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# Assessed exercises 4
# As before, each question has an associated function, with input arguments
# matching those specified in the question. Your functions will be tested
# against a range of different input values, against a model solution, to see if
# they produce the same answers.
import numpy as np
import numpy.random as npr
import numpy.linalg as npl

# At the end of lecture 4 we simulated some 2 dimensional data from a
# regression model. In this assignment we're going to try generalise it
# to higher dimensions.

# The first thing we'll need to do is simulate the variables  $x_i$  from a
# uniform distribution

# Q1 Write a function that takes  $n$ ,  $a_1$ ,  $a_2$  and  $s$  as inputs, and returns
# an array of length  $n$ , drawn from a uniform distribution  $U(a_1, a_2)$ . The seed should
# be set to  $s$ .
def exercise1(n, a1, a2, s):
    npr.seed(s)
    x_unif = npr.uniform(a1, a2, size = n)
    return x_unif

# A multiple linear regression model is defined as
#  $y = b_0 + b_1x_1 + b_2x_2 + b_3x_3 + b_4x_4 + \dots + b_px_p + \epsilon$ 
# where  $p$  is the dimension and  $\{x_1, x_2, \dots, x_p\}$  are the variables

# To fit a linear regression model to a dataset we use the standard equation
#  $b = (X^T X)^{-1} X^T y$ , to estimate the coefficients  $b = [b_0, b_1, \dots, b_p]$ 
# Here,  $y$  is the dataset (1D array) and  $X$  is a matrix where the first
# column is filled entirely with 1s and the subsequent columns are  $x_1, x_2$ , etc.

# Q2 Write a function that takes  $p$  and a list  $S$  as inputs, and returns
# a matrix  $X$ . Use your function from exercise one to create the  $x_1, x_2, \dots, x_p$ 
# variables, with  $n = 1000$ ,  $a_1 = 0$  and  $a_2 = 10$ . The input  $S = (s_0, s_1, \dots, s_p)$ 
# where  $s_i$  corresponds to the seed that should be used to create the  $x_i$ 
# Hint: Python treats all 1D arrays as row vectors. Instead Create the
# columns of  $X$  and return its transpose  $((X^T)^T = X)$ . Also, the function vstack
# is useful here.

def exercise2(p, S):
    n = 1000
    a1 = 0
    a2 = 10
    x_list = ([1]*1000)
    for i in range(len(S)):
        temp = exercise1(1000, 0, 10, S[i])
        x_list = np.vstack((x_list, temp))
    return x_list.T

# Q3 Write a function that takes the matrix  $X$  and vector  $y$  as input, and
# performs a multiple linear regression, using the standard equation

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# b = (X^T X)^{-1} X^T y, by calculating the inverse of (X^T X) and multiplying the result by (X^T y). The function should return the vector b, which fits for the intercept and slope parameters (b0, b1, b2, b3, b4)
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def exercise3(X,y):
    b = np.dot(np.linalg.inv(np.dot(X.T,X)),np.dot(X.T,y))
    return b
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# Q4 Write another function, with the same inputs and outputs, which uses the numpy.linalg.solve function rather than finding the inverse and then multiplying.
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def exercise4(X,y):
    temp_a = np.dot(X.T,X)
    temp_y = np.dot(X.T,y)
    b = np.linalg.solve(temp_a,temp_y)
    return b
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# Try testing your functions for different models, e.g.
# y = 3 + 2*x1 - x2 + 0.5*x3 - 0.1*x4 + npr.normal(0,1,n)
# where x1, x2, x3, x4 should be computed using the function exercise2
# with different seeds s1, s2, s3, s4. These seeds should be given as a list
# into exercise2 to create the matrix X. Running exercise3 and exercise4
# should give the same result, a vector (1D array) of length p+1, with entries
# equal to the coefficients defined in your multiple linear model,
# e.g. [3,2,-1,0.5,-0.1] for the above example. You can use %timeit to
# check whether exercise3 or exercise4 is quicker for fitting the regression model.
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This part is a test of the functions exercise3 and exercise4.
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s_test = (19,99,80)
x_test = exercise2(3,s_test)
test_y = npr.randint(10,100,size = 1000)
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# using exercise3 to generate matrix B
test_b1 = exercise3(x_test,test_y)
%timeit exercise3(x_test,test_y)
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#using exercise4 to get matrix B
test_b2 = exercise4(x_test,test_y)
%timeit exercise4(x_test,test_y)
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# the result shows 2 functions get the same matrix B and
#exercise4 needs less time than exercise3 to get matrix B
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