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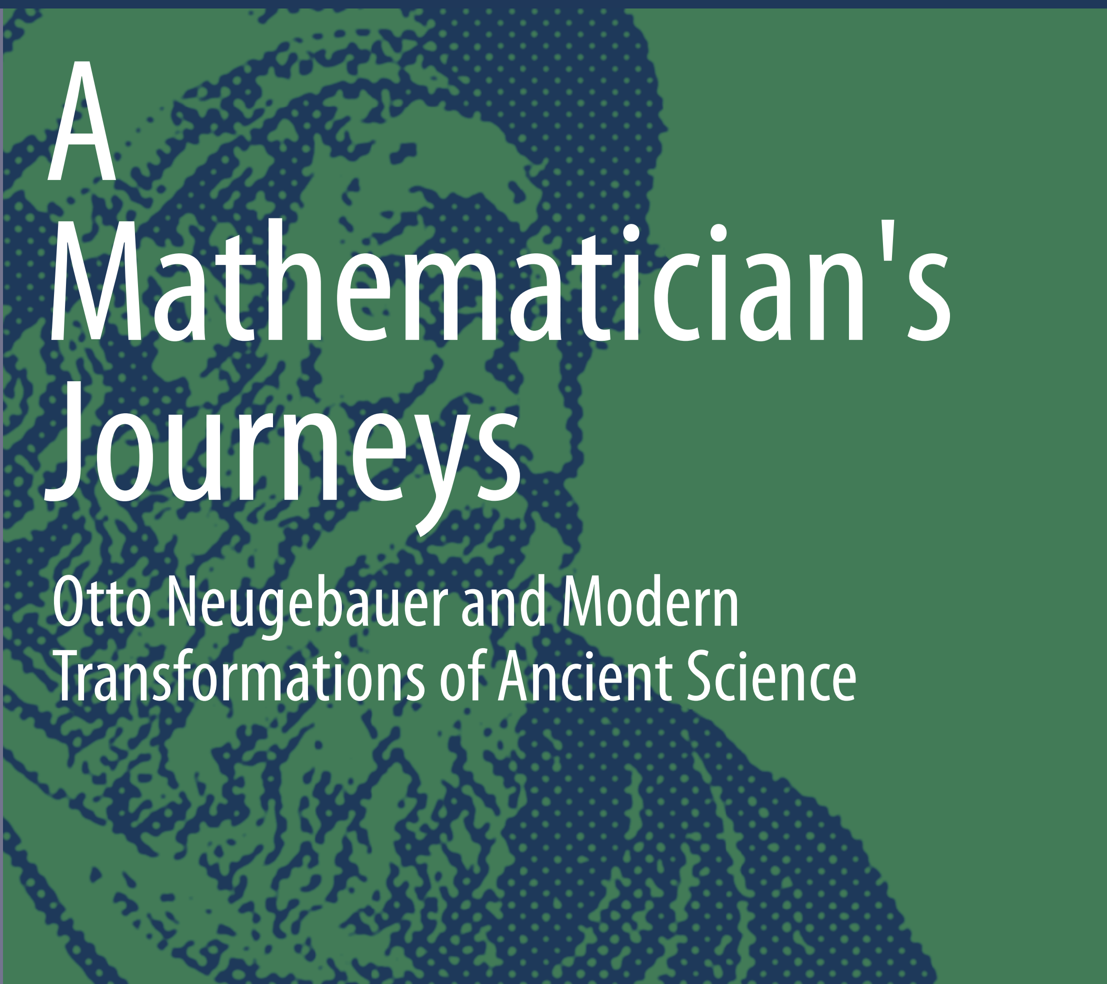

A Mathematician's Journeys

Otto Neugebauer and Modern Transformations of Ancient Science

Springer

A Mathematician's Journeys

Archimedes

NEW STUDIES IN THE HISTORY AND PHILOSOPHY OF SCIENCE AND TECHNOLOGY

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ISSN 1385-0180 ISSN 2215-0064 (electronic)
Archimedes
ISBN 978-3-319-25863-8 ISBN 978-3-319-25865-2 (eBook)
DOI 10.1007/978-3-319-25865-2

Library of Congress Control Number: 2015959787

Springer Cham Heidelberg New York Dordrecht London

Printed on acid-free paper

Springer International Publishing AG Switzerland is part of Springer Science+Business Media (www.springer.com)

Preface

Otto Neugebauer, more than any other scholar of recent times, shaped the way we perceive premodern science. Through his own hugely productive scholarship and his influence on three generations of colleagues who learned from him as students and collaborators, he inculcated both an approach to historical research on ancient and medieval mathematics and astronomy through precise mathematical and philological study of texts and a vision of these sciences as systems of knowledge and method that spread outward from the ancient Near Eastern civilizations, crossing cultural boundaries and circulating over a tremendous geographical expanse of the Old World from the Atlantic to India. It is impossible for a present-day historian working on these fields not to be constantly conscious of the power, and sometimes the limitations too, of Neugebauer's intellectual legacy.

Neugebauer's career was demarcated by repeated changes in his interests.[1] Born in 1899 in Innsbruck, he was schooled in the Akademisches Gymnasium at Graz, where he showed little enthusiasm for the classical languages but much for technical subjects including mathematics. After serving in the Austrian army in the First World War, he enrolled in turn at the Universities of Graz, Munich, and Göttingen, shifting the focus of his studies from engineering to physics to mathematics. As a doctoral student at the Mathematical Institute at Göttingen, although he studied a broad range of areas of contemporary mathematics, he turned his research entirely to the history of mathematics in antiquity, and the subject of his thesis was ancient Egyptian fractions. Despite his negligible original mathematical research, he earned the respect of his fellow mathematicians for the manifest rigor of his historical work and their indebtedness for his tireless service to the community, in particular as Courant's assistant in the administration of the Mathematical Institute and as the founding editor of the abstracting journal *Zentralblatt für Mathematik und ihre Grenzgebiete*.

[1] For biographical details we direct the reader to N. M. Swerdlow's outstanding article, Swerdlow 1993, as well as to the papers in this volume.

From the late 1920s through the late 1930s, Neugebauer's chief historical project was to study and publish the mathematical texts preserved on cuneiform tablets from ancient Babylonia, culminating in his three volume edition *Mathematische Keilschrift-Texte* (*MKT*, 1935–1937). Following the dismissals of Jewish faculty at Göttingen in 1933, Neugebauer took up a temporary post at the Mathematical Institute in Copenhagen. (Though not Jewish, Neugebauer was politically unacceptable to the Nazis and vice versa.) During his 5-year Danish sojourn, he came to be increasingly preoccupied with the history of ancient astronomy, beginning with late Babylonian tablets and Egyptian papyri. Mathematical astronomy was to be the subject closest to his heart for the rest of his life; among his numerous publications relating to it, the edition *Astronomical Cuneiform Texts* (*ACT*, 1955) and the *History of Ancient Mathematical Astronomy* (*HAMA*, 1975) stand out as indispensible resources for subsequent research.

Neugebauer resigned as editor of the *Zentralblatt* in late 1938 in response to the removal of Levi-Civita from the journal's editorial board and other racially motivated restrictions imposed on its operation. In the meantime, the collapse of the *Zentralblatt* as a genuine international journal led the American Mathematical Society to undertake a new abstracting journal, *Mathematical Reviews*, and Neugebauer was invited to be its editor, simultaneously being offered a position in the Mathematics Department at Brown University. He arrived in the USA in early 1939.

Brown's administration was greatly supportive of Neugebauer's research, and he was enabled to attract younger colleagues who shared interests and possessed complementary areas of expertise, beginning with the Assyriologist Abraham Sachs and the Egyptologist Richard Parker. A special department of History of Mathematics was founded in 1947 for Neugebauer and Sachs, 2 years after the publication of their joint volume *Mathematical Cuneiform Texts*, devoted to tablets, mostly in American collections, that had not been available for inclusion in *MKT*. The department later grew through the appointments of Gerald Toomer in 1965 and David Pingree in 1971; it was closed following Pingree's death in 2005 and the professorship in the history of the exact sciences in antiquity was transferred to the newly created Department of Egyptology and Ancient Western Asian Studies (renamed the Department of Egyptology and Assyriology in 2014). Through a steady flow of visitors and students, it built up a worldwide network of historians who were deeply influenced by Neugebauer. From 1950 until his death in 1990, Neugebauer also was a member of the Institute for Advanced Study at Princeton, where he worked for a part of every year in alternation with Brown.

Younger historians seldom have anything approaching the deep familiarity with older historiography that Neugebauer, through his vast reading, possessed, and it is not uncommon to find Neugebauer treated as the starting point for the fields most strongly represented in his work, especially ancient Near Eastern mathematics and astronomy. The quarter-century since his death gives us enough distance to consider afresh both the researches of others that preceded and laid the foundations for his contributions and the ways in which the study of the ancient exact sciences has taken new directions following his fundamental publications. Moreover, through documentary collections such as the Shelby White and Leon Levy Archives Center of the Institute for Advanced Study (which houses the largest holdings of Neugebauer's papers), new information is available concerning Neugebauer himself and in particular his intellectual formation at Graz, Munich, and Göttingen, helping us to understand his distinctively "mathematician's" approach to the history of science, what Swerdlow calls the "notable tension between the analysis of culturally specific documents… and the continuity and evolution of mathematical methods regardless of ages and cultures."[2]

In 2010, marking the twentieth anniversary of Neugebauer's death, a conference was held at the Institute for the Study of the Ancient World (ISAW), New York University, entitled "A Mathematician's Journeys: Otto Neugebauer between history and practice of the exact sciences."[3] The goal of the conference was to explore facets of Neugebauer's career, his impact on the history and practice of mathematics, and the ways in which his legacy has been preserved or transformed in recent decades, looking ahead to the directions in which the study of the history of science will head in the twenty-first century. This collection of papers includes a large part of the papers presented during the conference, several of them in considerably revised and expanded form. It has three principal focuses: the central interval of Neugebauer's career in the 1920s and 1930s during which he was most closely connected with the mathematical community while making himself, in turn, a mathematician, a historian of mathematics, and a historian of the exact sciences in the broader sense; the historiography of ancient Egyptian and Mesopotamian mathematics centering on Neugebauer's *Grundlagen der ägyptischen Bruchrechnung*, *MKT*, and its sequel *MCT* written in collaboration with Sachs; and the historiography of Babylonian astronomy centering on *ACT*. In the spirit of Neugebauer's own attention to less studied and less regarded texts as a means of better understanding the canonical landmark works of science, we hope that these papers will contribute to a more exact appraisal of the nature of Neugebauer's achievement and our relation to it.

[2] Swerdlow 1993, 141.

[3] The conference was organized by the editors of this collection together with John Britton, who, alas, died unexpectedly 5 months before it took place; we deeply missed his presence while editing the volume. The sponsors included New York University's Institute for the Study of the Ancient World and Courant Institute of Mathematical Sciences (New York), the Department of Egyptology and Ancient Western Asian Studies (now Egyptology and Assyriology) of Brown University (Providence), the Institute for Advanced Study (Princeton), the Center for International Research in the Humanities and Social Sciences (a joint research center between New York University and the Centre National de la Recherche Scientifique, France), and the CNRS research group Recherches Epistémologiques et Historiques sur les Sciences Exactes et les Institutions Scientifiques—now part of the CNRS research group SPHERE (Paris).

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Reference

Swerdlow, N. M. 1993. Otto E. Neugebauer (26 May 1899–19 February 1990). *Proceedings of the American Philosophical Society* 137.1: 138–165.

Otto Neugebauer (1899–1990), unidentified photographer, not later than 1936 (Courtesy of the Mathematisches Institut, Georg-August-Universität Göttingen). The photograph was published in *National Mathematics Magazine* 11, 1936, 16

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Otto Neugebauer and Ancient Egypt

Jim Ritter

Neugebauer at Göttingen

The Road to Göttingen

On August 21, 1919, Sergeant Otto Neugebauer, of the Austrian Alpine Artillery, left Cassino and the last of the series of Italian prisoner-of-war camps in which he had been interned since the end of the First World War, 9 months earlier, and headed back home to Graz in Austria.[1] He had lived there, since the death of his father before the War, in the house of his uncle and guardian Rudolf Schüssler and was now ready, at the age of nineteen, to resume his life. His uncle had already enrolled him, as a temporary measure and *in absentia*, in the Law Faculty at the Karl Franzen University of Graz for the Summer Semester[2] 1919, but Neugebauer was home in time to begin the Winter Semester 1919/1920 and could change his inscription to the Philosophical Faculty (i.e., Arts and Sciences) with an intended physics major.

[1] Information from Neugebauer's *Tagebuch* (The Shelby White and Leon Levy Archives Center, Institute for Advanced Studies: Otto Neugebauer papers/Box 13). I would like to thank the IAS Archivist, Christine Di Bella, for her generous help.

[2] Austrian, like German, universities operate on a two-semester system: "Winter Semester" and "Summer Semester". Traditionally, (with the corresponding month for Germany in parentheses) the first runs from the beginning of October to the end of February (March) while the second begins in March (April) and ends in September. Actual teaching time is generally:

Winter Semester: beginning of October (mid-October)—end of January (February)
Summer Semester: beginning of March (mid-April)—end of June (July)

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A. Jones et al. (eds.), *A Mathematician's Journeys*, Archimedes 45,
DOI 10.1007/978-3-319-25865-2_4

Besides the standard entrance-level courses in physics (experimental physics, electricity but also geophysics and physical chemistry) and mathematics (calculus and vector analysis) that he took that first semester, he also enrolled in a new course, "Introduction to Relativity Theory", offered by the young instructor (*Privatdozent*) of vector analysis, Heinrich Brell.[3] The subject was Einstein's new theory of gravitation, general relativity, having just been successfully tested by astronomers. This course—and its subject—so interested the young veteran that in the academic year 1920–1921 he took its equally newly-created continuation, "The Mathematical Foundations of Relativity Theory", by the invariant-theory specialist, Roland Weitzenböck. The same subject was certainly discussed in the "Seminar for Theoretical Physics", directed by the principal physics professor at Graz, Michael Radaković, a seminar in which Neugebauer was enrolled every semester after his first. By the summer of 1921, Neugebauer felt confident enough in the field to give a public two-part conference cycle on Hermann Weyl's recent first attempt at a unified field theory, conferences entitled "On the Extension of the General Theory of Relativity by Hermann Weyl" [*Über die Erweiterung der allgemeinen Relativitätstheorie durch Hermann Weyl*].[4] It is of interest to see, at the end of the lecture notes, a diagram created by the budding physicist (Fig. 2a), shown at the conclusion of his second lecture. It is, in fact, a résumé of the new theory, presented in the third edition of Neugebauer's source, Weyl's classic book *Raum Zeit Materie* (Weyl 1919: 251–253). Here Neugebauer has translated Weyl's three pages of discussion and calculation into a single diagram, showing how the Maxwell equations of electrodynamics (Equation 5, lower left) and the gravitational equations related to general relativity (Equation 9, lower right) arise out of a single choice for the action functional $\mathfrak{W}$ (Equation 1, top), and charts the logical connections among them. These classificatory diagrams were as innovative in theoretical physics as they were to become in Neugebauer's later work in the history of mathematics and astronomy, as we shall see. Another premonition of his later attitude is to be seen in the closing lines of the final lecture, where, referring to David Hilbert's views on axiomatization of mature theories,[5] Neugebauer says that "a completed science leads out beyond itself—or back to idealization" (*Eine vollendete Wissenschaft führt über sich selbst hinaus—oder zur Idealisierung zurück*). Though physics here is in his—and Hilbert's—mind, even after moving on to the history of ancient sciences, he will keep, as a central point of his work, the search for organizing principles from which alone, he will feel, any hope of a true understanding of ancient modes of thought can be found.

[3] The information about Neugebauer at Graz I owe to the kindness of Prof. Dr. Alois Kernbauer at the University Archives, Institut für Geschichte at the Karl-Franzens-Universität in Graz. For the Graz careers of Brell, Radaković and Weitzenböck see (Aigner 1985). Neugebauer's two public lectures are preserved in (The Shelby White and Leon Levy Archives Center, Institute for Advanced Studies: Otto Neugebauer papers/Box 13 "Vortrag über A.R.T. und Weyl"); the citation is from page 34.

[4] Otto Neugebauer papers, Box 1345, Courtesy of The Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, NJ, USA.

[5] For Hilbert's view on the role of axiomatization in physics and mathematics, see (Corry 2004).

Fig. 1 Otto Neugebauer's student identification photograph from Ludwig-Maximilian University in Munich, 1922 (Courtesy of the Archiv der Ludwig-Maximilians-Universität, Munich)

Graz had suffered greatly from the War and the subsequent economic crisis and the University had entered into a difficult period in terms of personnel and material; both Brell and Weitzenböck left at the end of the Summer Semester 1921, the first to the Montanistische Hochschule in Leoben[6] and the second to the University of Amsterdam. Though Radaković stayed on until his death in 1934, he was uninterested in the new physics of quantum theory and relativity.[7] Clearly now aiming at further work in the new physics and, more particularly, in relativity, Neugebauer saw that there was no longer a future for him in Graz and, at the end of the Summer Semester, he too decided to move on.

His choice of the Ludwig-Maximilian University in Munich for the Winter Semester of 1921/1922 made sense for a bright young man interested in the new theoretical physics that had revolutionized the face of physics in less than a decade.[8] In 1921, Munich was one of the main centers in Germany, indeed in the world, for this field and boasted some of the most authoritative researchers, grouped around the Institute of Arnold Sommerfeld, author of the 'bible' of the quantum theory of the time, *Atombau und Spekrallinien*.[9] Besides Sommerfeld himself, these included his doctoral student, Wolfgang Pauli, who had already written one of the earliest and

[6] Now the Montanuniversität Leoben for Mining and Metallurgy.

[7] Radaković's research interests centered on classical mechanics and its applications to ballistics and meteorology.

[8] I should like to thank Dr. phil. Claudius Stein for information about Neugebauer's semester at Munich.

[9] For the "Sommerfeld School" of Munich, see (Seth 2010) and (Eckert 2013).

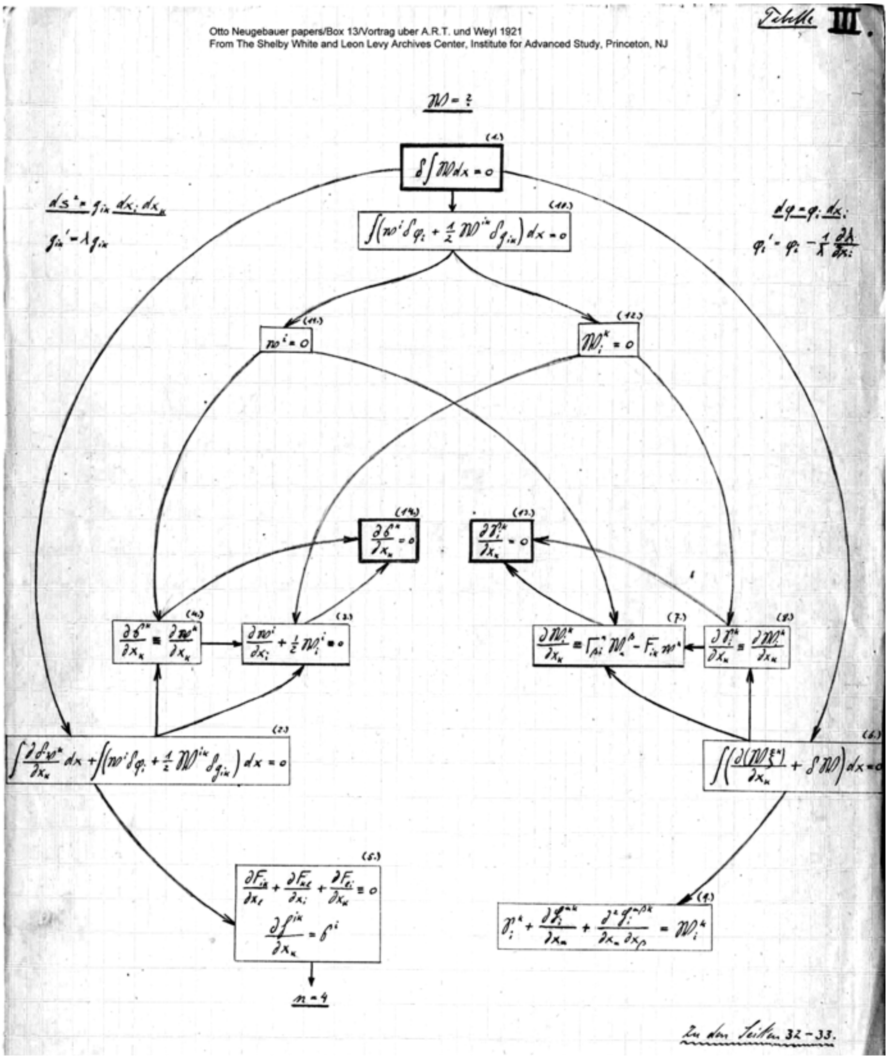

Fig. 2 (**a**) "Tabelle III" from Neugebauer's notes for his lecture on Weyl's unified theory, 1921 (Courtesy of the Shelby White and Leon Levy Archives Center, Institute for Advanced Study, Princeton, New Jersey)

Tabelle I.

'$\underline{h}$'-Rechnungen (Übersicht)

R 30	**M 19**
$10 = (\bar{\bar{3}} + \overline{10})\,x$ $x = 10 : (\bar{\bar{3}} + \overline{10})$	$(1 + \bar{2})\,x + 4 = 10$ $10 - 4 = 6 \quad x = 6 \cdot \bar{\bar{3}}$

M 25	**R 25**	**R 26**	**R 27**	**R 24**
$2x + x = 9$ $2 + 1 = 3$ $x = 9 : 3$	$x + \bar{2}x = 16$ $x = (16 : 3) \cdot 2$	$x + \bar{4}x = 15$ $x = (15 : 5) \cdot 4$	$x + \bar{5}x = 21$ $x = (21 : 6) \cdot 5$	$x + \bar{7}x = 19$ $x = (19 : 8) \cdot 7$

R 34	**R 32**	**R 33**	**R 31**	**R 29**[2])
$x + \bar{2}x + \bar{4}x = 10$ $x = 10 : (1 + \bar{2} + \bar{4})$	$x + \bar{3}x + \bar{4}x = 2$ $x = 2 : (1 + \bar{3} + \bar{4})$	$x + \bar{\bar{3}}x + \bar{2}x + \bar{7}x = 37$ $x = 37 : (1 + \bar{\bar{3}} + \bar{2} + \bar{7})$	$x + \bar{\bar{3}}x + \bar{2}x + \bar{7}x = 33$ $x = 33 : (1 + \bar{\bar{3}} + \bar{2} + \bar{7})$	$\left[\bar{3}\left((x + \bar{\bar{3}}x) + \bar{3}\,(x + \bar{\bar{3}}x)\right) = 10\right]$

K 3[3])	**R 28**
$x - (\bar{2} + \bar{4})\,x = 5$ $1 - (\bar{2} + \bar{4}) = \bar{4}$ $x = 5 \cdot 4$	$(x + \bar{\bar{3}}x) - \bar{3}\,(x + \bar{\bar{3}}x) = 10$ $10 : 10 = 1 \quad x = 10 - 1$[4])

B 1[6])
$x^2 + \left((\bar{2} + \bar{4})\,x\right)^2 = 100$ $1^2 + (\bar{2} + \bar{4})^2 = 1 + \bar{2} + \overline{16}$ $\sqrt{1 + \bar{2} + \overline{16}} = 1 + \bar{4}, \quad \sqrt{100} = 10$ $x_1 = 10 : (1 + \bar{4}) = 8$ $x_2 = (\bar{2} + \bar{4}) \cdot 8$

Fig. 2 (continued) (**b**) "Tabelle I" from Neugebauer's article on "Arithmetic and Calculational Techniques of the Egyptians" (Reference: Neugebauer 1930a, p. 307)

best surveys of general relativity and its extensions for the authoritative *Encyklopädie der mathematischen Wissenschaften* (Pauli 1921); Gregor Wentzel, Sommerfeld's Assistant, Adolf Kratzer, his former Assistant, while among his students at this time was Werner Heisenberg.[10]

Although Neugebauer had come to Munich to study modern theoretical physics, it was principally general relativity and its extensions that attracted him at this time, as it did so many other bright young scientists. But Munich was now concentrating on quantum theory, rather than relativity. Pauli had become disillusioned with relativity after the September 1920 meeting of the German Association of Natural Scientists and Physicians in Bad Nauheim: "None of the hitherto proposed theories—not even that of Einstein[11]—has so far succeeded in satisfactorily solving the problem of the elementary electrical quantum and it would be advisable to seek a deeper reason for this failure."[12] That deeper reason, Pauli felt, could only come

[10] Sommerfeld's book, first published in 1919, was to see eight thoroughly revised editions over the years. It was translated into English, Russian and French and used extensively as a textbook through the 1920s. Pauli's 237-page article on "Relativitätstheorie" (Pauli 1921), issued that same year as an independent monograph, was also to see a large number of reprints and translations and continues even today to serve as a textbook.

[11] The reference is to Einstein's first unified theory of 1919. For a discussion of this theory, and of Pauli's reaction to it and to that of Weyl, see (Goldstein and Ritter 2003: Section 2).

[12] *Keiner der bisherigen Theorien des Elektrons, auch nicht der Einsteinschen… ist es bisher gelungen, das Problem der elektrischen Elementarquante befriedigend zu lösen, und es liegt nahe,*

from quantum theory. That way lay the future and he persuaded the younger Heisenberg to follow him in abandoning relativity for the quantum. Since this accorded with Sommerfeld's own preference, when Neugebauer arrived a year later he found that Munich was a very exciting place to be indeed—if what one wanted was to do quantum physics.

Neugebauer, as was to become his way, did not follow the general trend. Like the others, he was inspired by Sommerfeld, but, unlike the others, his decision was to abandon physics entirely. As he succinctly put it a few years later, in his autobiographical note for his doctoral thesis: "Stimulated by lectures by Sommerfeld and such a pure mathematical content, I decided to devote myself to mathematics." [*Ich, angeregt durch Vorlesungen von Geheimrat Sommerfeld und solches rein mathematischen Inhaltes, beschloss mich der Mathematik zuzuwenden*].[13] But for mathematics it was Göttingen, not Munich, which was the shining city on the hill, and it was to Göttingen that he went at the end of that single Munich semester.

Mathematics at Göttingen

Otto Neugebauer transferred to the Georg-August University of Göttingen during the Summer Semester of the academic year 1922 when a new mathematics program was now to be in large part centered on the just-founded Mathematical Institute, under the direction of the recently returned Richard Courant, with his own special center of interest, the theory of differential equations.

Although the subjects on which Neugebauer was to be examined for his doctorate were Mathematical Analysis, Geometry and Physics,[14] judging by the courses he took, his own personal mathematical interests at this time lay more in pure mathematics and he took at least five courses with the analytic number theorist, Edmund Landau, one of the few active pure mathematicians at Göttingen.[15] Neugebauer's proverbial energy, as well as his abilities, both mathematical and administrative, were evident from the start. He was chosen by Courant to be his Assistant; first *außerplanmäßige* Assistant in October 1925, replacing Hellmuth Kneser who had been called to Greifswald, then promoted to *planmäßige* Assistant in June 1928, and finally Chief Assistant (*Oberassistent*) at the end of April 1930. He was also chosen to be one of the editorial assistants for the preparation of the first volume of Courant's classic textbook *Methods of Mathematical Physics* (Courant and Hilbert 1923).

nach einem tieferen Grund dieses Mißerfolges zu suchen. (Pauli apud Weyl 1920: 650)

[13] Lebenslauf, 22/9/1925, UAG Kur. P.A. Neugebauer, Otto, Band I, Hs N. I would like to express my deepest gratitude to Dr. Ulrich Hunger, at the Universitätsarchiv Göttingen for his most kind assistance.

[14] Bescheinigung, 23/4/1926, UAG Kur. P.A. Neugebauer, Otto, Band 1/II, Bescheinigung 23. April 1926.

[15] For Landau see (Schappacher 1987).

Egyptology at Göttingen

There was another prestigious intellectual center at Göttingen in the early 1920s: the Egyptological Seminar, the Göttinger Seminar für Ägyptologie, housed at this time in a small building, now the Michaelis-Haus. At some point before or after his arrival in Göttingen, Neugebauer had developed an appreciation for the culture of Ancient Egypt.[16] We know that he had read, in German translation[17] (Swerdlow 1993: 140), the most authoritative and influential book on Egyptian history and culture of the opening decades of the twentieth century: *A History of Egypt* by the American Egyptologist James Henry Breasted (1905). The book's section on Egyptian mathematics and astronomy, presumably Neugebauer's first view of the subject, is short and typical of the period's attitude on these subjects. For these reasons it is worth citing in full[18]:

> The science of the time, if we may speak of it as such at all, was such a knowledge of natural conditions as enabled the active men of this age to accomplish those practical tasks with which they were daily confronted. They had much practical acquaintance with astronomy, developed out of that knowledge which had enabled their ancestors to introduce a rational calendar nearly thirteen centuries before the rise of the Old Kingdom. They had already mapped the heavens, identified the more prominent fixed stars, and developed a system of observation with instruments sufficiently accurate to determine the positions of stars for practical purposes; but they had produced no theory of the heavenly bodies as a whole, nor would it ever have occurred to the Egyptian that such an attempt was useful or worth the trouble.
>
> In mathematics all the ordinary arithmetical processes were demanded in the daily transactions of business and government, and had long since come into common use among the scribes. Fractions, however, caused difficulty. The scribes could operate only with those having *one* as the numerator, and all other fractions were of necessity resolved into a series of several, each with *one* as the numerator. The only exception was two thirds, which they had learned to use without so resolving it. Elementary algebraic problems were also solved without difficulty. In geometry they were able to master the simpler problems, though the area of a trapezoid caused some difficulties and errors, while the area of the circle had been determined with close accuracy. The necessity of determining the content of a pile of grain had led to a roughly approximate result in the computation of the content of the hemisphere, and a circular granary to that of the cylinder. But no theoretical problems were discussed, and the whole science attempted only those problems which were continually met in daily life. (Breasted 1905: 100–101)

Non-theoretical, practical and elementary, Egyptian "science" was defined primarily by what it had not achieved. Nothing more needed to be said—and nothing more was said—on the subject in the more than 650 pages of Breasted's book. Only the remark on the rational calendar put in place "more than thirteen centuries before

[16] Interestingly enough, the Ludwig-Maximilians University in Munich was one of the few German universities to have offered courses in both Ancient Egyptian language and in Akkadian (Babylonian). The former, for specialists, were given by Friedrich von Bissing, then in his last semester in Munich and who had primarily archaeological interests. The Akkadian courses were given by Fritz Hommel who, in 1921–1922 offered a seminar on Old Babylonian texts. (LMU 1921). But we can find no trace of any interest expressed by Neugebauer in these areas before Göttingen.

[17] This was a revised version of the book, translated into German by the noted Heidelberg Orientalist Hermann Ranke in 1910 (Breasted 1910).

[18] We cite from the original English. No change was made to this section in the German edition.

the rise" of the earliest period of Egyptian written history seems, in the context, oddly positive and needs perhaps an elucidation, all the more so since, as we shall see, the question played an important role in Neugebauer's own later work. The previous year Eduard Meyer, the leading German ancient historian of that time, had published a monograph on Ancient Egyptian chronology (Meyer 1904) which brought much clarity to what had been a very confused subject. But from Meyer's point of view his most important contribution was elsewhere; the establishment of the exact date of the creation of the Egyptian calendar.[19]

> I do not hesitate to call the introduction of the Egyptian calendar on July 19, 4241 BC the first certain date in the history of the world.

This strangely precise (and much too early) date, accepted almost immediately by the larger Egyptological community, was to dominate—and mislead—Egyptian chronology up to the Second World War and Neugebauer's own work on the question.

At Göttingen, Neugebauer's interest in Egyptology was developed through his close contact with the current holder of the Chair of Egyptology, Kurt Heinrich Sethe, associate (*ausserordentlich*) professor since 1900, and full (*ordentlich*) professor since 1907. Arguably the greatest, and certainly, after his teacher Adolf Erman in Berlin,[20] the most influential Egyptological philologist of the period, Sethe was one of the rare Egyptologists to be interested in and—even rarer—to possess a full command of texts treating numbers and measures. Six years before Neugebauer's arrival in Göttingen, Sethe had published the (still) outstanding work on the subject: *On Numbers and Number Words among the Ancient Egyptians* (Sethe 1916) with the suggestive continuation of the title: *And What Is Be Learned from Them for Other Peoples and Languages: A Contribution to the History of Arithmetic and Language*. The innovative nature of this work was recognized immediately, at least among those few Egyptologists to whom the quantitative was not anathema. Although reviewed only twice[21]—by the German specialist on medical texts, Walter Wreszinski (1917), and by the English Egyptological philologist Battiscombe Gunn (1916)—the praise was laudatory in the extreme. Gunn, who with T. Eric Peet, was the only British Egyptologist to publish significant work on Egyptian mathematics,[22] wrote an unprecedented eight-page review in the major English-language Egyptological journal—and this for an enemy national in the middle of the First World War!—concluding:

[19] *… [S]tehe ich nicht an, die Einführung des ägyptischen Kalenders am 19. Juli 4241 v. Chr. als das erste sichere Datum der Weltgeschichte zu bezeichnen.* (Meyer 1904: 45). We shall return to this question in section "Eduard Meyer and the Sothic period".

[20] Erman, though he never worked himself on Egyptian mathematics, had as students a good number of those who did: Sethe in Germany, Breasted in America, Boris Aleksandrovich Turaev and Vasilij Vasil'evich Struve in the Soviet Union,…. (Erman 1929: 283).

[21] There is also (favorable) mention of the book by Günther Roeder (Roeder 1917: 281) and F. Llewellyn Griffith (Griffith 1917: 273) in their respective reviews of the year in Egyptology.

[22] Besides work on metrological questions, he wrote a 15 page review of Peet's edition of the Rhind papyrus (Gunn 1926) and, with Peet, was the first to analyze problems in detail from the then only partially-published Moscow mathematical papyrus (Gunn and Peet 1929).

> To have had from so great an authority no more than a detailed account of Egyptian numbers and number-words, satisfying the requirements of philology, semantics, and palaeography, would have been a most welcome accession to our science; but the author, owing to the compactness, so to speak, of his subject, has been able to deal with it comparatively, and has step by step adduced illustrative parallels, not only from the civilised peoples of ancient and modern times, but from primitive and illiterate races. In fact he has done what will be done one day for [religion, astronomy, medicine, etc.]. And in this he has marked an epoch: for the first time an Egyptologist of the very front rank has dealt with a part of his science not merely as a special contribution to Egyptology, but with a view to the light it throws upon similar phenomena among other peoples and languages. Because many of the conclusions he arrives at affect Indo-Germanic and Semitic number-lore, his book must be taken into account by whoever wishes in future to examine the origin and evolution of one of the most fascinating and curious fields of universal culture—the art of ciphering. (Gunn 1916: 279)

Neugebauer also was to appreciate this in Sethe's work, the comparative cross-cultural dimension of the history of the mathematical sciences remaining a central theme in his own research.

But Neugebauer had only a comparatively brief direct contact with Sethe. At the end of the Summer Semester of 1923, Sethe left Göttingen for the University of Berlin where he had been called to replace Erman in the chair of Egyptology. His successor at Göttingen, and former student, Hermann Kees, was chosen in large part because of his interest in Egyptian religious texts and beliefs, a subject which had become the center of Sethe's own interests for some years now. So, although Neugebauer kept in epistolary contact with Sethe, he was essentially on his own at Göttingen after 1923 in his interest in Egyptian mathematical texts and numeration.

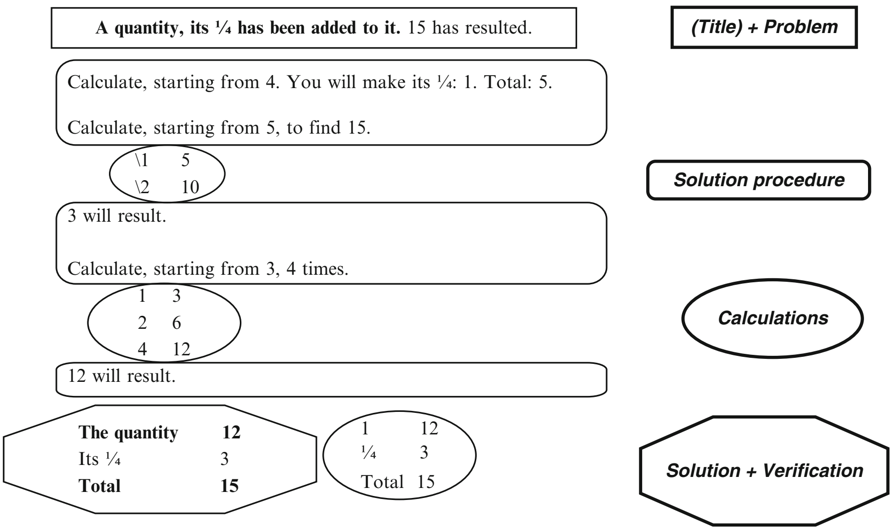

Fig. 3 Translation of Problem 26 of the Rhind Papyrus in the British Museum (BM 10057 + 10058)

The State of Egyptian Mathematical Historiography in 1923

Into what intellectual landscape did Neugebauer wander when he began his initiation into Egyptian language and culture? In particular what was known and understood about Egyptian mathematics by, say, late 1923? One important point needs to made at the outset concerning this domain, one that in large part determines its boundaries and nature and thus the range of possibilities for any research program in the field: the extreme paucity of sources and the elementary nature of their contents.

What mathematical texts were known to exist when Neugebauer began to interest himself in Ancient Egypt is easily resumed:

1 complete papyrus (Rhind papyrus), containing some 80 problems and a few tables, known since its first publication in 1877 (Eisenlohr 1877);
1 partially preserved papyrus (Moscow papyrus) of which only one problem had then been published (Turaev 1917)[23];
6 papyrus fragments, found in 1888–1889 at the town site of Lahun in the Fayum (Lahun fragments), containing the remains of 6 problems and 1 table (Griffith 1898);
2 papyrus fragments of unknown origin, with the remains of 4 problems, held in the Berlin Museum (Berlin fragments) and published at the beginning of the century (Schack-Schackenburg 1900, 1902);
2 wooden exercise tablets, supposedly found in the city of Akhmîm and purchased for the Cairo Museum (Akhmîm tablets), published in 1901 (Daressy 1901: p. 95–96, pl. LXII–LXIV) though not correctly understood until 1923 (Peet 1923b).

Thus from the whole pharaonic period—that is, the beginning of the third millennium BC to the end of the first millennium BC—we possess a total of two papyri, nine small fragments from perhaps three other papyri, a small leather roll, two wooden school tablets, and a potsherd. This was the full extent of known sources in 1923—and, to a great degree, it remains so today. True, the British Museum Leather Roll, containing two copies of a list of calculations, was unrolled in 1927 (Glanville 1927) and the Moscow Papyrus fully published in 1930 (Struve 1930). A few further minor discoveries and publications have followed: a school exercise on an ostracon from the Eighteenth Dynasty tomb of Senenmut at Deir el-Bahri (Hayes 1942: n° 153) and a fragment of papyrus with two incomplete problems (Imhausen and Ritter 2004: 91), found among the others at Lahun in 1890, but for some reason omitted in the original publication. But not another single pharaonic period mathematical source has come to light in all the years since, though post-pharaonic

[23] Though the Moscow papyrus had not yet been published in 1923, photographs of the complete papyrus were in Sethe's possession at Göttingen as part of his work on the Berlin Egyptian Dictionary.

Egyptian mathematical texts (Greek and Roman period texts in demotic writing) began to appear in 1959.

What was then known was the basic structure of Egyptian mathematical texts, that is, their division into tables on the one hand, and problems on the other; this last involving the posing of a problem, its solution, including the carrying out of individual calculations, and finally a verification of the final answer. A typical Egyptian problem is given in translation in Fig. 3, with the various parts of the problem indicated.[24] The solution procedure is expressed as a step-by-step algorithm involving one arithmetic command in each sentence.

Each step can be (though is not necessarily) followed by an explicit calculation effecting the operation. In the example here there is a calculation for each of the last two steps, division, then multiplication. The calculations are always carried out in a two-column arrangement, the division, $15 \div 5$ being carried out as follows:

\1	5
\2	10

The first column is initialized with 1 and the divisor 5 is placed facing it in the second column. The idea is to find entries adding up to the dividend, 15, in the second column, the sum of the corresponding entries in the first column then providing the answer. Various techniques, such as doubling, halving, finding the 2/3, etc. can be applied as required to the columns together.[25] Here one doubling has been carried out to generate the second line and this is sufficient since $5+10=15$ and these two lines are therefore checked (\); the corresponding entries of the first column 1 and 2 will then add up to the answer, 3. Multiplication is similar though inverse in the sense that one factor is placed at the head of the second column, the other factor then being sought in the first column with the corresponding entries of the second column providing the result.

Fig. 4 Translation of the beginning of the beginning of the "2/*n*" table for doubling fractions (from the papyrus UC 32159, Reference: Imhausen and Ritter 2004, p. 95)

2	3	$\overline{\overline{3}}$	2				
	5	$\overline{3}$	1 $\overline{\overline{3}}$	$\overline{15}$	$\overline{3}$		
	7	$\overline{4}$	1 $\overline{2}$ $\overline{4}$	$\overline{28}$	$\overline{4}$		
	9	$\overline{6}$	1 $\overline{2}$	$\overline{18}$	$\overline{2}$		
	11	$\overline{6}$	1 $\overline{\overline{3}}$ $\overline{6}$	$\overline{66}$	$\overline{6}$		
	13	$\overline{8}$	1 $\overline{2}$ $\overline{8}$	$\overline{52}$	$\overline{4}$	$\overline{104}$	$\overline{8}$
	15	$\overline{10}$	1 $\overline{2}$	$\overline{30}$	$\overline{2}$		
	17	$\overline{12}$	1 $\overline{3}$ $\overline{12}$	$\overline{51}$	$\overline{3}$	$\overline{68}$	$\overline{4}$
	19	$\overline{12}$	1 $\overline{2}$ $\overline{12}$	$\overline{76}$	$\overline{4}$	$\overline{114}$	[$\overline{6}$]
	21	$\overline{14}$	1 $\overline{2}$	$\overline{42}$	$\overline{2}$		

[24] The problem is number 26 of the Rhind papyrus, using the standard numbering, first established in its first edition (Eisenlohr 1877). The translation is my own.

[25] The techniques mentioned in most popularizations of Egyptian mathematics are erroneously limited to halving and doubling only. For a more accurate presentation see (Ritter 1995: 50–60).

The tables, used are primarily used as an aid in the carrying out of various steps in the calculations, where these are difficult or time-consuming. An example is given in Fig. 4, the beginning of the so-called $2/n$ table, used for the doubling of odd fractions.[26] Since neither a non-unit fraction (with the exception of 2/3) nor a repeated unit fraction is permitted, two or more distinct fractions must be sought whose sum is the desired doubled fraction. In the table, the numbers are arranged in three or more columns. The first column shows the divisor n (in the first entry only it shows both "dividend" 2 and divisor 3). This is followed by columns that alternately shows fractions of the divisor and their values (as a series of unit fractions). As an example, the third line starts with the divisor 7 in the first column, as it is the double of 1/7 that is to be expressed in unit fractions. This is followed in the second column by 1/4 and $1+1/2+1/4$ and in the third by 1/28 and 1/4. This is be understood as 1/4 of 7 is $1+1/2+1/4$, and 1/28 of 7 is 1/4. Since $1+1/2+1/4$ and 1/4 add together to equal 2, the series of unit fractions representing the double of 1/7 is $1/4+1/28$.

Neugebauer in Copenhagen I

In the spring of 1924, Neugebauer was invited to Copenhagen by the Danish mathematician Harald Bohr, the younger brother of the physicist Niels. This was the first, though as we shall see, far from the last time that the Danish capital was to play a major role in Neugebauer's life.

Bohr had been a frequent visitor to Göttingen in the preceding decade and a half. A quintessential arithmetic-analyst,[27] working on the summability of Dirichlet series and the Riemann zeta function conjecture, he had found a congenial teacher, then colleague in the older Göttingen number-theorist Edmund Landau, with whom he had coauthored a number of articles over the years. But in the year 1923 Bohr had found what was to be his true life's work, a domain of which he was essentially the creator, the theory of "almost periodic functions".[28] He decided to prepare an introduction in German to the new discipline, which was to be published in the Swedish international mathematical journal *Acta Mathematica*. Over the next 3 years, the "introduction" grew into a 258 page classic, published as three separate papers (Bohr 1924, 1925, 1926). Foreseeing the length, Bohr felt that his German,

[26] For a discussion of the uses of tables see (Imhausen and Ritter 2004: 95) and the references indicated there. We possess two copies of this table: one occupying almost all of the recto of the Rhind Papyrus (running from the double of 1/5 to the double of 1/101), the other illustrated here, being one of the Lahun fragments (doubling from 1/3 to 1/21).

[27] The term is Salomon Bochner's in (Bochner 1952).

[28] Bohr's original definition of an almost periodic function: A (complex-valued) continuous function f on $\mathbf{R}$ is called *almost periodic* if for any $\varepsilon>0$, every interval on $\mathbf{R}$ of length greater than a given $I(\varepsilon)$ contains at least one point $\tau(\varepsilon)$ such that $|f(x+\tau)-f(x)|\leq\varepsilon$ for all x. That is to say, an almost periodic function is one which, on a sufficiently long interval, comes arbitrarily close again to any of its already attained values.

though fluent, would need to be vetted by a native speaker, well-versed in mathematics. As Neugebauer had just finished helping Courant with the editorial work on the latter's *Methoden der mathematischen Physik,* the Director of the Institute could now spare his Assistant for a few months to do similar work with Bohr.

Neugebauer's passage to Denmark produced two results, apart from Bohr's first two articles in *Acta*: one was a collaboration with Bohr on a mathematical project in the domain of almost periodic functions that bore fruit some 2 years later in a joint article—Neugebauer's only publication in mathematics—on ordinary linear differential equations with an almost-periodic source term (Bohr and Neugebauer 1926).

The second result was of considerably greater importance for Neugebauer's career. Harald Bohr was the editor (together with Tommy Bonnesen) of the major Danish mathematical journal *Matematisk Tidsskrift B*, a publication of the Danish Mathematical Society.[29] The Society also published *Matematisk Tidsskrift A*, more general and aimed at mathematics teachers. In 1923 Section B of the Journal had received for review a copy of T. Eric Peet's new edition of the Rhind Papyrus but had no reviewer, competent both in mathematics and in Egyptology, in view. Neugebauer's arrival in Copenhagen offered an unlooked-for opportunity that Bohr was quick to seize. As he explained it himself in an introductory footnote to Neugebauer's review:

> When the Journal's Series B accepted the new edition of "Ahmes' Reckoning Book" for review, the young German mathematician O. Neugebauer, who was living in this town and who has been engaged in a detailed study of Ancient Egyptian culture, was so kind as to accept my invitation to write a review for the Journal. Since however the Ancient Egyptian reckoning book presents such an extensive interest to everyone interested in mathematics, the Journal's editors have preferred to print the review in Series A, thereby making it available to a wider circle of readers.[30]

This, Neugebauer's first publication, already reveals what were to become the three leitmotifs of his early interest in ancient mathematics and, more specifically, Egyptian mathematics:

> "Egyptian mathematics was a simple affair …", at least for us, if we consider the mathematical core of their problems. But against this there stands, at first glance, a dreadfully awkward and cumbersome calculational apparatus. While the basic number system is purely decimal (without place notation), that used for multiplication with integers is *dyadic*….[31] (Neugebauer 1925: 67),

[29] Still publishing, but since 1953 under the title of *Mathematica Scandinavica* and under the editorship of all five of the Mathematical Societies in Scandinavia.

[30] *Da Tidsskriftets Afdeling B modtog den nye Udgave af »Ahmes Regnebog« til Anmeldelse, opholdt sig her i Byen den unge tyske Matematiker Hr. O. Neugebauer der har beskæftiget sig indgaaende med den gamle ægyptiske Kultur, og som var saa elskværdig at imødekomme min Opfordring om at skrive en Anmeldelse til Tidsskriftet. Da den gamle ægyptiske Regnebog jo imidlertid frembyder saa stor Interesse for enhver matematisk interesseret, har Tidsskriftets Redaktioner foretrukket at lade Anmeldelsen trykke i Afdeling A for derved at gøre den tilgængelig en større Læsekreds.* (*apud* Neugebauer 1925: 66 note *).

[31] *"Egyptian mathematics was a simple affair …", mindestens für uns, wenn wir den mathematischen Kern ihrer Probleme betrachten. Dem steht aber ein auf den ersten Blick erschreckend umständlicher und schwerfälliger Rechenapparat gegenüber. Während das zu Grunde gelegte*

- a concentration on calculational questions,
- a claim, still undeveloped, of the purely additive character of Egyptian mathematics,
- an emphasis on the necessity of avoiding distorting retrospective analyses using contemporary mathematical concepts.

Now Kurt Sethe had, the previous year, already written a review of the new Peet translation (Sethe 1924), and this too in a mathematical journal, the very prestigious *Jahresbericht der Deutschen Mathematiker-Vereinigung* (the *Yearly Report of the German Mathematical Society*). Sethe's article, cited in Neugebauer's own review, focusses on the same main issues as Neugebauer's, that is principally calculational techniques with fractions. Discussions of solution procedures for both men were essentially restricted to comments on the geometric area problems and this choice of focus certainly represents the mainstream of interest in the domain at the time.

The conclusion of this very positive review also puts forward, for the first time in the Neugebauer corpus, what he considered to be the primary qualities of good historical writing:

> In its strictly material way, in its avoidance of all artificial hypotheses, and the sure understanding of historical possibilities, it serves as an excellent guide for the reader.[32] (Neugebauer 1925: 70)

It is in precisely in the name of these values that Neugebauer himself will undertake his reconstruction of the domain of the history of ancient mathematics in the years to follow.

Neugebauer on Egyptian Mathematics and Astronomy

Egyptian Fractions—The Thesis

Neugebauer had arrived at an insight during the preparation of the review of Peet's edition of the Rhind papyrus which he now saw as the central point of a possible thesis. He had remarked in the review, what Peet had not seen—nor any previous commentator—that there was a possible explanation for a strange phenomenon occurring in Problem 48 of that papyrus. This text presents a circle inscribed in a square of side 9 *ḫt* (*khet*), followed by two calculations: the squaring of 9 and the squaring of 8. In view of preceding problems it is clear that here we have the

Zahlensystem ein rein dezimales ist (ohne Stellenwert der Zeichen) ist die bei der Multiplikation mit ganzen Zahlen zur Anwendung kommende Methode eine d y a d i s c h e….

The initial English citation is from the book under review (Peet 1923a: Preface). The reference to "dyadic" here is the doubling technique used in explicit calculations.

[32] *In ihrer streng sachlichen Art, in der Vermeidung aller künstlichen Hypothesen und dem sicheren Verständnis für historische Möglichkeiten ist sie geeignet dem Leser ein vorzüglicher Führer zu sein.*

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calculation of the area of the two figures; that of the circle being calculated as the equivalent of a square of side 8/9 of the circle's diameter. Now the general practice in mathematical papyri is to use an abstract system of numeration[33] (without metrological units). Here however there is a unique use of the unit of area*, sṯꜣt (setjat),* consistently attached to one of the factors. For instance, the calculation for the square reads:

\1	9 *setjat*
2	18 *setjat*
4	36 *setjat*
\8	72 *setjat*

Previous commentators had assumed that the Egyptians, like ourselves, would have conceptualized multiplication as a matter of that type; i.e., the calculation of an area as *length* × *length* = *area*, each factor then having potentially the units of length, only the result being in units of area. Taking the labeling of one of the factors by *setjat* seriously, Neugebauer saw it otherwise, as *pure number* × *area* = *area*:

> with the calculation of the surfaces there comes the clear impression of the original significance of an operation. Determining the surface of a square of side 9 cubits,[34] 9 square cubits will be taken nine times, much more correctly so than any mechanical calculation with "dimensions".[35] (Neugebauer 1925: 69)

The revelation for Neugebauer was that here was clear evidence that the Egyptians did *not* conceptualize as we do, that there was an *additive* basis to their way of thinking that could be of tremendous importance in establishing the evolution of mathematical thought. Such an insight was one of the determining factors in Neugebauer's change of direction in the period 1924–1925. From this point on he turned away from mathematical research and would not become a small fish in a large pond; with the training he had received from the Göttingen mathematicians on one side and Sethe on the other, he would bring a unique combination of gifts to the small but equally significant domain of the history of mathematics and there rout the sloppy thinking and fantastical reconstructions that, as he saw it, had plagued this domain for so long.

The Peet review had illuminated the two legs upon which a thesis could be written: the centrality of the fraction as the core of Egyptian mathematics and the insight that the additive idea lay at the basis of that culture's mathematical thinking. The thesis itself, "The Foundations of Egyptian Fractional Calculations" (*Die Grundlagen der ägyptischen Bruchrechnung*), was finished in the summer of 1926. The colors are announced from the opening page:

[33] For this system see (Ritter 2001: 121).

[34] Neugebauer makes an error here, the *setjat* is a square *khet*, not a square cubit *(meḥ)*.

[35] *…bei der Berechnung von Flächengrössen zeigt sich die deutliche Empfindung für die ursprüngliche Bedeutung einer Operation. Um die Fläche eines Quadrates von 9 Ellen Seitenlänge zu bestimmen werden 9 QuadratelIen neunmal genommen, also viel korrekter als ein mechanisches rechnen mit "Dimensionen".*

> the most important single result of this work is the insight into the exclusively additive foundations of Egyptian mathematics, which gives to the entire further development its specific character.[36] (Neugebauer 1926: 1)

And the larger significance of this is in the opening epigram, taken from the 1869 Tübingen Inaugural Lecture of a mathematician and historian of mathematics whom Neugebauer viewed with respect, Hermann Hankel:

> Whoever knows the history of mathematics and has an eye open for the typical character of an epoch cannot overlook the influence that a period and a tradition have exercised on the development of mathematical science. Were I allowed to document these facts in detail here, then you Gentlemen would recognize in the state of mathematics of any epoch the reflection of all the traditions which characterize that period. For even mathematics too is a science which is carried on by *men*, and every period, like every people, has only *one* spirit.[37] (Hankel 1869: 25)

Neugebauer saw in the additive structure of Egyptian mathematics just that culture-specific trait which characterizes the Egyptian spirit and would provide an explanation for their "dreadfully awkward" choice of calculational techniques.

Radical changes had come to history in recent times, Neugebauer argued in the opening lines of his thesis:

> It is not only Greek science that succumbed to the magic that a millennial past had cast over all Egyptian thought; modern [historical] science too has had to gradually learn to approach things "in an unprejudiced manner", and so to understand how they came to be. Next to the demand not to consider all phases of a process as simultaneous and equivalent for our understanding, there is another: so far as possible to guard against the uncritical transfer of common modern concepts and experiences to ancient conditions.[38] (Neugebauer 1926: 1)

With the demand for an "unprejudiced manner" to determine truly how things "came to be", one detects an echo of the famous motto of the positivist school in history, put forward by the great German historian Leopold von Ranke in his first book:

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[36] *Das wichtigste prinzipielle Ergebnis der vorliegenden Arbeit ist die Einsicht in die ausschließlich additive Grundlage der ägyptischen Mathematik, welche der gesamten weiteren Entwicklung ihr spezifisches Gepräge gibt.*

[37] *Wer die Geschichte der Mathematik kennt und ein offenes Auge für den typischen Charakter einer Zeit hat, kann den Einfluss nicht übersehen, den Zeitcharakter und Volkseigenthümlichkeit auf die Entwicklung der mathematischen Wissenschaft ausgeübt haben. Wäre es mir erlaubt, diese Thatsache hier ausführlich zu begründen, so würden Sie, hochverehrte Herren, in dem Zustande der Mathematik in jeder Epoche den Reflex aller der Eigenthümlichkeiten erkennen, welche jene Zeit charakterisiren. Es ist eben Mathematik auch eine Wissenschaft, die von M e n s c h e n betrieben wird, und jede Zeit, sowie jedes Volk hat nur E i n e n Geist.*

Though Neugebauer cites only the last sentence, he clearly has in mind the whole paragraph.

[38] *Nicht nur die griechische Wissenschaft ist dem Zauber erlegen, den eine tausendjährige Vergangenheit über alles ägyptische Denken gebreitet hatte; auch die moderne Wissenschaft hat erst allmählich lernen müssen, „vorurteilslos" an die Dinge heranzutreten und sie so zu verstehen, wie sie geworden sind. Neben die Forderung, nicht alle Phasen eines Prozesses wie Gleichzeitiges und für unser Verständnis Gleichwertiges zu betrachten, tritt die andere, sich. soweit als irgend möglich davor zu hüten, uns geläufige moderne Begriffe und Anschauungen auf antike Verhältnisse kritiklos zu übertragen.*

> To history has been assigned the office of judging the past, of instructing the present for the benefit of future ages. To such high offices this work dare not aspire: it wants only to show what actually happened—*wie es eigentlich gewesen*.[39] (Ranke 1824: Vorrede)

If Neugebauer's history is informed by the positivist movement, his view of mathematics is firmly in the foundationalist tradition of the Göttingen school; the unification of mathematics under the aegis of set theory and the securing of its foundation through formalized logic.[40] This had already been the case in his work on Weyl's unified field theory and marked, for Neugebauer, the radical changes in recent mathematics:

> The mathematics of the last century too has experienced a major change; its "arithmetization"[41] has made great strides and investigations of its logical foundations have reached a decisive stage. Both directions have sharpened the capacity to single out the conceptual core of mathematical theorems and operations. Clearly history too must try to recognize the relation in which the concepts which are original to a given historical development stand to those concepts which, to modern ways of thinking, must have occupied this place from a purely logical point of view.[42] (Neugebauer 1926: 1)

The core that Neugebauer had detected is the additive structure, which he saw as essential for the development of mathematics.

The implication of all this was clear for Neugebauer: the time had come to rethink the history of mathematics in the light of these two revolutions in modern thought. Previous histories of mathematics had suffered from their unawareness of how this modern historical rigor, largely philological in origin, together with a steadfast refusal of anachronism, had outdated their traditional approach. This was particularly visible for Neugebauer in the few standard studies on ancient, particularly Egyptian, mathematics:

> I need only point to the arbitrary constructions of a M[oritz] Cantor or a [Friedrich] Hultsch. The critical and careful view of the historian of mathematics has, on this point, not succeeded in keeping up with contemporary philological work."[43] (Neugebauer 1926: 1)

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[39] *Man hat der Historie das Amt, die Vergangenheit zu richten, die Mitwelt zum Nutzen zukünftiger Jahre zu belehren, beigemessen; so hoher Ämter unterwindet sich gegenwärtiger Versuch nicht: er will bloß zeigen, wie es eigentlich gewesen.*

[40] For mathematics at Göttingen see (Rowe 2004).

[41] For the arithmetization program in mathematics see (Petri and Schappacher 2007) and (Jahnke and Otte 1981).

[42] *Auch die Mathematik der letzten Jahrhunderte hat eine große Wandlung erfahren; ihre „Arithmetisierung" hat große Fortschritte gemacht und die Untersuchungen über ihre logischen Grundlagen sind in ein entscheidendes Stadium getreten. Beide Richtungen haben den Blick dafür geschärft, den begrifflichen Kern mathematischer Sätze und Operationen herauszuschälen. Es ist klar, daß auch die Geschichte gerade der Anfänge der Mathematik danach streben muß, das Verhältnis zu erkennen, in dem die Begriffe, die in der gegebenen geschichtlichen Entwicklung die ursprünglichen sind, zu jenen Begriffen stehen, die nach modernen Anschauungen diesen Platz in rein logischer Hinsicht einnehmen müssten.*

[43] *…ich brauche etwa nur auf die willkürlichen Konstruktionen von M. Cantor oder Hultsch hinzuweisen. Kritik und Sorgfalt der Historiker der Mathematik haben es in diesem Punkte nicht vermocht, mit der gleichzeitigen philologischen Arbeit Schritt zu halten.*

Neugebauer's view of the correct way of proceeding is clear from the way the thesis is organized. The first chapter of the thesis is far from being an empirical summary of Egyptian calculational practices, gathering together observations of the kind he made on Problem 48. It starts with a chapter on the "Conceptual Foundations of Egyptian Mathematics" where as he stated in a later article,

> pure linguistic and psychological considerations, … showed that the original number concept is not limited to '*natural*' integers, but includes, as equally legitimate elements, precisely the "natural" fractions, while the 'algorithmic' fractions appear first as the inevitable result of a true *calculational technique* ('division').…
>
> The linguistic-psychological motivation naturally reaches deeper than the so-to-speak empirical, that inferred from the calculational formalism….[44] (Neugebauer 1930a: 336 n. 115)

For Neugebauer this additive spirit infuses all calculational operations of the Ancient Egyptians. Addition, subtraction, and multiplication, of course, but also division which is nothing other than a multiplicative test and thus ultimately an addition as well. All efforts to reconstruct anachronistically a multiplicative thinking for the Egyptians are thus necessarily condemned to failure—this criticism was particularly aimed at previous attempts by Eisenlohr, Cantor and Hultsch to find a prime-number concept in the Rhind papyrus.

Turning then to the numbers appearing in Egyptian calculations, Neugebauer distinguishes two types: natural [*natürlich*] and algorithmic [*algorithmisch*]. The former includes the integers and a small number of (unit) fractions used in everyday life, such as 1/2, 1/3, 1/4,…, 1/8. These comprise the fundamental core of Egyptian numbers. All other fractions make up the algorithmic class and constitute the only *extension* of the number concept developed in Ancient Egypt.[45]

The second part of the thesis is dedicated to a detailed analysis of the calculations with fractions appearing in the Rhind papyrus. He shows how the various techniques, particularly the use of "auxiliaries" [*Hilfszahlen*], a name given to a particular calculational technique involving the addition of fractions, flow naturally from such an additive spirit.

This dismissal of the work of two of the leading historians of mathematics of the period (e.g., Cantor 1894 and Hultsch 1895) is not just the typical iconoclastic enthusiasm of the young doctoral student but remained a constant in Neugebauer's attitude to those who held views he considered insufficient or outdated. See section "The Neugebauer style" below. It has to be placed in the context of a general polemic against Cantor's cultural history, see (Lützen and Purkert 1993).

[44] *…rein sprachliche und psychologische Überlegungen … zeigten, daß sich der ursprüngliche Zahlbegriff nicht auf den der „*natürlichen*"* g a n z e n Zahlen *beschränkt, sondern als gleichberechtigte Elemente eben die „natürlichen" Brüche mit umfaßt, während die „algorithmischen" Brüche erst als zwangsläufiges Resultat einer wirklichen* Rechentechnik *erscheinen („Division")….*

Die sprachlich-psychologische Motivierung reicht natürlich tiefer als die sozusagen empirische aus dem Rechenformalismus erschlossene…

[45] For fractions, this corresponds more or less to the general distinction introduced in (Benoit et al. 1992: 11) between "special-status fractions" and "*quantièmes*", but there it is applied to other cultures as well and based on a distinction in the *written* forms of the elements of the two classes (special signs for the special-status fraction vs. systematic constructability for the *quantièmes*).

An example is given in Fig. 4, taken from the verification in Problem 34 of the Rhind papyrus. The transcription incidentally shows two major notational innovations introduced for the first time into the study of ancient mathematics by Neugebauer in his thesis: 1° the representation of fractions $1/n$ by a bar over the integer n (and a double bar over 3 for 2/3). and 2° the use of boldface type to represent numbers and words written in red ink in the Egyptian texts. Both conventions are now universally adopted.

The calculation shows a multiplication of 5 1/2 1/7 1/14 by 1 1/2 1/4, knowing that the answer should be equal to 10. Starting with the first factor and producing the second factor by successive halving, the problem is now to show that the sum of all the fractions in the second column is indeed equal to 10. The scribe has summed the integers and the halves and quarters to produce 9 1/2 1/8. The complement needed to reach 10, i.e. 1/4 1/8, must now be shown to be equal to the sum of the remaining six fractions from column 2. To do this the scribe chooses the smallest fraction, here 1/56 and puts down the integer 1 in red ink underneath (the "auxiliary"); the other fractions are then attributed numbers on the basis of their relation to 1/56. The two fractions 1/28 for example are given the auxiliary 2 since they are the double of 1/56. The sum of the auxiliaries are added, their sum here is 21. Finally to show their equality with 1/4 1/8, the corresponding auxiliaries of these last, namely 14 and 7 respectively are added together. Since their sum is also 21 the scribe has determined that the sum of the six remaining fractions are indeed equal to 1/2 1/4, and thus that the result of the multiplication is indeed 10.

But the main emphasis in the second section of the thesis is on an attempt to use the additive principle to determine the origin of the Rhind $2/n$ table; i.e., finding the algorithm that would have been used by the Egyptians to decompose fractions of the form $2/n$ for odd n between 5 and 101 into sums of unit fractions. Neugebauer's failure to find a single unique method for the construction of the table, a failure common to all approaches to this question both before him and since, leads him to postulate a complicated developmental history, in which several stages, separated in time and not well integrated one with the other, would explain the difficulties.

To finally establish his thesis on the non-existence of any multiplicative concept in Egyptian mathematics, Neugebauer has to confront the commonly appearing term Egyptian *sep*, translated as 'times', as in *sep* 2 ('twice'), *sep* 3 ('three times'), etc. and as such, generally used in the expression for the operation of multiplication: *waḥ-tep m* p *sep* q('Calculate, starting with p, q times') as we have seen in our example above (section "The state of Egyptian mathematical historiography in 1923"). In the thesis he suggests a way of dealing with this and refers the reader to a forthcoming article (Neugebauer 1926: 5). This article duly appeared the following year (Neugebauer 1927); a short note in which, taking a cue from Sethe's discussion of the word *sep* (Sethe 1916: 46), Neugebauer points out that in the syntactic structure of phrases involving *sep* and a number, *sep* is treated as an object, so saying *sep* 5 "five times" in Egyptian is no more multiplicative than saying "five apples" [my example]. Thus for Neugebauer the last objection to a completely additive conception of number is answered.

The thesis, defended on 21 April 1926 at Göttingen, had been reviewed by the two Egyptologists, Kurt Sethe (now in Berlin) and Hermann Kees, and by Richard Courant, who wrote the résumé of the opinions expressed.[46] In his summary of the reports by the Egyptologists, he stressed the recognition on their part that the thesis marked a turning point from the Egyptological perspective:

> Both of the reports offered from our colleagues, Sethe in Berlin and Kees, show that the accompanying work of Herr Neugebauer represents a quite decisive advance in the unriddling of an important question and deserves high recognition as an achievement.[47] (Courant apud UAG Math. Net. Prüf. Neugebauer, Otto: 19 April 1926: 1)

But when Courant turns to the mathematical point of view, he sounds distinctly less enthusiastic—at least about the subject matter.

> From a mathematical point of view, it is naturally not a question of deep or difficult arithmetic problems; rather one can compare the solved exercises [of the Rhind papyrus] to the disentanglement of a complicated spectrum and the disclosure of the number-theoretic regularities contained therein.[48] (Courant apud UAG Math. Net. Prüf. Neugebauer, Otto: 19 April 1926: 1)

In closing, Courant takes up a motivation, shared by Neugebauer himself, that of "reclaiming" a central role for contemporary mathematicians in the comprehension and appreciation of past mathematics:

> It is not the first time that a mathematician has achieved something essential for Egyptology; the beginning of scientific Egyptology, as is well known, is closely linked to the names of some great French mathematicians. I am especially happy that again today a scholar, whom I know to be a mature mathematician, takes up the tradition.[49] (Courant apud UAG Math. Net. Prüf. Neugebauer, Otto: 19 April 1926: 2)

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[46] UAG Math. Nat. Prüf. Neugebauer, Otto: 19 April 1926.

[47] *Die bei den von Kollegen S e t h e in Berlin und Kollegen K e e s erstatteten Gutachten zeigen, dass die beiliegende Arbeit von Herrn N e u g e b a u e r vom Standpunkte der Ägyptologie aus einen ganz entschiedenen Fortschritt in der Enträtselung einer wichtigen Frage bedeutet und als Leistung hohe Anerkennung verdient.*

[48] *In mathematischer Hinsicht handelt es sich natürlich nicht um tiefe oder schwierige arithmetische Probleme, vielmehr kann man die gelöste Aufgabe mit der Entwirrung eines komplizierten Spektrums und der Aufdeckung der darin enthaltenen zahlentheoretischen Gesetzmässigkeiten vergleichen.*

The reference here is to the then current semi-empirical methods of analyzing atomic spectra by using the "old" Bohr-Sommerfeld quantum theory to derive numerical relations among the various lines of atomic spectra. The Göttingen physics department, with Max Born and his student Friedrich Hund, was then at the forefront of just such an approach; see (Hund 1927).

[49] *Es ist nicht das erste Mal, dass ein Mathematiker wesentliches für die Ägyptologie geleistet hat; die Anfänge der wissenschaftlichen Ägyptologie sind ja bekanntlich eng mit den Namen einiger grosser französischer Mathematiker verbunden. Es freut mich ganz besonders, dass heute wieder ein mir als reifer Mathematiker bekannter Gelehrter die Tradition aufnimmt.*

"French mathematicians" refers to those mathematicians and engineers who participated in Napoleon's Egyptian Expedition or in the edition of the monumental *Description de l'Égypte* which codified its scientific results: Gaspard Monge, Jean-Joseph Fourier, Edmé François Jomard, and Pierre Simon Girard.

One hears here an echo of the “nostrification” for which Göttingen was well-known: claiming for mathematics (and where possible, Göttingen mathematics) that which had been done in other domains.[50]

Neugebauer’s received his doctorate with his thesis adjudged “excellent (*ausgezeichnet*)” and his oral defense “very good (*sehr gut*)”.[51] For the completion of the Sethean program there remained now only a comparative study of the Mesopotamian material, a good, quick Habilitation subject. After a summer partially passed in the South of France with mathematician friends from Göttingen, Hans Hopf and the Russian Pavel Sergeyevich Alexandrov,[52] Neugebauer turned to the study of Sumerian with Anton Deimel in Rome.

Egyptian Mathematics—The Quellen und Studien *Articles*

The period following Neugebauer’s successful thesis defense marked a period of particularly intense intellectual and organizational activity in his life. Besides his research work, he continued as Courant’s Assistant, but also developed important ties with the Berlin scientific publisher, Julius Springer, with whom he would in the following years develop and act as editor for no less than four important scientific series. In 1929, with the Kiel historians of Greek mathematics, Otto Toeplitz and Julius Stenzel, he founded the “Sources and Studies on the History of Mathematics, Astronomy and Physics” (*Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*) series on the history of science.[53] Two years later, he became the founding editor of the major review journal for mathematics, the “Journal for Mathematics and Its Neighboring Areas” (*Zentralblatt für Mathematik und ihre Grenzgebiete*)[54] and in 1932, its associated monograph series “Results in Mathematics and Its Neighboring Areas” (*Ergebnisse der Mathematik und ihrer Grenzgebiete*). Finally in 1933, together with the engineer Wilhelm Flügge, he created the *Zentralblatt für Mechanik*.

But Egypt was not forgotten. Neugebauer continued his work on the unpublished Moscow mathematical papyrus, to which he had access thanks to Sethe’s photographs and the Russian contacts of his friend Alexandrov. In 1928, he spent time in Leningrad with Vasilij Vasil’evich Struve, who was to edit the papyrus as the first

[50] For the example of such an annexation in the case of Minkowski and special relativity, see (Walter 1999).

[51] UAG Kur. P.A. Neugebauer, Otto: Band 1/II, Bescheinigung 23. April 1926.

[52] See (Bečvářová and Netuka 2010: 16).

[53] This consisted of two parts, series A to publish editions of ancient texts—the first volume was Struve’s edition of the Moscow papyrus (Struve 1930)—and series B for research articles in the subject.

[54] Now called *Zentralblatt MATH*, this was created to replace the venerable but ailing *Jahrbuch über die Fortschritte der Mathematik*. For the history of these two journals see (Reinhard Siegmund-Schultze 1993).

volume of *Quellen und Studien A*. Most importantly, he was working on a pair of long articles, some 120 pages in all, on Egyptian mathematics to be published in the first volume of *Quellen und Studien B* and which would turn out to be, in a sense, his final word on Egyptian mathematics.

At the end of the introduction to the first of these articles, which basically resumes and extends his thesis, Neugebauer acknowledges his sources of inspiration.[55] The first, as we have seen already, is Sethe, to whom the article is dedicated "in admiration and gratitude":

> I must in conclusion recall three books which have been of particular influence… The first is SETHE's *On Numbers and Number Words*…, which in particular first opened up for me the possibility of historical ways of looking at concepts of number and fraction which were not founded on mere "intuition".[56] (Neugebauer 1930a: 303)

The remaining two influences on the young Neugebauer in these first publications are the historian of Egyptian art Heinrich Schäfer and the French anthropologist Lucien Lévy-Bruhl. As Neugebauer goes on to explain:

> Then [follows] H. SCHÄFER's *On Egyptian Art, Particularly Drawing*…, especially through its principal insights into the Egyptian conception of "perspective" style, and finally LÉVY-BRUHL's *How Natives Think*, which SCHÄFER's work completes in a much more general manner.[57] (Neugebauer 1930a: 303)

It was probably Sethe who introduced Neugebauer to the seminal work of Schäfer, then director of the Egyptian Museum in Berlin, as Sethe and Schäfer were longtime friends, having been students together under Erman in Berlin.[58] And as

[55] Though in large part composed in the autumn of 1928, Neugebauer states in a note (Neugebauer 1930a: 301) at the beginning of the article that as he cites the Moscow Mathematical Papyrus, he had wanted to wait until the official publication of that text in (Struve 1930).

[56] *Ich muß zum Abschluß dreier Werke gedenken, die von bestimmendem Einfluß auf die eigentlichen Grundlagen meiner Anschauungen geworden sind …. Das erste S e t h e s „Von Zahlen und Zahlworten"…, das mir überhaupt erst die Möglichkeit einer nicht auf bloße „Intuition" gegründeten geschichtlichen Betrachtungsweise des Zahl- und Bruchbegriffs erschlossen hat.*

[57] *Dann H. S c h ä f e r s „Von ägyptischer Kunst besonders der Zeichenkunst"…, vor allem durch seine prinzipiellen Einsichten über die ägyptische Auffassung der „perspektivischen" Darstellungsweise und schließlich L é v y-B r u h l s „Fonctions mentales dans les sociétés inférieures"…, das S c h ä f e r s Werk in ganz allgemeiner Hinsicht ergänzt.*

The earliest, and best, English-language review of the version of Schäfer's book used by Neugebauer (Schäfer 1919) is to be found in (Davies 1921), with a detailed summary of the first edition. Only a much later, considerably revised (first by the author, then by the editor and the translator) version was translated into English in 1974 under the title *Principles of Egyptian Art*. Lévy-Bruhl's now very perjorative-sounding French title (Fonctions mentales dans les sociétés inférieures) is here rendered into English by that under which the authorized English-language version of the book was published in 1925.

[58] As Erman later recalled about them: "And within the individual generations [of my students] harmony and friendship ruled, and I can only picture many of my students in their youth, like… Sethe and Schäfer, as pairs." [*Auch innerhalb der einzelnen Generationen herrschte Eintracht und Freundschaft, und manche meiner Schüler wie…Sethe und Schäfer kann ich mir in ihrer Jugend nur als Paare denken.*] (Erman 1929: 283).

Neugebauer himself points out, Schäfer led him to Lévy-Bruhl and the ‘primitive mind’.

The impact of these formative influences can be seen in the conclusions that Neugebauer himself put forth as central to his project.

> I would like to set up a sentence of Lévy-Bruhl’s as almost a leitmotiv for the setting for pre-Greek, and particularly Egyptian, mathematics: “Their mentality does not lend itself well to the operations familiar to us; but through means which are proper to itself, it knows how to obtain, up to a certain point, the same results.”[59] (Neugebauer 1930a: 303)

In his thesis Neugebauer had emphasized two aspects of the importance of Sethe’s 1916 groundbreaking work, the possibility of achieving a level of *rigor* in philological analysis comparable to that in mathematics and the introduction of a true *historicity* of number concepts. In the 1928–1930 paper, he was equally influenced by another aspect, the importance of cross-cultural *comparativeanalysis* to tease out possible laws of mathematical development. In the pursuit of this third point of the program, he went to Rome to study Assyriology with Anton Deimel in view of doing in that field what he had begun in the Egyptological domain, but with the essentially important addition of a potentially much larger corpus of unpublished texts to edit and study. On 12 November 1927 he obtained his *Habilitation* with a thesis on the origins of the sexagesimal system in Mesopotamia and was launched on the massive undertaking of the *Mathematische Keilschrifttexte* edition.

If his thesis had been an attempt to found new ideas about the nature of mathematical activity on a historical basis, the new pair of articles on the subject seek to analyze the totality of known Egyptian mathematics, divided into “Arithmetic and Calculation of the Egyptians” [*Arithmetik und Rechentechnik der Ägypter*] (Neugebauer 1930a) and “The Geometry of Egyptian Mathematical Texts” [*Die Geometrie der ägyptischen mathematischen Texte*] (Neugebauer 1931). Such a division for him now however is purely conventional, denoting merely a separation into domains of *application* of one and the same fundamental Egyptian—but also pre- and early Greek—mathematical nucleus, a “unitary developmental field for the mastery of calculational (that is, ‘arithmetic-algebraic’) problems” [*einheitliches Entwicklungsfeld zur Beherrschung rechnerischer (d. h. „arithmetisch-algebraischer“) Aufgaben*] (Neugebauer 1930a: 302).

The main thrust of his work is now to unveil the organizational principles at work in the Egyptian mathematical texts. The addition of “algebraic” to “arithmetic” in the above quote is symptomatic of a certain shift in his thinking since the thesis. The organizing principle that he will adopt in these two articles is algebraic and thematic. Thus, for example, the *ʿḥʿ (aḥa)* problems (those using this term meaning

[59] *Einen Satz L é v y—B r u h l s möchte ich geradezu als Leitmotiv für die Einstellung zur vorgriechischen, insbesondere ägyptischen Mathematik, hinstellen: „Leur mentalité se prête mal aux opérations qui nous sont familières; mais, par des procédés qui lui sont propres, elle sait obtenir, jusqu’à un certain point, les mêmes résultats“.*

The citation from Lévy-Bruhl is to be found at (Lévy-Bruhl 1922: 205), in the chapter entitled “The pre-logical mentality and its relation to numeration”.

'quantity' in their formulation, like Problem 26 of the Rhind papyrus, Fig. 3), with examples in the Rhind, Moscow, 'Kahun' (Lahun) and Berlin texts, represent a particular method, essentially algebraic in nature,[60] and expressible in modern form as a linear equation $x + ax + bx + c = d$, where a and b are fractions, and c and d integers. As is common with Neugebauer, at least since his work in Graz on unified theories, recognizing the correct underlying organizational principle allows a classification of the material. The Neugebauer classificatory diagram for the *aḥa* problems is reproduced here as Fig. 2b, organized, not by the specific form of the equation, but by the "consistent application" [*konsequent Anwendung*] (Neugebauer 1930a: 308) of the solution method.

Egyptian 'geometric' problems as laid out in the second article (Neugebauer 1931) are in fact metrological texts and thus akin to the arithmetic problems treated in the first article. Neugebauer's organizational principle in their treatment however is by geometric form: figures, surfaces, volumes grouped under their particular forms, as was traditional in their treatment by previous authors. The solution methods being straightforward for the most part, they all serve as a means of classification. But Neugebauer's net is thrown wide and he includes (published) material not previously brought together in this context, including a late second millennium literary school text (Papyrus Anastasi I), a Ptolemic-period papyrus in Greek that he felt reflected earlier Egyptian practice, and architectural diagrams taken over from Schäfer's work. *Exhaustivity*, the marshaling of *all* available evidence, was another trademark of Neugebauer which marked already his early Egyptian work.

These two articles were to be the last substantive work that Neugebauer ever published on the subject of Egyptian mathematics. When he spoke of it later, either in his *Vorgriechische Mathematik* (Neugebauer 1934) or in *The Exact Sciences in Antiquity* (Neugebauer 1957) or in a small number of occasional book reviews, it was never a question of more than a selection of the points he had already made in full before 1931.

The Return to Egypt—Neugebauer and Egyptian Astronomy

On January 26, 1932 Otto Neugebauer was named Associate (*außerordentlicher*) Professor[61] and seemed set to create a world center for the study of ancient science at Göttingen. Almost exactly a year later, Adolf Hitler was named Chancellor of Germany and on April 7, 1933 his government promulgated the "Law for the Restoration of the Civil Service" [*Gesetz zur Wiederherstellung des Berufsbeamtentums*], banning from public employment, and thus from employment at any German university, all non-Aryans and political opponents of Nazism. Neugebauer, being neither a Jew nor a Communist, was thus not directly affected

[60] Neugebauer specifically opposes the interpretation of these problems as examples of a 'false position' method, common in earlier work (Neugebauer 1930a: 309).

[61] UAG Kur. P.A. Neugebauer, Otto: Band III, Document 11.

but the writing was clearly on the wall and, with the idea of awaiting a more favorable climate in Germany, he activated his Copenhagen contacts. Harald Bohr obtained for him an offer of a position at the University of Copenhagen. On October 18, 1933 he asked for a 1-year leave of absence from his position at the University to be effective from the beginning of 1934. This was granted on November 1, 1933, with the condition that he relinquish his Göttingen salary for this period.[62] He was to renew this formality every year until May 28, 1936 when, it being clear that far from abating, the Nazi pressure on academic life was increasing, he announced that he would no longer ask for an extension of his leave of absence and thus resigned from his position at the University of Göttingen.[63]

Once settled in Copenhagen for the Second Semester of 1933/34, Neugebauer taught a course on "pre-Greek (i.e., Egyptian and Babylonian) mathematics",[64] which, written up under the same title (Neugebauer 1934), would become the first volume of a planned trilogy: *Lectures on the History of the Ancient Mathematical Sciences* [*Vorlesungen über Geschichte der antiken mathematischen Wissenschaften*], with a second volume planned on pre-Greek mathematical astronomy and a third on Greek science. The treatment of Egyptian mathematical texts in the first volume is essentially a résumé of the two *Quellen und Studien* articles and will constitute the last contributions of Neugebauer to Egyptian mathematics, aside from the later reworking of the same material in his popular *The Exact Sciences in Antiquity* (Neugebauer 1957).

With the second volume of the *Vorlesungen* in view, Neugebauer had been turning his attention more and more to ancient astronomical texts. Naturally he was principally interested in the abundant and advanced Babylonian material. However he did find the time, in conjunction with a colleague, Aksel Volten, of the Egyptological Institute of the University of Copenhagen and specialist in Demotic,[65] to publish, in what was to be the last fascicule of the last volume of *Quellen und Studien*, a Roman period Egyptian astronomical text, papyrus Carlsberg 9, found in a temple library in Tebtunis and dating from around 144 AD (Neugebauer and Volten 1938).[66] Unlike most Egyptian astronomical papyri which are mythological in content it deals with a 25-year lunar cycle, serving to connect the religious lunar calendar with the civil solar one, presented in the form of tables generated by a numerical subtraction algorithm. Though it contains a list of the zodiacal signs, a borrowing

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[62] Note from Schnoering at the Ministry to Kurator Valentiner. UAG Kur. P.A. Neugebauer, Otto: Band I, Document II 16.

[63] Letter from Kurator Valentiner to Neugebauer. UAG Kur. P.A. Neugebauer, Otto, Band I, Document II 17.

[64] "Über vorgriechische Mathematik", course notes in The Shelby White and Leon Levy Archives Center, Institute for Advanced Studies: Otto Neugebauer papers.

[65] Demotic refers to both a very late stage (seventh century BC to fifth century AD) of the ancient Egyptian language, and to its corresponding written form, an extremely cursive form of hieratic.

[66] Succinctly reedited in (Neugebauer and Parker 1969: Text vol. p. 220–225, Plate vol. p. 65).

from Hellenistic astronomy, it was recognized by Neugebauer as stemming from an indigenous Egyptian tradition, a rarity among extant late Egyptian sources.[67]

One other late text, from the same Tebtunis temple library, connects up to an even older native Egyptian tradition: papyrus Carlsberg 1, that Neugebauer and the director of the Egyptological Institute, Hans Ostenfeld Lange, published as a monograph a little over a year later (Lange and Neugebauer 1940).[68] The papyrus contains a hieratic text with a demotic commentary; large parts of this text exist also in hieroglyphic form on the ceilings of the royal tombs of Seti I and Ramesses IV, dating back to the thirteenth and twelfth centuries BC respectively. This time, the context is mythological: the sky goddess Nut and the appearance and disappearance of the stars. Though all aspects of the text were treated in the study, it was clear that Neugebauer's main interest in it lay in "Chapter E" the marking of the hours of any night by the culmination (in the earlier versions, the rising) of one of a group of thirty-six constellations, known today by the Greek name of "decans".

Though the *Vorlesungen* were never completed,[69] the material, in an enormously expanded form, was published in various forms over the following years by Neugebauer. In the Egyptian case this was in the shape of a dozen articles culminating and subsumed in the monumental three volumes of *Egyptian Astronomical Texts*, written with his Egyptological partner at Brown University, Richard A. Parker (Neugebauer and Parker 1960, 1964, 1969).[70]

The Neugebauer Style

Once the final decision to abandon mathematics (as a research but not as a social and intellectual milieu) had been taken, Neugebauer's future agenda was fixed. There were to be two main tasks: one negative, clearing the dead wood of outworn ideas and overly speculative theories, often accepted simply because the necessary compelling arguments and marshaling of evidence had not been carried out; the other positive, replacing these with accurate, justifiable—and justified—explanations, using the intensive rigor of the new philology and the new mathematics and the extensive breadth of Sethe's cross-cultural comparative method. We have already seen a number of examples of the positive side of this approach in the case of Egypt. The negative, critical aspect was to create for Neugebauer an image of a

[67] The original of the text dates back to the fourth century BC (Parker 1950: 24–29).

[68] See now (Neugebauer and Parker 1960: 36–94, pls. 36–42).

[69] Swerdlow (Swerdlow 1993: 147) argues plausibly that Neugebauer no longer felt that its original program of presenting Egypt and especially Mesopotamia as simply prolegomena to Greek science was in any way adequate or pertinent.

[70] Unlike his work on Egyptian and Babylonian mathematics or on Babylonian astronomy, Neugebauer never published alone a major article or monograph on Egyptian astronomy; all are cosigned with an Egyptologist, Volten, Lange, Parker. It is unclear if and to what extent he ever learned Demotic in which the majority of the astronomical texts were written.

redoubtable polemicist, always ready to take on other prominent historians, dead—like Moritz Cantor or Eduard Mayer—or alive. To understand the dynamic of Neugebauer's interventions into questions of ancient science, we shall review here two examples of the Neugebauer interventionist style and of its reception in the professional milieux which dealt with Ancient Egyptian mathematics and astronomy.

Eduard Meyer and the Sothic Period

Since the star Sirius constitutes one of the decans, that marking, in principal, the first day of the year, that is, the beginning of the first month of the Inundation season, Neugebauer's studies of the papyri Carlsberg 1 and Carlsberg 9 led him directly to the nature of the relationship between the Egyptian civil and lunar calendars; and this in turn led to the question of the 'Sothic cycle' so important for Eduard Mayer's Egyptian chronology.

Eduard Meyer, who, as we have seen (section "Egyptology at Göttingen"), believed that he had fixed the first certain date in Antiquity, July 19, 4241 BC, had presented Egyptology with a major problem. His argument turned on the so-called "Sothic cycle", the relation between the Egyptian civil year of 365 days and the astronomically observed annual heliacal rising of the bright star Sothis (our Sirius). There is a discrepancy of ¼ of a day per year between the Egyptian civil calendar of 365 days and the time for the Earth to make one complete revolution around the Sun, and thus $4 \times 365 = 1460$ years between coincidences of the astronomical heliacal rising of Sirius and any fixed date of the civil calendar. Based on the few dated mentions of the gap between the Egyptian New Year and the rising date of Sirius in Egyptian texts, Meyer assumed that the Egyptians must have fixed their civil calendar on a date when the rising of the Nile (the first day of the first month of the Inundation Season, our July 19) and the rising date of Sirius coincided. Since these occurrences are separated by 1460 years, the only acceptable candidates for that event were 2781 BC or 4241 BC. The former could be ruled out since it was known that the civil calendar was in use before the Old Kingdom (during which the first date falls) and thus the minimal date for the introduction was 4241 BC. Since Mayer assumed that such a fixing of the calendar year required repeated and accurate observations of the rising of Sirius, he had to postulate the existence of a corps of trained astronomers in Egypt at a time a full millennium earlier than previously accepted dates for the invention of writing and the creation of a centralized state. This in turn put pressure on Egyptologists to push back all historical dates in Egypt to fill the thousand-year void, an essentially impossible task.

In his work on ancient Egyptian astronomy, Neugebauer had shown how the astronomical papyri's cycles could have been discerned and the tables constructed—by means of numerical schema deriving from dead reckoning over long periods of time, with no need for precise astronomical observation.

Installed in Copenhagen, Neugebauer had had his interest in Egypt revitalized through the influence of two men; the Egyptologist Aksel Volten whom we have already met as Neugebauer's co-author of the edition of the papyrus Carlsberg 9 and an old friend from Göttingen, the Croatian-born mathematician Willy Feller.[71] Feller too had been an Assistent of Courant from 1925 to 1928, obtaining his doctorate with the latter in the same year as Neugebauer, then had gone as a *Privatdozent* to the University of Kiel. Refusing to take the Nazi oath in 1933, Feller had come to Copenhagen for a year, working with Herbert Busemann on differential equations and becoming friendly with Harald Bohr—and of course meeting up again with Neugebauer when the latter arrived in 1934. Offered a professorship at the University of Stockholm in the Winter Semester of that year, Feller nonetheless kept in close touch with his Copenhagen friends and colleagues. In 1939, when Neugebauer left the no-longer safe haven of Copenhagen for Brown University in Providence, Rhode Island, Feller too was offered a professorship there and became the executive editor of Mathematical Reviews when his friend founded that American replacement for his Zentralblatt, from which Neugebauer and numerous other anti-Nazi editors had resigned in 1938. It was in discussions with Feller, who shared a strong interest in Ancient Egypt, that the idea of taking on Meyer's isolated date arose and it was Feller who pushed Neugebauer into publication in 1939:

> I would like to emphasize that the core of this work came out of general discussions with Dr. W. Feller of Stockholm. I must also thank Dr. Feller for the bibliography on the Nile as well as the checking of the manuscript and the proofs.[72] (Neugebauer 1939a: 169)

In Neugebauer's typical manner, the article is divided into two parts, the negative and the positive. First, under the rather menacing epigram "Lasciate ogni speranza",[73] he attacks the very possibility that at the date proposed by Meyer, some thousand years before the invention of writing in Egypt and the establishment of a State, there could have existed a body of astronomers capable of the observations and calculations necessary for the construction of such a calendar. Furthermore, in only 8 years, the calendar would have been already 2 days out of synchronization, a fact hardly to be missed by such a hypothetical body of scientists. In short the Sothic cycle and the 365-day civil year were originally two independent systems for the Egyptians.

Having disposed of Meyer's correlation of the civil and Sothic year, Neugebauer passed to the replacement theory. He pointed out that dead reckoning, i.e., counting the number of days between successive inundations, would have permitted the establishment of a 365 day civil year by simply averaging over this interval for some reasonable time, say 50 years—the variability of the inundation would have become visible only over some much longer period of time during which the Sothic cycle could have been recognized as providing a tighter link with the rising of the Nile.

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[71] For Feller (born Vilibald Srećko Feller) see (Birnbaum et al. 1970).

[72] *Ich möchte hervorheben, daß der Kern dieser Arbeit in gemeinsamer Diskussion mit Dr. W. Feller, Stockholm, entstanden ist. Ebenso habe ich Herrn Dr. Feller für die Literaturnachweise über den Nil sowie für Durchsicht von Manuskript und Korrekturen zu danken.*

[73] *Lasciate ogni speranza voi ch'intrate!*, "Abandon all hope, you who enter!" is the final line of the inscription Dante places at the entrance to Hell in his *Divina Commedia* (canto 3, line 9).

Only thereafter did the question of the relative lapse between the Sothic and the civil date become of interest to Egyptians.

The article caused an immediate stir in the Egyptological community. Jean Capart, the eminent Belgian Egyptologist and historian of art, immediately contacted Neugebauer in his capacity as editor of the Belgian journal *Chronique d'Égypte*, requesting a shortened translation into French. This was very quickly forthcoming under the title of "La période sothique" and Capart prefaced it in enthusiastic terms:

> Professor O. Neugebauer has had the kindness to sum up for *Chronique d'Égypte* his recently published study on the "Sothic period". At the beginning of this article he had written, concerning those, I suppose, whose opinion he combats: *Lasciate ogni speranza*. He might equally well have written: *Chronologia egyptiaca liberata*.[74] For Professor Neugebauer's proof liberates Egyptian historical studies from the Procrustean bed to which Ed. Meyer had felt it necessary to bind them. (Capart *apud* Neugebauer 1939b: 258)

The "first certain date in the history of the world" had vanished. It was not only Capart who was excited; the German Egyptologist Alexander Scharff, giving a talk in the summer of 1939 to the Bavarian Academy of Sciences, began by stating:

> We can never be sufficiently grateful to the … mathematician O. Neugebauer for a physically small but intellectually momentous work, in which, with compelling conclusions of amazing simplicity, he abolishes the astronomical foundations of the entire edifice of Egyptian chronology that Ed. Meyer had once constructed with such enormous astuteness.[75] (Scharff 1939: 3)

But it was Neugebauer's negative argument that had really swept the field. His proposed replacement did not in itself fix any particular period as a candidate for the introduction of the calendar since it was only Meyer's hypothesis of the existence of an organized body of professional astronomers in fifth millennium Egypt that Neugebauer opposed. In fact, Neugebauer was rather tempted to simply leave the date there where Meyer had placed it, around 4200 BC. It was the Egyptologists who saw the liberation as one of being able to significantly lower the date of the introduction of the calendar; indeed this was the major point of Scharff's lecture, cited above, and by the time Neugebauer was settled in the US at Brown he had finally become convinced (Neugebauer 1942: 401 n. 17) that it was likely to have been introduced during the period of State formation at the beginning of the third millennium BC.[76]

[74] Latin for "Egyptian chronology liberated".

[75] *Wir können dem … Mathematiker O. Neugebauer gar nicht dankbar genug sein für eine umfänglich kleine, inhaltlich bedeutungsschwere Arbeit, in der er mit zwingenden Schlüssen von verblüffender Einfachheit die astronomischen Grundlagen des ganzen einst von Ed. Meyer mit gewaltigem Scharfsinn erdachten Gebäudes der ägyptischen Chronologie aufhebt.*

[76] That no "clearing of the ground' is ever truly established once and for all is made clear by the fact that some celebrations of the centennial of Meyer's publication in 2004 trot out the full hypothesis, initial date and all, e.g., (Zulian 2004), where, interestingly, Neugebauer is completely ignored.

Kurt Vogel and the Eye of Horus

Though Neugebauer had been the first trained mathematician[77] to turn to a professional study of Ancient Egyptian mathematics, he was not to remain so long. Kurt Vogel,[78] 11 years older than Neugebauer, had also studied mathematics and physics seriously—at Erlangen and Göttingen—and, after his service in the War, had become a teacher of mathematics at the Maximilians-Gymnasium in Munich, where indeed he remained until his retirement. He too had a long-standing interest in Ancient Egypt and had also learned the language while at Göttingen. In 1927 he decided to go back for a doctorate under the University of Munich Egyptologist Wilhelm Spiegelberg, with a view to working in the area of Egyptian mathematics. His 1929 thesis (Vogel 1929a) was entitled "The Foundations of Egyptian Arithmetic in Relation to the 2:*n* Table of the Rhind Papyrus" [*Die Grundlagen der ägyptischen Arithmetik in ihrem Zusammenhang mit der 2:n-Tabelle des Papyrus Rhind*]. At the University, his work attracted the patronage of mathematician Constantin Carathéodory and of the historian of mathematics, Heinrich Wieleitner. The latter was associated with the venerable *Archive for the History of Mathematics, the Sciences and Technology* [*Archiv für Geschichte der Mathematik, der Naturwissenschaften und der Technik*] and opened its pages to Vogel. Over the following 3 years he published some ten articles on Egyptian mathematics before turning his attention to Greek and Babylonian subjects.[79]

The similarities to Neugebauer's own history, the choice of thesis topic, the striking similarities in range of subjects and even titles—Vogel published his 1958–1959 synthesis of ancient mathematics under the title *Vorgriechische Mathematik* (Vogel 1958/1959)—bear witness to what had developed as a fierce rivalry that persisted between the two over a long period. From their respective fortresses of *Quellen und Studien* and *Archiv*, they traded negative reviews of each other's work—and of that of each other's protégés—with occasional forays into enemy territory. Upon invitation by the editors, Neugebauer published his (essentially very critical) essay review of Vogel's thesis in the pages of *Archiv* (Neugebauer 1930b), while Vogel published a string of reviews in Neugebauer's *Zentralblatt für Mathematik*. When the London Leather Roll was finally unrolled and published in 1927 (Glanville 1927), the first (though minor) addition to the Egyptian mathematical corpus in over a quarter of a century, both men rushed into print (Vogel 1929b; Neugebauer 1929) with predictably opposed interpretations.

One typical example of this opposition can be seen at work in the exchange, a year later, over the question of the so-called 'Horus-eye fractions' in Egyptian mathematics.[80] In 1911, the Berlin Egyptologist Georg Möller was preparing the second

[77] Though T. Eric Peet had majored in mathematics as an Oxford undergraduate, he had never practiced it, having turned to archeology immediately after graduation.

[78] What follows is based on (Mahoney and Schneider 1986) and (Folkerts 1983).

[79] Most of these have been reprinted in (Folkerts 1988).

[80] A detailed discussion of these "fractions" and their current status can be found in (Ritter 2003).

volume of his great hieratic sign-list, that is the list of ordinary Egyptian written signs and since it was generally believed that hieratic signs were in all cases cursive forms of pictographic originals, Möller had organized his sign-list by hieroglyphic form, organized by class of objects represented. In particular he believed he had shown that a series of seven signs representing the dimidiated fractions—1/2, 1/4, 1/8, …, 1/64—of the basic unit of volume, the *ḥeqat*, used to measure grain came originally from component parts of the Eye of Horus, a frequently encountered religious symbol in Ancient Egypt (Möller 1911). This had become received opinion by 1930 when Neugebauer looked into the matter (Neugebauer 1930c). On the negative side, he first passed in review the mathematical uses of these signs in conjunction with ordinary fractions in the mathematical texts and presented an argument that they were not fractions but integer multiples of a known smaller capacity unit, the *ra*. This is a direct outcome of the additive basis of Egyptian mathematics, for in such a system, rather than fractional parts of a unity—1 *ḥeqat* measure—it is more natural to think of dyadic parts of the *ḥeqat*'s equivalent of 320 *ra*, i.e., 160, 80, 40, 20, 10, 5. Then he assembled arguments to show that the *ḥeqat*-"Horus-eye fractions" system was originally separate from and independent of the *ra* capacity measure system. Finally, turning to the form of the hieroglyphic signs, the sole basis for Möller's identification, he marshaled all the then-available evidence to show that, at least for 1/16 and 1/32, this identification simply did not stand up to scrutiny.[81] In its place Neugebauer argued for the signs in question coming from an abbreviated form of the writing of their equivalents, 20 *ra* and 10 *ra*, thus arguing for a secondary origin of the Horus-eye fractions as integral multiples of a smaller basic unit.

The response was immediate—and from Vogel. In the very next issue of the same journal with his "On the Question of the Parts of the Bushel" (Vogel 1930), he countered with a series of arguments against Neugebauer's refusal to see the Horus-eye signs as fractions on grounds largely already presented in his own work and in reviews of those of his rival, while Neugebauer in turn, in "Once More, the Parts of the Bushel" (Neugebauer 1932) brought out his earlier criticisms of Vogel and reiterated his own position. And there the "exchange" ended.

In this case, unlike the result of the Neugebauer—Meyer "debate", nobody listened to either Neugebauer's critiques of Möller's hypothesis nor to his counterproposal for the origin of the supposed 'Horus-eye fractions'. Möller's hypothesis was simply repeated in virtually every reference to Ancient Egyptian capacity measurement for over 70 years. There is, in fact, only one single mention of Neugebauer's argument in either the Egyptological or in the history of science literature in all this time, a passing reference at the end of an article on the religious aspects of the Eye of Horus (Müller-Wollermann 1986).

Looking at the ways in which Neugebauer was heard—or ignored—by the Egyptological community in his attempts to address them directly on contentious issues points to the limitations of Neugebauer's program to bring rigor to the field

[81] In his edition of the Rhind papyrus, Peet had already expressed skepticism about Möller's hieroglyphic equivalents for these capacity measures (Peet 1923a: 25–26).

of Egyptology. He was listened to when he offered something that was already on the agenda of the Egyptologists; in the case of Meyer's chronology, this was the unease they felt with an overlong chronology and here Neugebauer's critique was welcomed enthusiastically, though not his comfort with Meyer's early date in general. On the other hand, Möller's suggestion of a religious origin for metrological measures fit well with the general theological and humanistic interests of the Egyptologists and Neugebauer's (and even Peet's) objections found no traction and evoked not the slightest interest.[82]

The Essential Tension

That interest in fundamental questions that drew Otto Neugebauer as a young veteran first to unified theories based on the new theory of relativity, then to pure mathematics, led him finally to a search for the founding principles of mathematical and logical thought in humanity's earliest preserved records. In the early twentieth century, in terms of known texts, that could only mean Ancient Egypt. But more than that, Neugebauer also had a personal interest in that culture. He is of course well-known for his later harsh judgement that

> Egypt provides us with the exceptional case of a highly sophisticated civilization which flourished for many centuries without making a single contribution to the development of the exact sciences. (Neugebauer 1975: II 559)

Yet he would still write, many years after he had abandoned Egypt as a research domain, that

> of all the civilizations of antiquity, the Egyptian seems to have been the most pleasant.… There is probably no other country in the ancient world where cultivated life could be maintained through so many centuries in peace and security. (Neugebauer 1957: 71)

But it was not only the culture that had attracted him to the Nile Valley. The contact at Göttingen with the one Egyptologist who most represented the attempt to bring rigor to the study of Egyptology determined in large part not only Neugebauer's decision to leave pure mathematics for a study of its history but also the manner of interpellating the ancient texts. An appreciation of the cultural context, a sensitivity to the foundational questions lying behind calculational techniques and notational conventions, an appreciation of the possibilities of classificatory and structural approaches: in all this Neugebauer was a product of Göttingen, not only of Courant and Hilbert but of Sethe too.

If we ask to what extent and how Neugebauer influenced the domain of Egyptological research, the reply is more nuanced. Unlike the situation in Assyriology, there are no generic references to Neugebauer's work in either the Egyptological nor the in history of Egyptian science communities today, no standard references to his publications. Recognition of his work on fractions is limited,

[82] For more detail on this question see (Ritter 2003).

at most, to citing the corresponding ten pages of *The Exact Sciences in Antiquity* as a general source for the novice, with no mention made of the general historical and foundational motivation behind it. It was not that he was either ignored or unappreciated by the Egyptologists of the period. T. Eric Peet for example, in a 1934 critical review of a book on Egyptian art, bemoaned the absence in it of an appreciation of the new and exciting work being done in the field.

> Much new light has recently been thrown on Egyptian drawing and sculpture, and on Egyptian mathematics, by the ability of some writers, notably Schafer in the case of drawing and sculpture, and Gunn, Neugebauer, Vogel,…in the case of mathematics, to clear their minds effectively of the modern point of view, to go back behind the Greek and to see the subject as the Egyptians saw it. (Peet 1934: 120)

That Peet was able to appreciate this in Neugebauer is all the more striking since many at the time—and even more so later—saw in his use of elementary algebraic symbolism in the analysis of Egyptian mathematics not a mere notational convenience, the organizing principles for him being the solution methods employed, but the actual ancient conceptual framework. Ironically this made Neugebauer seem to be yet another anachronistic modernizer, a position against which he fought so hard in his early days.

Even when he was listened to by Egyptologists in the 1920s and 1930s he was always seen as essentially an outsider, "der Mathematiker" in Scharff's words cited earlier. This was not entirely a misreading; there was always an essential tension in Neugebauer's work. As Noel Swerdlow very finely observed in his obituary of his colleague and friend (Swerdlow 1993), there were always two aspects to Neugebauer: the historian sensitive to the specificities of given historical and cultural contexts and the practicing mathematician, concerned with the rigor of analysis and the essential unity of all mathematics. From the beginning Neugebauer attempted to synthesize these two aspects: " It has been my endeavor," he explained in the Introduction to his thesis, "so far as it was in my power, to emphasize here both tendencies—those of the historical as well as of the mathematical sciences." [*Es war mein Bestreben, beide Tendenzen, sowohl die der historischen wie der mathematischen Wissenschaften, soweit es in meinen Kräften stand, hier zur Geltung zu bringen.* Neugebauer 1926: 1]. It was not that he had any illusions about the modern mathematical interest of the extant Egyptian material but he could hold at bay within himself the impatience of the professional mathematician because he was interested in other, more fundamental questions about the nature of all mathematical cognition, questions that the Ancient Egyptian material like any other could supply if approached in the right manner. But the paucity of the Egyptian material compared to the Mesopotamian, and its more elementary nature meant that the early "desire to investigate the logico-conceptual foundations of mathematics of one of the most interesting people in Antiquity" [*die logisch-begrifflichenGrundlagen der Mathematik eines der interessantesten Völker des Altertums*, Neugebauer 1926: *Vorwort*] gave way to the sheer intellectual challenge of the sophisticated application of arithmetic methods and their application to physical phenomena in

Mesopotamian mathematical astronomy. And to *this* Neugebauer, Egypt had nothing more to offer.

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