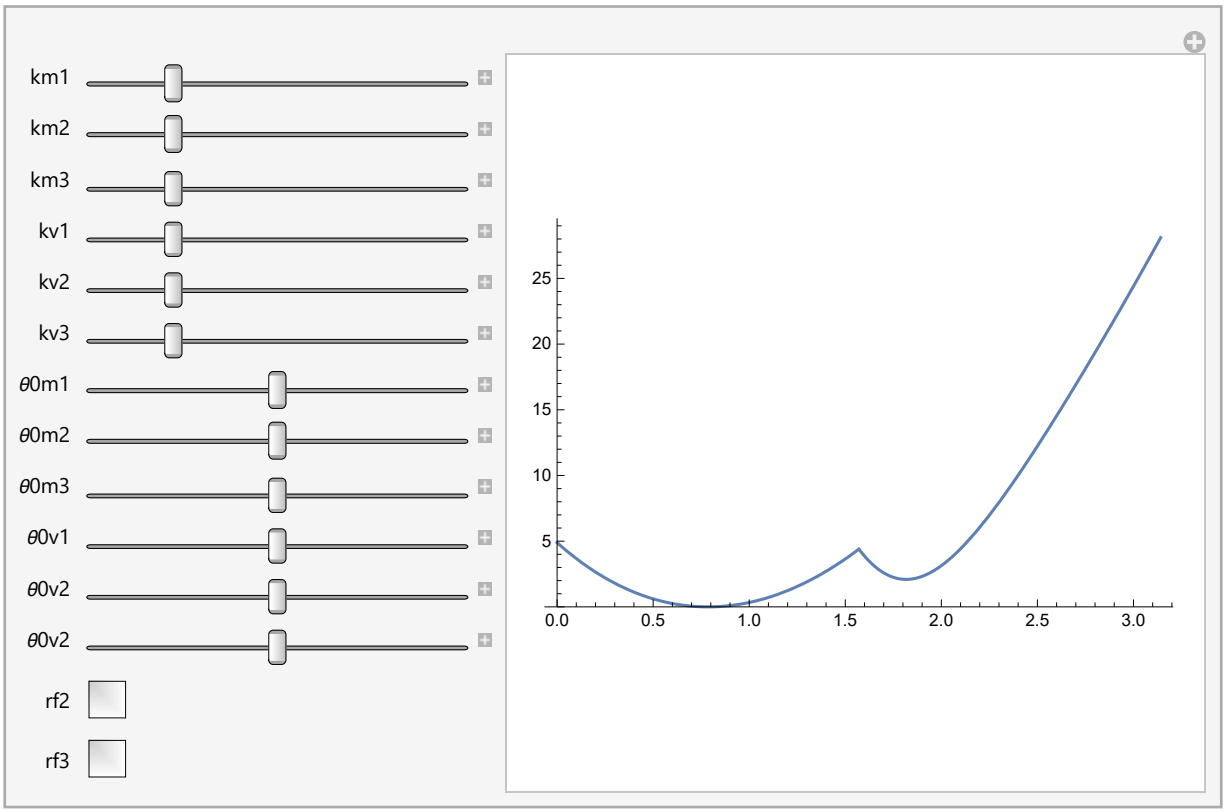
In[16]:=
```
Manipulate[Plot[{U[{km1, km2}, {kv1, kv2}, {θ0m1, θ0m2}, {θ0v1, θ0v2}, {False, False}, θ],
   U[{km1, km2}, {kv1, kv2}, {θ0m1, θ0m2}, {θ0v1, θ0v2}, {False, True}, θ]}, {θ, 0, π},
  PlotRange → {0, Automatic}, PlotLegends → Automatic], {{km1, 1}, 0, 5, 0.5},
 {{km2, 1}, 0, 5, 0.5}, {{kv1, 1}, 0, 5, 0.5}, {{kv2, 1}, 0, 5, 0.5},
 {{θ0m1, π / 4}, 0, π / 2, π / 24}, {{θ0m2, π / 4}, 0, π / 2, π / 24},
 {{θ0v1, π / 4}, 0, π / 2, π / 24}, {{θ0v2, π / 4}, 0, π / 2, π / 24}]
```

Out[16]=



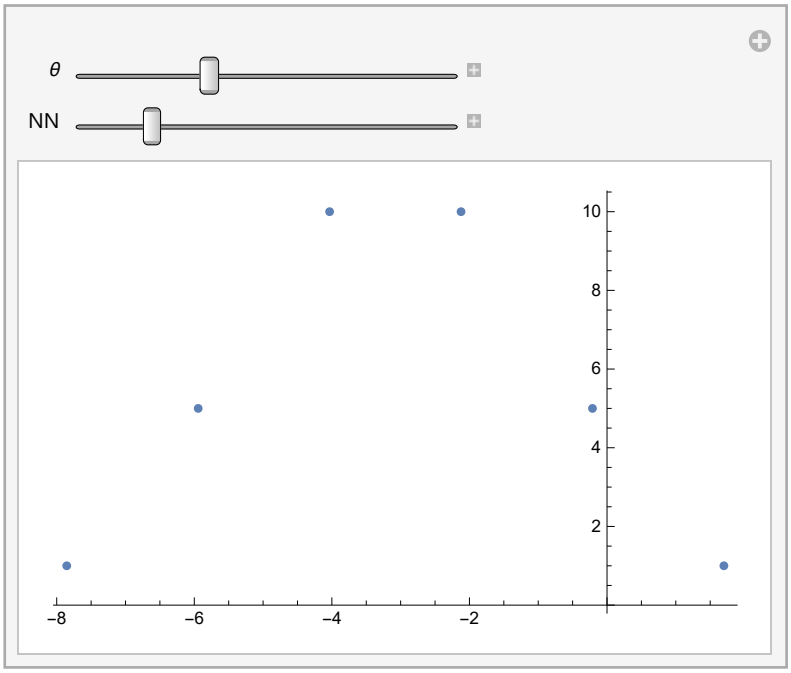
In[17]:=
```
Manipulate[Plot[U[{km1, km2, km3}, {kv1, kv2, kv3}, {θ0m1, θ0m2, θ0m3},
    {θ0v1, θ0v2, θ0m3}, {False, rf2, rf3}, θ], {θ, 0, π}, PlotRange → {0, Automatic}],
  {{km1, 1}, 0, 5, 0.5}, {{km2, 1}, 0, 5, 0.5}, {{km3, 1}, 0, 5, 0.5}, {{kv1, 1}, 0, 5, 0.5},
  {{kv2, 1}, 0, 5, 0.5}, {{kv3, 1}, 0, 5, 0.5}, {{θ0m1, π / 4}, 0, π / 2, π / 24},
  {{θ0m2, π / 4}, 0, π / 2, π / 24}, {{θ0m3, π / 4}, 0, π / 2, π / 24},
  {{θ0v1, π / 4}, 0, π / 2, π / 24}, {{θ0v2, π / 4}, 0, π / 2, π / 24},
  {{θ0v2, π / 4}, 0, π / 2, π / 24}, {rf2, {False, True}}, {rf3, {False, True}}]
```

Out[17]=



In[18]:=
```
Manipulate[ListPlot[
  Table[{(NN - n) * ω[θ] + n * ω[π - θ], Factorial[NN] / (Factorial[NN - n] * Factorial[n])},
   {n, 0, NN}]], {{θ, π / 4}, 0, 3 * π / 4, π / 32}, {{NN, 5}, 1, 25}]
```

Out[18]=

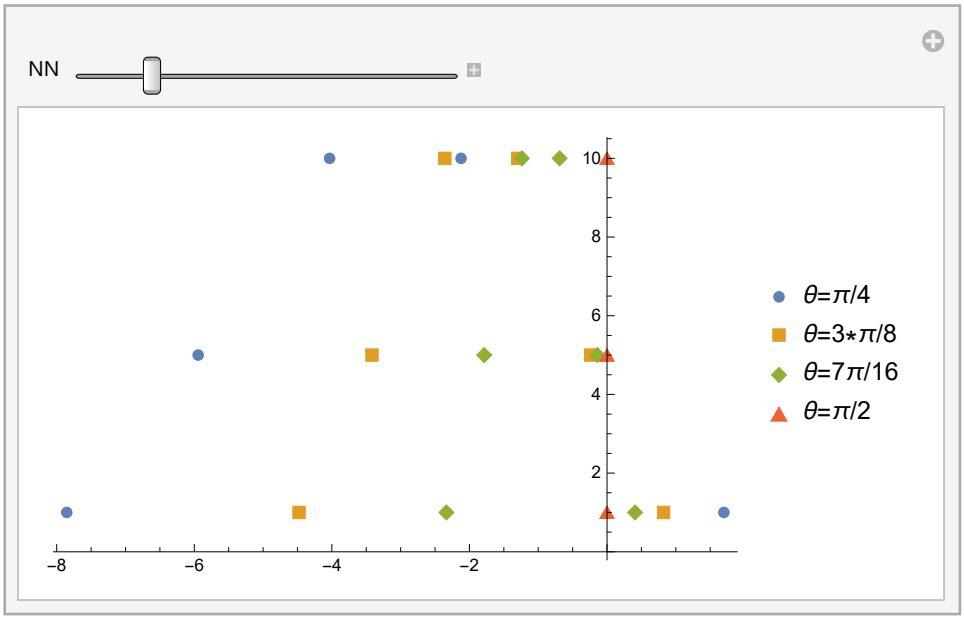


In[19]:=
```
Clear[θ]
```

In[20]:=
```
ConfigsTable[NN_, θ_] := Table[{(NN - n) * ω[θ] + n * ω[π - θ],
    (Factorial[NN] / (Factorial[NN - n] * Factorial[n]))}, {n, 0, NN}];
```

In[21]:=
```
p = Manipulate[ListPlot[{ConfigsTable[NN, π / 4], ConfigsTable[NN, 3 * π / 8],
     ConfigsTable[NN, 7 * π / 16], ConfigsTable[NN, π / 2]}, PlotMarkers → Automatic,
   PlotLegends → {"θ=π/4", "θ=3*π/8", "θ=7π/16", "θ=π/2"}], {{NN, 5}, 1, 25}]
```

Out[21]=



In[22]:=
```
Export["~/google-drive/Temp/2020-07-14/omega-vs-nconfigs_N-14.png",
 ListPlot[{ConfigsTable[14, π / 4], ConfigsTable[14, 3 * π / 8],
   ConfigsTable[14, 7 * π / 16], ConfigsTable[14, π / 2]}, PlotMarkers → Automatic,
  PlotLegends → {"θ=π/4", "θ=3π/8", "θ=7π/16", "θ=π/2"}, AxesLabel → {Ω, NΩ}]
]
```

Export: Directory C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-14\ does not exist.

Export: Cannot open ~/google-drive/Temp/2020-07-14/omega-vs-nconfigs_N-14.png.

Out[22]= $Failed

In[23]:=
```
UExprA = Simplify[1 / 2 * (γv[θ] - γv0) ^ 2, 0 ≤ θ < π / 2]
```

Out[23]=
$$\frac{1}{2}\left(\gamma v0 - 2\,\text{ArcCsc}\left[\sqrt{1+\text{Cos}[\theta]^2}\,\right] + 2\,\text{ArcCsc}\left[\sqrt{1+\text{Cos}[\theta]^2}\,\text{Csc}[\theta]\,\right]\right)^2$$

In[24]:=
```
dUExprA = FullSimplify[D[UExprA, {θ, 2}], 0 ≤ θ < π / 2]
```

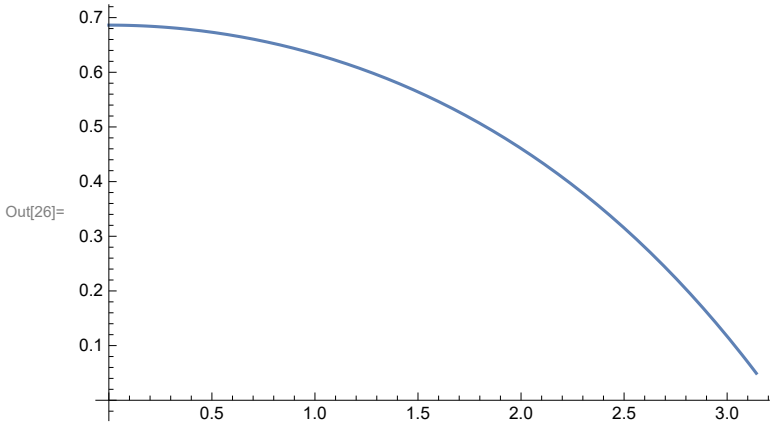
Out[24]=
$$\frac{1}{\left(3+\text{Cos}[2\,\theta]\right)^2} 4\left(-2+\left(\gamma v0 - 2\,\text{ArcCsc}\left[\sqrt{1+\text{Cos}[\theta]^2}\,\right] + 2\,\text{ArcCsc}\left[\sqrt{1+\text{Cos}[\theta]^2}\,\text{Csc}[\theta]\,\right]\right)\text{Cos}[\theta]\right)$$
$$\left(-5+\text{Cos}[2\,\theta] + 4\sqrt{2}\,\text{Sin}[\theta]\right)$$

In[25]:=
```
NMinimize[{dUExprA /. {γv0 → π / 4}, 0 ≤ θ < π / 2}, {θ}][[1]]
```

Out[25]= 0.653947

In[26]:=
```
Plot[NMinimize[{dUExprA /. {γv0 → γvv}, 0 ≤ θ < π/2}, {θ}][[1]], {γvv, 0, π}]
```

Out[26]=



In[27]:=
```
(*Export["~/Dev/origami-paper-1/PRL/figs/convexA.csv",Table[
  {N[γvv],NMinimize[{dUExprA/.{γv0→γvv},0≤θ<π/2},{θ}][[1]]},{γvv,0,3*π/2,π/24}]]*)
```

In[28]:=
```
UExprB = Simplify[1/2 * (γv[θ] - γv0)^2, π/2 < θ ≤ 3 * π / 4]
```

Out[28]=
$$\frac{1}{2}\left(\gamma v0 - 2\left(\text{ArcCos}\left[\frac{\text{Sin}[\theta]}{\sqrt{1+\text{Cos}[\theta]^2}}\right] + \text{ArcSec}\left[\sqrt{1+\text{Cos}[\theta]^2}\right]\right)\right)^2$$

In[29]:=
```
dUExprB = Simplify[D[UExprB, {θ, 2}], π/2 < θ ≤ 3 * π / 4]
```

Out[29]=
$$\frac{1}{\left(3+\text{Cos}[2\theta]\right)^2}$$
$$4\left(-2+\gamma v0\,\text{Cos}[\theta] - 2\,\text{ArcCos}\left[\frac{\text{Sin}[\theta]}{\sqrt{1+\text{Cos}[\theta]^2}}\right]\text{Cos}[\theta] - 2\,\text{ArcSec}\left[\sqrt{1+\text{Cos}[\theta]^2}\right]\text{Cos}[\theta]\right)$$
$$\left(-5+\text{Cos}[2\theta] - 4\sqrt{2}\,\text{Sin}[\theta]\right)$$

In[30]:=

In[31]:=
```
(*Export["~/Dev/origami-paper-1/PRL/figs/convexB.csv",
 Table[{N[γvv],NMinimize[{dUExprB/.{γv0→γvv},π/2<θ≤3*π/4},{θ}][[1]]},
  {γvv,0,3*π/2,π/24}]]*)
```

In[32]:=
```
solveA = Solve[dUExprA == 0, γv0]
```

Out[32]=
$$\left\{\left\{\gamma v0 \to 2\left(\text{ArcCsc}\left[\sqrt{1+\text{Cos}[\theta]^2}\right] - \text{ArcCsc}\left[\sqrt{1+\text{Cos}[\theta]^2}\,\text{Csc}[\theta]\right] + \text{Sec}[\theta]\right)\right\}\right\}$$

In[33]:=

Visualization

single unit

In[34]:=
```
refTri = {{0, 0, 0}, {1, 0, 0}, {1, 1, 0}};
rvec = Array[r, 3];
A[r_] := {{0, -r[[3]], r[[2]]}, {r[[3]], 0, -r[[1]]}, {-r[[2]], r[[1]], 0}};
```

In[37]:=
```
A[rvec] + Transpose[A[rvec]]
```

Out[37]= $\{\{0, 0, 0\}, \{0, 0, 0\}, \{0, 0, 0\}\}$

In[38]:=
```
Q[rvec_, θ_] := (1 - Cos[θ]) * rvec⊗rvec + Cos[θ] * IdentityMatrix[3] + Sin[θ] * A[rvec]
FullSimplify[Q[rvec, θ].Transpose[Q[rvec, θ]], rvec.rvec == 1]
```

Out[39]= $\{\{1, 0, 0\}, \{0, 1, 0\}, \{0, 0, 1\}\}$

In[40]:=
```
rotate[points_, u_, θ_] := Transpose[Q[u, θ].Transpose[points]];
```

In[41]:=
```
transX[points_, c_] := Module[{x = points},
  For[i = 1, i ≤ Length[x], i++,
   x[[i]] = x[[i]] + c;
  ];
  x
 ]
```

In[42]:=
```
θToψ = {θ → π / 2 - ψ};
ψToθ = {ψ → π / 2 - θ};
rm = {0, 1, 0}
r1 = {1 / Sqrt[2], -1 / Sqrt[2], 0}
rm1 = Simplify[Q[r1, ψ].rm]
```

Out[44]= $\{0, 1, 0\}$

Out[45]= $\left\{\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right\}$

Out[46]= $\left\{\frac{1}{2}\left(-1 + \text{Cos}[\psi]\right), \text{Cos}\left[\frac{\psi}{2}\right]^2, \frac{\text{Sin}[\psi]}{\sqrt{2}}\right\}$

In[47]:=
```
θv = θ /. θToψ
r2 = Simplify[{-Sin[θv] / Sqrt[2], -Sin[θv] / Sqrt[2], -Cos[θv]}]
```

Out[47]= $\frac{\pi}{2} - \psi$

Out[48]= $\left\{-\frac{\text{Cos}[\psi]}{\sqrt{2}}, -\frac{\text{Cos}[\psi]}{\sqrt{2}}, -\text{Sin}[\psi]\right\}$

In[49]:=
```
rm2 = FullSimplify[Q[r2, ϕ1].rm1]
```

Out[49]= $\left\{\frac{1}{2}\left(-\text{Cos}[\phi 1] + \text{Cos}[\psi] + \text{Sin}[\phi 1]\ \text{Sin}[\psi]\right),\right.$
$\left.\frac{1}{2}\left(\text{Cos}[\phi 1] + \text{Cos}[\psi] + \text{Sin}[\phi 1]\ \text{Sin}[\psi]\right), \frac{-\text{Cos}[\psi]\ \text{Sin}[\phi 1] + \text{Sin}[\psi]}{\sqrt{2}}\right\}$

In[50]:=
```
θϕToTri[θ_, ϕ_] := Transpose[Q[r2 /. {ψ → π / 2 - θ}, ϕ].Q[r1, π / 2 - θ].Transpose[refTri]]
```

In[51]:=
```
θϕToRm[θ_, ϕ_] := Module[{tri, rmv},
  tri = θϕToTri[θ, ϕ];
  rmv = tri[[3]] - tri[[2]];
  rmv
 ]
```

In[52]:=
```
compat = Simplify[Solve[(θϕToRm[θ, ϕ1])[[1]] == 0, {ϕ1}]] /. {c1 → 0}
```

Out[52]=
$$\left\{\left\{\phi 1 \to \text{ArcTan}\left[-\sqrt{2}\,\text{Cos}[\theta]^2+\text{Sin}[\theta],\,-\text{Cos}[\theta]\left(\sqrt{2}+\text{Sin}[\theta]\right)\right]\right\},\right.$$
$$\left.\left\{\phi 1 \to \text{ArcTan}\left[\sqrt{2}\,\text{Cos}[\theta]^2+\text{Sin}[\theta],\,\text{Cos}[\theta]\left(\sqrt{2}-\text{Sin}[\theta]\right)\right]\right\}\right\}$$

In[53]:=
```
reflXY[points_] := Module[{x = points, temp},
   For[i = 1, i ≤ Length[x], i++,
    temp = x[[i]][[1]];
    x[[i]][[1]] = x[[i]][[2]];
    x[[i]][[2]] = temp;
   ];
   x
  ];
reflX[points_, idx_] := Module[{x = points},
   For[i = 1, i ≤ Length[x], i++,
    x[[i]][[idx]] = -x[[i]][[idx]];
   ];
   x
  ];
```

In[55]:=
```
θϕToTri2[θ_, ϕ_] :=
 Transpose[Q[-r2 /. {ψ → π/2 - θ}, ϕ].Q[r1, π/2 - θ].Transpose[reflXY[refTri]]]
```

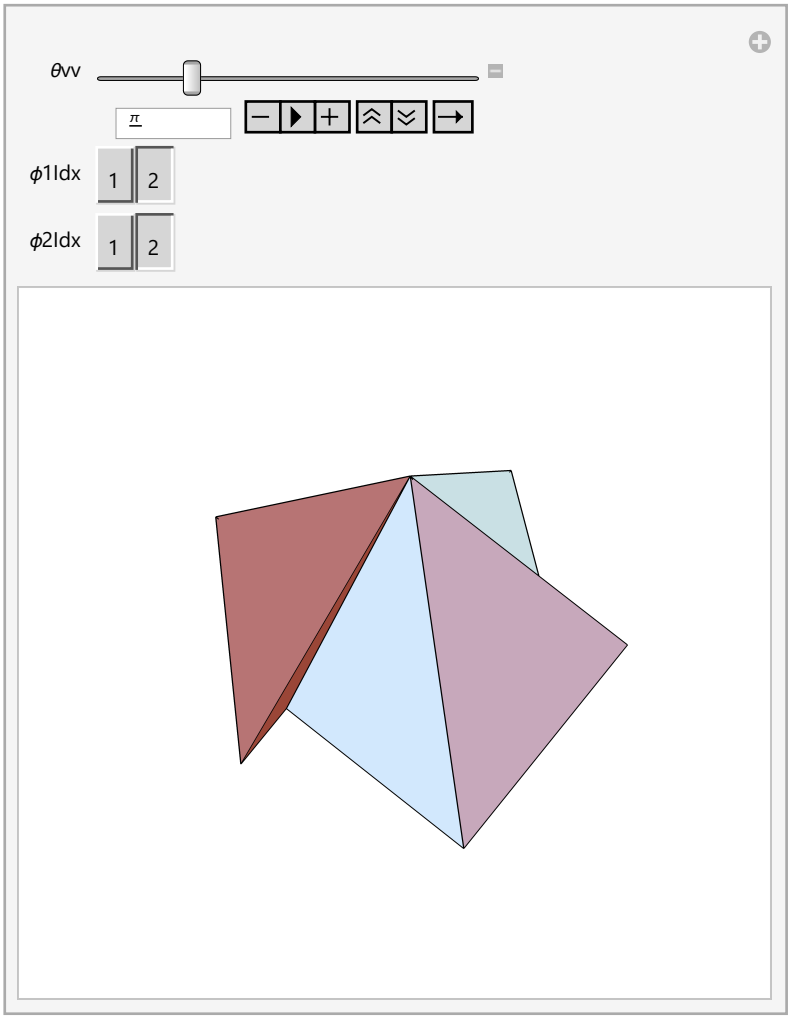
In[56]:=
```
θϕToTri2[π/2, 0]
```

Out[56]=
```
{{0, 0, 0}, {0, 1, 0}, {1, 1, 0}}
```

In[57]:=
```
refl1[points_, θ_] := transX[reflX[transX[points, {-Sin[θ], 0, 0}], 1], {Sin[θ], 0, 0}];
refl2[points_, θ_] := transX[reflX[transX[points, {0, -Sin[θ], 0}], 2], {0, Sin[θ], 0}];
```

In[59]:=
```
Manipulate[
 ϕ1vv = (ϕ1 /. compat[[ϕ1Idx]]) /. {θ → θvv};
 ϕ2vv = (ϕ1 /. compat[[ϕ2Idx]]) /. {θ → θvv};
 aTri1 = θϕToTri[θvv, ϕ1vv];
 aTri2 = θϕToTri2[θvv, ϕ2vv];
 aTri3 = refl1[aTri1, θvv];
 aTri4 = refl1[aTri2, θvv];
 aTri5 = refl2[aTri1, θvv];
 aTri6 = refl2[aTri2, θvv];
 aTri7 = refl2[aTri3, θvv];
 aTri8 = refl2[aTri4, θvv];
 Graphics3D[{Triangle[aTri1], Triangle[aTri2], Triangle[aTri3], Triangle[aTri4],
   Triangle[aTri5], Triangle[aTri6], Triangle[aTri7], Triangle[aTri8]}, Boxed → False],
 {{θvv, π/12}, 0, 3 * π / 4, π/24}, {{ϕ1Idx, 1}, {1, 2}}, {{ϕ2Idx, 1}, {1, 2}}
]
```

Out[59]=



In[60]:=
```
Clear[Waterbomb]
```

In[61]:=
```
Waterbomb[θv_, ϕ1Idx_, ϕ2Idx_] :=
  Module[{aTri1, aTri2, aTri3, aTri4, aTri5, aTri6, aTri7, aTri8, ϕ1v, ϕ2v, tris, k},
   ϕ1v = (ϕ1 /. compat[[ϕ1Idx]]) /. {θ → θv};
   ϕ2v = (ϕ1 /. compat[[ϕ2Idx]]) /. {θ → θv};
   aTri1 = θϕToTri[θv, ϕ1v];
   aTri2 = θϕToTri2[θv, ϕ2v];
   aTri3 = refl1[aTri1, θv];
   aTri4 = refl1[aTri2, θv];
   aTri5 = refl2[aTri1, θv];
   aTri6 = refl2[aTri2, θv];
   aTri7 = refl2[aTri3, θv];
   aTri8 = refl2[aTri4, θv];
   tris = {aTri1, aTri2, aTri3, aTri4, aTri5, aTri6, aTri7, aTri8};
   {
    Catenate[{{EdgeForm[]}, Table[Triangle[tri], {tri, tris}]}],
    Catenate[{{EdgeForm[{Black}]}, Catenate[Table[
        If[k == 5 || k == 1 || k == 3 || k == 7,
         {Line[{tris[[k]][[1]], tris[[k]][[3]]}], Line[{tris[[k]][[2]],
            tris[[k]][[3]]}], Line[{tris[[k]][[1]], tris[[k]][[2]]}]},
         {Line[{tris[[k]][[1]], tris[[k]][[3]]}], Line[
           {tris[[k]][[2]], tris[[k]][[3]]}]}
        ]
        , {k, 1, Length[tris]}]]}]
   }
  ];
```

In[62]:=
```
θToCompat[θ_] := If[θ ≤ π / 2, 2, 1]
```

In[63]:=
```
(*Export["symm-folding.gif",
 Manipulate[Graphics3D[Waterbomb[θv,If[θv≤π/2,2,1],If[θv≤π/2,2,1]],Boxed→False],
  {{θv,0},0,3*π/4,π/128}]]*)
```

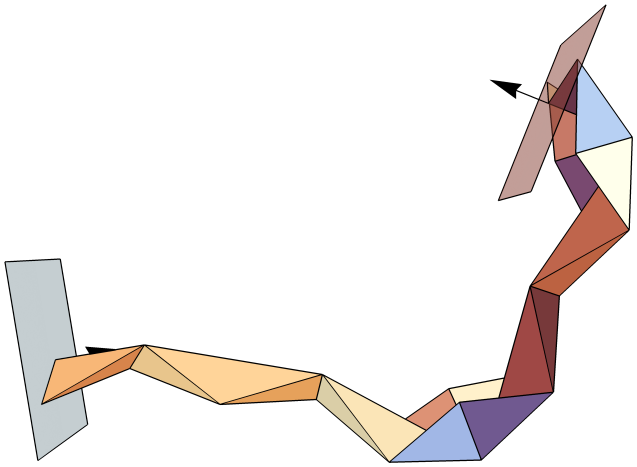
In[64]:=
```
WaterbombStrip[θv_, switches_] := Module[{WBs, θi, WBi, i, j, ci,
    ui, ωi, uhati, Σω, WB1Tris, WBNTris, WB1u, WB1v, WBNu, WBNv, WB1n, WBNn},
   Σω = ω[θv];
   WBs = {Waterbomb[θv, θToCompat[θv], θToCompat[θv]]};
   For[i = 2, i ≤ Length[switches], i++,
    θi = If[switches[[i]], θv, π - θv];
    WBi = Waterbomb[θi, θToCompat[θi], θToCompat[θi]];
    ci = WBs[[i - 1]][[1]][[4 + 1]][[1]][[1]] - WBi[[1]][[2 + 1]][[1]][[1]];
    ui = WBs[[i - 1]][[1]][[8 + 1]][[1]][[1]] - WBs[[i - 1]][[1]][[4 + 1]][[1]][[1]];
    uhati = ui / Sqrt[ui.ui];
    Σω = Σω + ω[θi];
    For[j = 2, j ≤ Length[WBi[[1]]], j++,
     WBi[[1]][[j]][[1]] = transX[rotate[WBi[[1]][[j]][[1]], uhati, Σω], ci];
    ];
    For[j = 2, j ≤ Length[WBi[[2]]], j++,
     WBi[[2]][[j]][[1]] = transX[rotate[WBi[[2]][[j]][[1]], uhati, Σω], ci];
    ];
    Σω = Σω + ω[θi];
    AppendTo[WBs, WBi];
```

```
    ];
    WB1Tris = WBs[[1]][[1]];
    WBNTris = Last[WBs][[1]];
    WB1u = WB1Tris[[7]][[1]][[2]] - WB1Tris[[7]][[1]][[1]];
    WB1v = WB1Tris[[3]][[1]][[2]] - WB1Tris[[3]][[1]][[1]];
    WB1n = WB1u × WB1v;
    WBNu = WBNTris[[5]][[1]][[2]] - WBNTris[[5]][[1]][[1]];
    WBNv = WBNTris[[9]][[1]][[2]] - WBNTris[[9]][[1]][[1]];
    WBNn = WBNu × WBNv;
    {WBs, {EdgeForm[Black], Line[{WB1Tris[[3]][[1]][[1]], WB1Tris[[3]][[1]][[2]]}],
      Line[{WB1Tris[[7]][[1]][[1]], WB1Tris[[7]][[1]][[2]]}],
      Line[{WBNTris[[5]][[1]][[1]], WBNTris[[5]][[1]][[2]]}],
      Line[{WBNTris[[9]][[1]][[1]], WBNTris[[9]][[1]][[2]]}]},
     {EdgeForm[Black], Arrow[{WB1Tris[[3]][[1]][[2]], WB1Tris[[3]][[1]][[2]] - WB1n}],
      Arrow[{WBNTris[[5]][[1]][[2]], WBNTris[[5]][[1]][[2]] + WBNn}]},
     {Opacity[0.5], Polygon[rotate[{{0, 2.0, 1.5}, {0, 2.0, -0.5},
         {0, -0.5, -0.5}, {0, -0.5, 1.5}}, {0, 1, 0}, -ω[θv]]]},
     {Opacity[0.5], Polygon[transX[rotate[{{0, 2.0, 1.5}, {0, 2.0, -0.5},
          {0, -0.5, -0.5}, {0, -0.5, 1.5}}, {0, 1, 0}, Σω], WBNTris[[5]][[1]][[1]]]]]}
    }
   ]
WaterbombStripPlanes[θv_, switches_] := Module[{WBs, θi, WBi, i, j, ci,
    ui, ωi, uhati, Σω, WB1Tris, WBNTris, WB1u, WB1v, WBNu, WBNv, WB1n, WBNn},
   Σω = ω[θv];
   WBs = {Waterbomb[θv, θToCompat[θv], θToCompat[θv]]};
   For[i = 2, i ≤ Length[switches], i++,
    θi = If[switches[[i]], θv, π - θv];
    WBi = Waterbomb[θi, θToCompat[θi], θToCompat[θi]];
    ci = WBs[[i - 1]][[1]][[4 + 1]][[1]][[1]] - WBi[[1]][[2 + 1]][[1]][[1]];
    ui = WBs[[i - 1]][[1]][[8 + 1]][[1]][[1]] - WBs[[i - 1]][[1]][[4 + 1]][[1]][[1]];
    uhati = ui / Sqrt[ui.ui];
    Σω = Σω + ω[θi];
    For[j = 2, j ≤ Length[WBi[[1]]], j++,
     WBi[[1]][[j]][[1]] = transX[rotate[WBi[[1]][[j]][[1]], uhati, Σω], ci];
    ];
    For[j = 2, j ≤ Length[WBi[[2]]], j++,
     WBi[[2]][[j]][[1]] = transX[rotate[WBi[[2]][[j]][[1]], uhati, Σω], ci];
    ];
    Σω = Σω + ω[θi];
    AppendTo[WBs, WBi];
   ];
   WB1Tris = WBs[[1]][[1]];
   WBNTris = Last[WBs][[1]];
   WB1u = WB1Tris[[7]][[1]][[2]] - WB1Tris[[7]][[1]][[1]];
   WB1v = WB1Tris[[3]][[1]][[2]] - WB1Tris[[3]][[1]][[1]];
   WB1n = WB1u × WB1v;
   WBNu = WBNTris[[5]][[1]][[2]] - WBNTris[[5]][[1]][[1]];
   WBNv = WBNTris[[9]][[1]][[2]] - WBNTris[[9]][[1]][[1]];
   WBNn = WBNu × WBNv;
   {
    {EdgeForm[Black], Arrow[{{0, 0, 0}, -WB1n}], Arrow[{{0, 0, 0}, WBNn}]},
```

```
    {Opacity[0.5], Polygon[rotate[{{0.0, -1.0, 1.0},
        {0.0, -1.0, -1.0}, {0.0, 1.0, -1.0}, {0.0, 1.0, 1.0}}, {0, 1, 0}, -ω[ϴv]]]},
    {Opacity[0.5], Polygon[rotate[{{0.0, -1.0, 1.0}, {0.0, -1.0, -1.0},
        {0.0, 1.0, -1.0}, {0.0, 1.0, 1.0}}, {0, 1, 0}, Σω]]}
   }
  ]
```

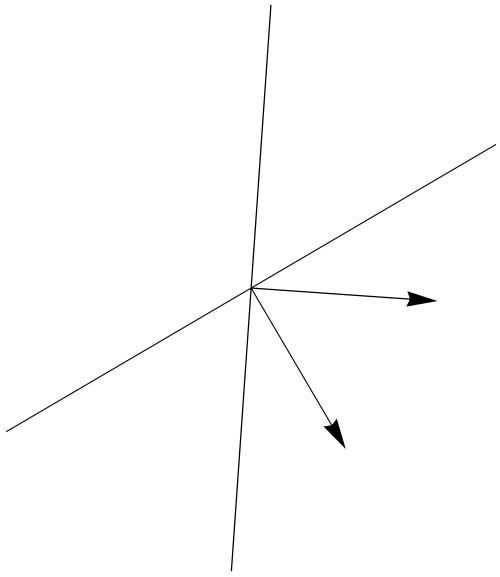
In[66]:=
```
Graphics3D[WaterbombStrip[3 * π / 8, {True, True, False, True, False}],
 Boxed -> False, ViewPoint -> Front]
```

Out[66]=



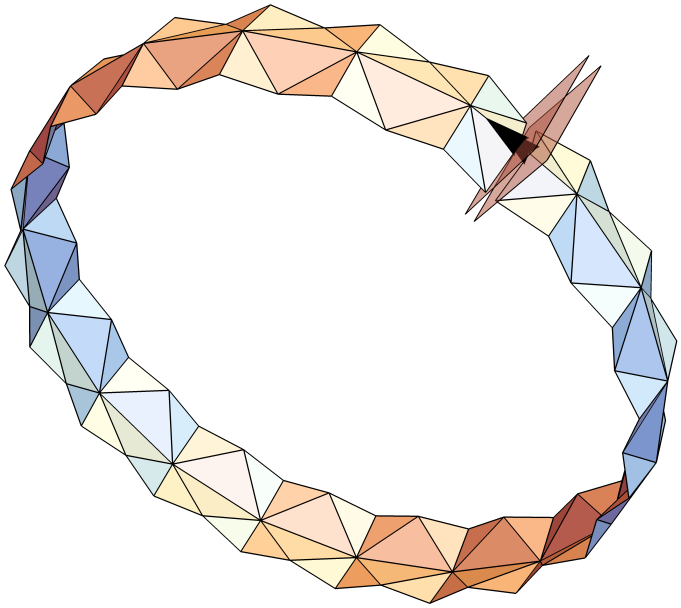
In[67]:=
```
Graphics3D[WaterbombStripPlanes[π / 3, {True, True, False, True}],
 Boxed -> False, ViewPoint -> Front]
```

Out[67]=



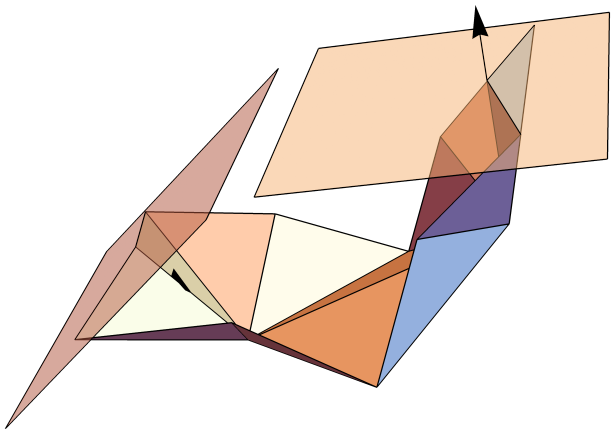
In[68]:=
```
Graphics3D[WaterbombStrip[π/3 + π/24, {True, True, True, True, True, True, True, True,
    True, True, True, True, True, True, True, True, True, True, True, True}], Boxed → False]
```

Out[68]=



In[69]:=
```
Graphics3D[WaterbombStrip[7 * π / 12, {True, True}], Boxed → False]
```

Out[69]=



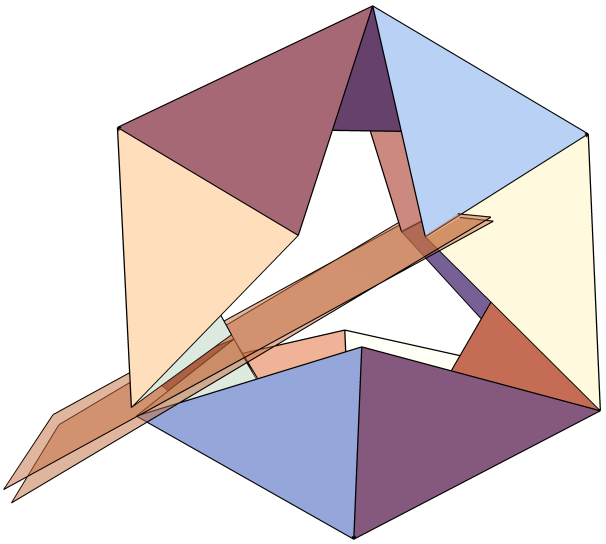
In[70]:=
```
(*Export["up-down_to_down-up.gif",
 Animate[Graphics3D[WaterbombStrip[θv,{True,False}],Boxed→False],
  {θv,π/4,3*π/4,π/64},AnimationRepetitions→Infinity,AnimationRunning→True]]*)
```

In[71]:=
```
(*Export["up-down_to_down-up_energy.gif",
 Animate[Show[Plot[U[{Sqrt[2],Sqrt[2]},{1,1},{π/4,π/4},{π/4,π/4},{True,False},θ],
    {θ,π/4,3*π/4},PlotRange→{0,Automatic}],
   ListPlot[{{θv,U[{Sqrt[2],Sqrt[2]},{1,1},{π/4,π/4},{π/4,π/4},{True,False},θv]}}]],
  {{θv,π/4},π/4,3*π/4,π/64},AnimationRepetitions→Infinity,AnimationRunning→True]]*)
```

In[72]:=
```
(*Animate[Graphics3D[WaterbombStrip[θv,{True,False}],Boxed→False];
 Plot[U[{Sqrt[2],Sqrt[2]},{1,1},{π/4,π/4},{π/4,π/4},{True,False},θ],
  {θ,π/4,3*π/4},PlotRange→{0,Automatic}],{θv,π/4,3*π/4,π/64},
 AnimationRepetitions→Infinity,AnimationRunning→False]*)
```

In[73]:=
```
Graphics3D[WaterbombStrip[5 * π / 8 + π / 64 + π / 128, {True, True, True}], Boxed → False]
```

Out[73]=



In[74]:=
```
WaterbombStripIncompat[θv_, switches_] := Module[{WBs, θi, WBi, i, j, ci, ui, ωi, uhati},
  WBs = {Waterbomb[θv, θToCompat[θv], θToCompat[θv]]};
  For[i = 2, i ≤ Length[switches], i++,
   θi = If[switches[[i]], θv, π - θv];
   WBi = Waterbomb[θi, θToCompat[θi], θToCompat[θi]];
   ci = WBs[[i - 1]][[4]][[1]][[1]] - WBi[[2]][[1]][[1]];
   ui = WBs[[i - 1]][[8]][[1]][[1]] - WBs[[i - 1]][[4]][[1]][[1]];
   uhati = ui / Sqrt[ui.ui];
   For[j = 1, j ≤ Length[WBi], j++,
    WBi[[j]][[1]] = transX[WBi[[j]][[1]], ci];
   ];
   AppendTo[WBs, WBi];
  ];
  WBs
 ]
```

In[75]:=
```
Graphics3D[WaterbombStripIncompat[π / 3, {True, False}], Boxed → False, ViewPoint → Front]
```

... Part: Part 4 of

$$\left\{\left\{\text{EdgeForm}[], \ll 7 \gg, \text{Triangle}\left[\left\{\left\{\sqrt{3}, \sqrt{3}, 0\right\}, \left\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\right\}, \left\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\right\}\right]\right\}, \left\{\text{EdgeForm}[\{\blacksquare\}], \ll 19 \gg, \text{Line}\left[\left\{\left\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \ll 1 \gg, \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\right\}, \{\ll 1 \gg\}\right\}\right]\right\}\right\} \text{ does not exist.}$$

... Thread: Objects of unequal length in

$$\left\{\text{EdgeForm}[], \ll 7 \gg, \text{Triangle}\left[\left\{\left\{\sqrt{3}, \sqrt{3}, 0\right\}, \left\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\right\}, \left\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\right\}\right]\right\} + \{-\blacksquare\} \text{ cannot be combined.}$$

... Part: Part 8 of

$$\{\{\text{EdgeForm}[], \ll 7 \gg, \text{Triangle}[\{\{\sqrt{3}, \sqrt{3}, 0\}, \{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\}, \{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\}\}], \{\text{EdgeForm}[\{\blacksquare\}], \ll 19 \gg, \text{Line}[\{\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \ll 1 \gg, \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\}, \{\ll 1 \gg\}\}]\}\}$$ does not exist.

... Part: Part 4 of

$$\{\{\text{EdgeForm}[], \ll 7 \gg, \text{Triangle}[\{\{\sqrt{3}, \sqrt{3}, 0\}, \{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\}, \{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\}\}], \{\text{EdgeForm}[\{\blacksquare\}], \ll 19 \gg, \text{Line}[\{\{\sqrt{3} - \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] - \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg], \ll 1 \gg, \frac{1}{2}\text{Power}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \frac{1}{2}\text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg] + \text{Plus}[\ll 2 \gg]\,\text{Plus}[\ll 2 \gg]\}, \{\ll 1 \gg\}\}]\}\}$$ does not exist.

... General: Further output of Part::partw will be suppressed during this calculation.

... Power: Infinite expression $\frac{1}{0}$ encountered.

... Infinity: Indeterminate expression 0 ComplexInfinity encountered.

... Infinity: Indeterminate expression 0 ComplexInfinity encountered.

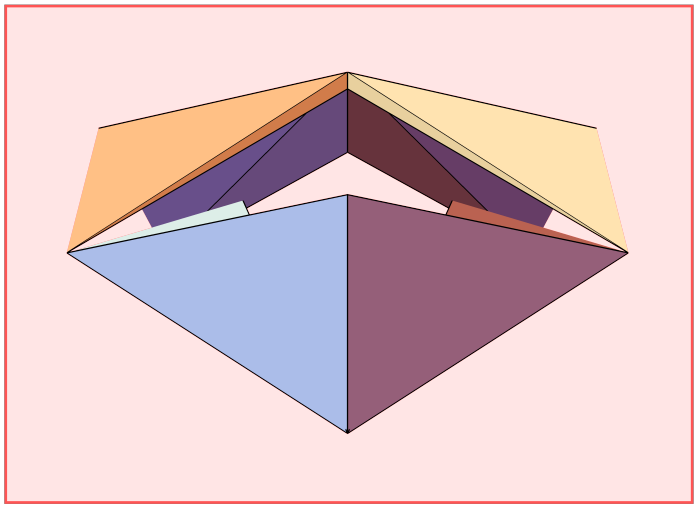
... Infinity: Indeterminate expression 0 ComplexInfinity encountered.

... General: Further output of Infinity::indet will be suppressed during this calculation.

Thread: Objects of unequal length in

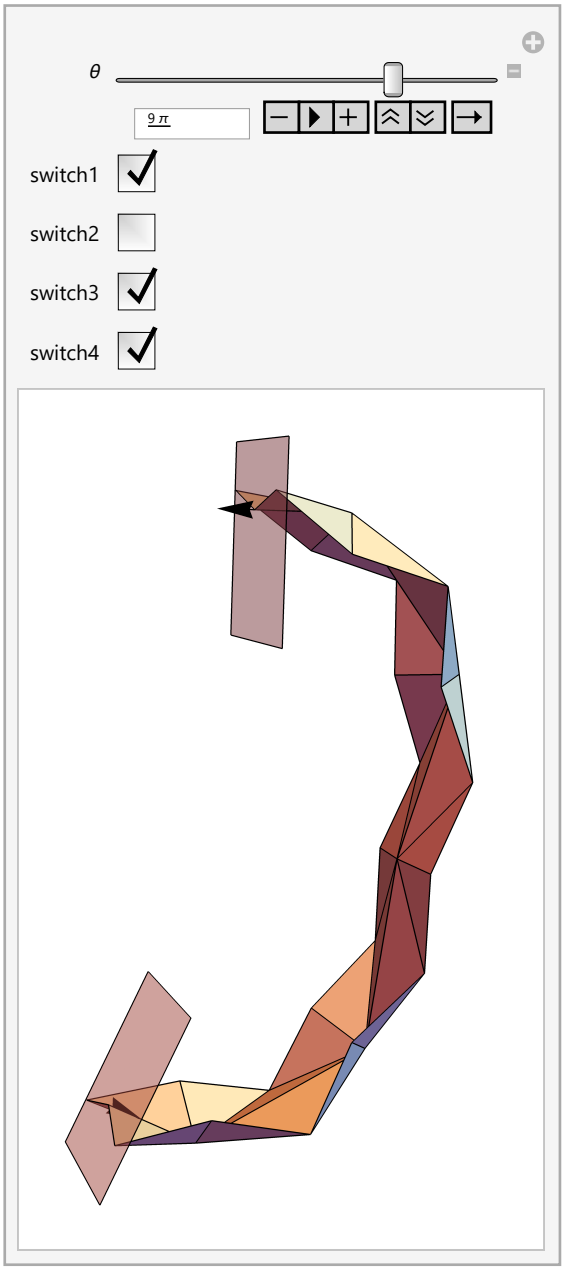
{■} + {-■} + {EdgeForm[], ≪7≫, Triangle[{{{√3, √3, 0}, {√3 - 1/2 Plus[≪2≫] Plus[≪2≫] - Plus[≪2≫] Plus[≪2≫] - 1/2 Power[≪2≫] Plus[≪2≫], √3 - Plus[≪2≫] Plus[≪2≫] - 1/2 Plus[≪2≫] Plus[≪2≫] - 1/2 Power[≪2≫] Plus[≪2≫], 1/2 Power[≪2≫] Plus[≪2≫] + 1/2 Plus[≪2≫] Plus[≪2≫] + Plus[≪2≫] Plus[≪2≫]}, {√3 - 1/2 Plus[≪2≫] Plus[≪2≫] - Plus[≪2≫] Plus[≪2≫] - 1/2 Plus[≪2≫] Plus[≪2≫] - Plus[≪2≫] Plus[≪2≫] - Power[≪2≫] Plus[≪2≫], √3 - 1/2 Plus[≪2≫] Plus[≪2≫] - Plus[≪2≫] Plus[≪2≫] - 1/2 Plus[≪2≫] Plus[≪2≫] - Plus[≪2≫] Plus[≪2≫] - Power[≪2≫] Plus[≪2≫], Power[≪2≫] Plus[≪2≫] + 1/2 Plus[≪2≫] Plus[≪2≫] + Plus[≪2≫] Plus[≪2≫] + 1/2 Plus[≪2≫] Plus[≪2≫] + Plus[≪2≫] Plus[≪2≫]}}]} cannot be combined.

Out[75]=



In[76]:=
```
Manipulate[Graphics3D[WaterbombStrip[θ, {True, switch1, switch2, switch3, switch4}],
  Boxed → False], {{θ, π/2}, 0, 3 * π / 4, π/128}, {switch1, {True, False}},
 {switch2, {True, False}}, {switch3, {True, False}}, {switch4, {True, False}}]
```

Out[76]=



In[77]:=
```
reflWaterbomb[wb_] := Module[{newWb, i},
   newWb = {};
   For[i = 1, i ≤ Length[wb], i++,
    AppendTo[newWb, Triangle[reflX[wb[[i]][[1]], 1]]];
   ];
   newWb
  ];
rotWaterbomb[wb_, uhat_, ω_] := Module[{newWb, i},
   newWb = {};
   For[i = 1, i ≤ Length[wb], i++,
    AppendTo[newWb, Triangle[rotate[wb[[i]][[1]], uhat, ω]]];
   ];
   newWb
  ];
```

In[79]:=
```
wb = Waterbomb[π / 2, 1, 1]
```

Out[79]=
```
{{EdgeForm[], Triangle[{{0, 0, 0}, {1, 0, 0}, {1, 1, 0}}],
  Triangle[{{0, 0, 0}, {0, 1, 0}, {1, 1, 0}}], Triangle[{{2, 0, 0}, {1, 0, 0}, {1, 1, 0}}],
  Triangle[{{2, 0, 0}, {2, 1, 0}, {1, 1, 0}}], Triangle[{{0, 2, 0}, {1, 2, 0}, {1, 1, 0}}],
  Triangle[{{0, 2, 0}, {0, 1, 0}, {1, 1, 0}}], Triangle[{{2, 2, 0}, {1, 2, 0}, {1, 1, 0}}],
  Triangle[{{2, 2, 0}, {2, 1, 0}, {1, 1, 0}}]},
 {EdgeForm[{■}], Line[{{0, 0, 0}, {1, 1, 0}}], Line[{{1, 0, 0}, {1, 1, 0}}],
  Line[{{0, 0, 0}, {1, 0, 0}}], Line[{{0, 0, 0}, {1, 1, 0}}], Line[{{0, 1, 0}, {1, 1, 0}}],
  Line[{{2, 0, 0}, {1, 1, 0}}], Line[{{1, 0, 0}, {1, 1, 0}}], Line[{{2, 0, 0}, {1, 0, 0}}],
  Line[{{2, 0, 0}, {1, 1, 0}}], Line[{{2, 1, 0}, {1, 1, 0}}], Line[{{0, 2, 0}, {1, 1, 0}}],
  Line[{{1, 2, 0}, {1, 1, 0}}], Line[{{0, 2, 0}, {1, 2, 0}}], Line[{{0, 2, 0}, {1, 1, 0}}],
  Line[{{0, 1, 0}, {1, 1, 0}}], Line[{{2, 2, 0}, {1, 1, 0}}], Line[{{1, 2, 0}, {1, 1, 0}}],
  Line[{{2, 2, 0}, {1, 2, 0}}], Line[{{2, 2, 0}, {1, 1, 0}}], Line[{{2, 1, 0}, {1, 1, 0}}]}}
```

In[80]:=
```
reflWaterbomb[wb]
```

Out[80]=
```
{Triangle[EdgeForm[]], Triangle[EdgeForm[{-■}]]}
```

In[81]:=
```
Length[WaterbombStrip[π / 2, {True, True, True}]]
```

Out[81]= 5

In[82]:=
```
(*Manipulate[Graphics3D[rotWaterbomb[Waterbomb[θ,1,1],{0,1,0},ω[θ]]],
 {{θ,π/2},0,3*π/4,π/64}]*)
```

In[83]:=
```
(*Manipulate[
 WBStripTemp=WaterbombStrip[θ,{True,switch1,switch2,switch3,switch4}];
 WBStrip1={};
 For[j=1,j≤Length[WBStripTemp],j++,
  AppendTo[WBStrip1,rotWaterbomb[WBStripTemp[[j]],{0,1,0},ω[θ]]]
 ];
 WBStrip2={};
 For[j=1,j<=Length[WBStrip1],j++,
  AppendTo[WBStrip2,reflWaterbomb[WBStrip1[[j]]]];
 ];
 Obj=PolyhedronData["Icosahedron","GraphicsComplex"];
 Obj[[1]]=transX[Obj[[1]],{0.0,1.0,z}];
 Graphics3D[Catenate[{WBStrip1,WBStrip2}],Boxed→False],
 {{θ,π/2},0,3*π/4,π/128},{switch1,{True,False}},{switch2,{True,False}},
 {switch3,{True,False}},{switch4,{True,False}},{{z,0},-10,10,0.1}
]*)
```

In[84]:=

Distance metrics

In[85]:=
```
tmp = WaterbombStrip[π / 3, {True, False}]
```

Out[85]=
```
{ ··· 1 ··· }
```
large output | show less | show more | show all | set size limit...

In[86]:=
```
tmp[[1]][[1]][[1]][[1]]
```

Out[86]=
```
EdgeForm[ ]
```

In[87]:=
```
WaterbombStripDiam[wbGraphic_] :=
  Module[{diam, nWBs, nTris, nPts, iWB, jWB, iTri, jTri, iPt, jPt, dx, wb},
   wb = wbGraphic[[1]];
   diam = 0;
   nWBs = Length[wb];
   nTris = Length[wb[[1]][[1]]] - 1;
   nPts = Length[wb[[1]][[1]][[2]][[1]]];
   For[iWB = 1, iWB ≤ nWBs, iWB++,
    For[iTri = 2, iTri ≤ nTris + 1, iTri++,
     For[iPt = 1, iPt ≤ nPts - 1, iPt++,
      For[jWB = iWB, jWB ≤ nWBs, jWB++,
       For[jTri = iTri, jTri ≤ nTris + 1, jTri++,
        For[jPt = iPt + 1, jPt ≤ nPts, jPt++,
         dx = wb[[iWB]][[1]][[iTri]][[1]][[iPt]] -
           wb[[jWB]][[1]][[jTri]][[1]][[jPt]];
         If[diam < Sqrt[Dot[dx.dx]], diam = N[Sqrt[Dot[dx.dx]]], True];
        ];
       ];
      ];
     ];
    ];
   ];
   diam
  ];
```

In[88]:=
```
WaterbombStripDiam[WaterbombStrip[π / 2, {True, False, False, True, True, True}]]
```

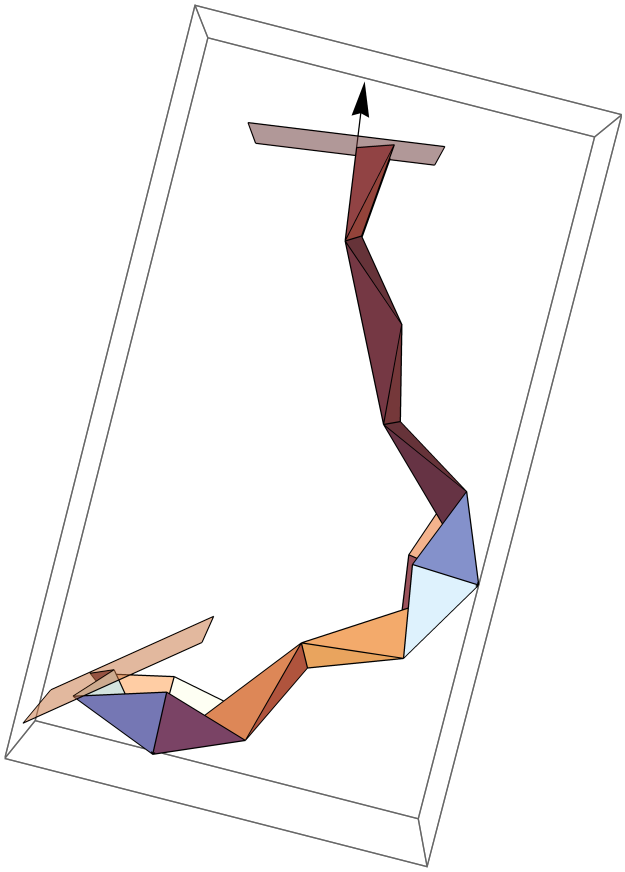
Out[88]=
```
12.0416
```

In[89]:=
```
cases = {
    {3 * π / 8, {True, True, True, False, False}},
    {3 * π / 8, {True, True, False, True, False}},
    {3 * π / 8, {True, True, False, False, True}},
    {3 * π / 8, {True, False, True, True, False}},
    {3 * π / 8, {True, False, True, False, True}},
    {3 * π / 8, {True, False, False, True, True}},
    {5 * π / 8, {True, True, False, False, False}},
    {5 * π / 8, {True, False, True, False, False}},
    {5 * π / 8, {True, False, False, True, False}},
    {5 * π / 8, {True, False, False, False, True}}
   };
For[k = 1, k <= Length[cases], k++,
 Export["wb-strip_n0-2_n1-3_k-" <> ToString[k] <> ".pdf",
  Graphics3D[WaterbombStrip[cases[[k]][[1]], cases[[k]][[2]]], Boxed -> False]];
 Export["wb-strip-front_n0-2_n1-3_k-" <> ToString[k] <> ".pdf",
  Graphics3D[WaterbombStrip[cases[[k]][[1]], cases[[k]][[2]]],
   Boxed -> False, ViewPoint → Front]];
 Export["wb-strip-planes_n0-2_n1-3_k-" <> ToString[k] <> ".pdf",
  Graphics3D[WaterbombStripPlanes[cases[[k]][[1]], cases[[k]][[2]]], Boxed -> False]];
 Ω = Sum[ω[If[cases[[k]][[2]][[l]], cases[[k]][[1]], π - cases[[k]][[1]]]], {l, 1, 5}];
 Export["wb-strip-planes-front_n0-2_n1-3_k-" <> ToString[k] <> ".pdf",
  Graphics3D[WaterbombStripPlanes[cases[[k]][[1]], cases[[k]][[2]]],
   Boxed -> False, ViewPoint → Front]];
 Ω = Sum[ω[If[cases[[k]][[2]][[l]], cases[[k]][[1]], π - cases[[k]][[1]]]], {l, 1, 5}];
 Print["k: ", k, ", diam: ",
  WaterbombStripDiam[WaterbombStrip[cases[[k]][[1]], cases[[k]][[2]]]], " Ω=", N[Ω]];
]
```

```
k: 1, diam: 7.27464 Ω=-1.29755

k: 2, diam: 7.00042 Ω=-1.29755

k: 3, diam: 5.41274 Ω=-1.29755

k: 4, diam: 7.36168 Ω=-1.29755

k: 5, diam: 5.38738 Ω=-1.29755

k: 6, diam: 5.44434 Ω=-1.29755

k: 7, diam: 7.29586 Ω=-1.29755

k: 8, diam: 6.51865 Ω=-1.29755

k: 9, diam: 7.2423 Ω=-1.29755

k: 10, diam: 8.42891 Ω=-1.29755
```

In[91]:=
```
Graphics3D[WaterbombStrip[5 * π / 8, {False, False, True, False, False}]]
```

Out[91]=



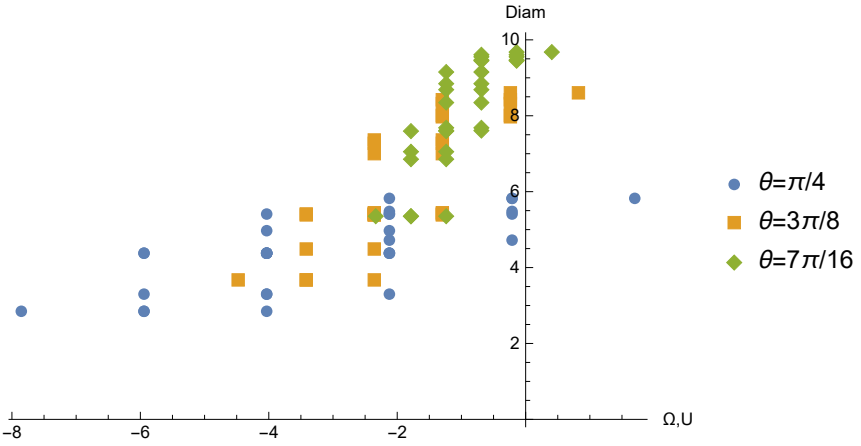
In[92]:=
```
WaterbombStripDiam[WaterbombStrip[5 * π / 8, {True, False, False, False, True}]]
```

Out[92]=
```
8.42891
```

In[93]:=
```
WaterbombStripDiamTable5[θ_] := Module[{ret, i, j, cases, case, Ω, d},
    ret = {};
    cases = Flatten[Table[{a, b, c, d, e}, {a, {True, False}}, {b, {True, False}},
       {c, {True, False}}, {d, {True, False}}, {e, {True, False}}]];
    For[i = 1, i ≤ Length[cases] / 5, i++,
     case = Table[cases[[j]], {j, 5 * (i - 1) + 1, 5 * (i - 1) + 5}];
     Ω = Sum[ω[If[case[[k]], θ, π - θ]], {k, 1, 5}];
     d = WaterbombStripDiam[WaterbombStrip[θ, case]];
     AppendTo[ret, {Ω, d}];
    ];
    ret
   ];
```

In[94]:=
```
p = ListPlot[{WaterbombStripDiamTable5[π / 4], WaterbombStripDiamTable5[3 * π / 8],
   WaterbombStripDiamTable5[7 * π / 16]}, PlotLegends → {"θ=π/4", "θ=3π/8", "θ=7π/16"},
  AxesLabel → {"Ω,U", "Diam"}, PlotMarkers -> Automatic]
```

Out[94]=



In[95]:=
```
Export["~/google-drive/Temp/2020-07-24/omega-vs-shape_N-5.pdf", p];
```

Export: Directory C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\ does not exist.

Export: Cannot open ~/google-drive/Temp/2020-07-24/omega-vs-shape_N-5.pdf.

In[96]:=
```
N[WaterbombStripDiamTable5[angles[[1]]]]
```

Part: Part specification angles[[1]] is longer than depth of object.

Part: The expression If[angles[[1]] ≤ π/2, 2, 1] cannot be used as a part specification.

ReplaceAll:
{{{ϕ1 → ArcTan[Times[≪3≫] + Sin[≪1≫], −Cos[≪1≫] Plus[≪2≫]]}, {ϕ1 → ArcTan[Times[≪2≫] + Sin[≪1≫], Cos[≪1≫] Plus[≪2≫]]}}[[If[angles[[1]] ≤ π/2, 2, 1]]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

Part: The expression If[angles[[1]] ≤ π/2, 2, 1] cannot be used as a part specification.

ReplaceAll:
{{{ϕ1 → ArcTan[Times[≪3≫] + Sin[≪1≫], −Cos[≪1≫] Plus[≪2≫]]}, {ϕ1 → ArcTan[Times[≪2≫] + Sin[≪1≫], Cos[≪1≫] Plus[≪2≫]]}}[[If[angles[[1]] ≤ π/2, 2, 1]]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

Part: The expression If[angles[[1]] ≤ π/2, 2, 1] cannot be used as a part specification.

General: Further output of Part::pkspec1 will be suppressed during this calculation.

ReplaceAll:
{{{ϕ1 → ArcTan[Times[≪3≫] + Sin[≪1≫], −Cos[≪1≫] Plus[≪2≫]]}, {ϕ1 → ArcTan[Times[≪2≫] + Sin[≪1≫], Cos[≪1≫] Plus[≪2≫]]}}[[If[angles[[1]] ≤ π/2, 2, 1]]]} is neither a list of replacement rules nor a valid dispatch table, and so cannot be used for replacing.

General: Further output of ReplaceAll::reps will be suppressed during this calculation.

Out[96]=
```
{{5. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {If[3.14159 - 1. angles[[1]] ≤ 1.5708,
    ωA /. {θ → 3.14159 - 1. angles[[1]]}, ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  4. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
    ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  4. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
    ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  4. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
    ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  4. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {4. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
   If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
    ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
  4. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
 {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
```

```
    3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {4. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {2. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    3. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {4. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {3. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    2. If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {4. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {4. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
      ωB /. {θ → 3.14159 - 1. angles[[1]]}] +
    If[angles[[1]] ≤ 1.5708, ωA /. {θ → angles[[1]]}, ωB /. {θ → angles[[1]]}], 0.},
  {5. If[3.14159 - 1. angles[[1]] ≤ 1.5708, ωA /. {θ → 3.14159 - 1. angles[[1]]},
     ωB /. {θ → 3.14159 - 1. angles[[1]]}], 0.}}
```

In[97]:=
```
angles = {π / 4, 3 * π / 8, 7 * π / 16};
labels = {"piDiv4", "3piDiv8", "7piDiv16"};
For[i = 1, i ≤ Length[angles], i++,
 Print[
  "~/google-drive/Temp/2020-07-24/wb_strip_" <> labels[[i]] <> ".csv", angles[[i]], i];
 Export["~/google-drive/Temp/2020-07-24/wb_strip_" <> labels[[i]] <> ".csv",
  N[WaterbombStripDiamTable5[angles[[i]]]]];
]
```

~/google-drive/Temp/2020-07-24/wb_strip_piDiv4.csv$\frac{\pi}{4}$1

... Export: Directory C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\ does not exist.

... OpenWrite: Cannot open C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\wb_strip_piDiv4.csv.

In[100]:=
```
WaterbombStripDiamTable5D[θ_] := Module[{ret, i, j, cases, case, Ω, d, N0, N1},
   ret = {};
   cases = Flatten[Table[{a, b, c, d, e}, {a, {True, False}}, {b, {True, False}},
      {c, {True, False}}, {d, {True, False}}, {e, {True, False}}]];
   For[i = 1, i ≤ Length[cases] / 5, i++,
    case = Table[cases[[j]], {j, 5 * (i - 1) + 1, 5 * (i - 1) + 5}];
    N0 = Sum[If[case[[k]], 0, 1], {k, 1, 5}];
    N1 = Sum[If[case[[k]], 1, 0], {k, 1, 5}];
    Ω = Sum[ω[If[case[[k]], θ, π - θ]], {k, 1, 5}];
    d = WaterbombStripDiam[WaterbombStrip[θ, case]];
    AppendTo[ret, {Ω, d, N0, N1}];
   ];
   ret
  ];
```

In[101]:=
```
Export["~/google-drive/Temp/2020-07-24/wb_strip_" <> labels[[1]] <> ".csv",
 N[WaterbombStripDiamTable5D[angles[[1]]]]]
Export["~/google-drive/Temp/2020-07-24/wb_strip_" <> labels[[2]] <> ".csv",
 N[WaterbombStripDiamTable5D[angles[[2]]]]]
Export["~/google-drive/Temp/2020-07-24/wb_strip_" <> labels[[3]] <> ".csv",
 N[WaterbombStripDiamTable5D[angles[[3]]]]]
```

... Export: Directory C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\ does not exist.

... OpenWrite: Cannot open C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\wb_strip_piDiv4.csv.

Out[101]= $Failed

... Export: Directory C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\ does not exist.

... OpenWrite: Cannot open C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\wb_strip_3piDiv8.csv.

Out[102]= $Failed

... Export: Directory C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\ does not exist.

... OpenWrite: Cannot open C:\Users\grasinmj\Documents\~\google-drive\Temp\2020-07-24\wb_strip_7piDiv16.csv.

Out[103]= $Failed