Little Piplup

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Contents

1 Settings

1.1 C++

2 Data Structures

2.1 Segment Tree

To deal with queries on intervals, we use segment tree.

```
int arr[SIZE];
int tree[TREE_SIZE];
int makeTree(int left,int right,int node)
{
    if (left == right)
        return tree[node] = arr[left];
    int mid = (left + right) / 2;
    tree[node] += makeTree(left, mid, node * 2);
    tree[node] += makeTree(mid + 1,right, node * 2 +1);
    return tree[node];
}
```

Updating segment Tree

```
void update(int left,int right,int node, int change_node ,int diff)
{
    if (!(left <= change_node &&change_node <= right))
        return; //No effect on such nodes.
    tree[node] += diff; // This part must be changed with tree function.
    if (left != right)
    {
        int mid = (left + right) / 2;
        update(left, mid, node * 2, change_node, diff);
        update(mid +1,right, node * 2 +1, change_node, diff);
    }
}
```

Answering queries via segment tree

```
/*
Our Search range : start to end
Node has range left to right
We may answer query in O(log n) time.
*/
int Query(int node, int left, int right, int start, int end)
{
    if (right < start || end < left)
        return 0; //Node is out of range
    if (start <= left && right <= end)
        return tree[node]; //If node is completely in range
    int mid = (left + right) / 2;
    return Query(node * 2, left, mid, start, end)
    +Query(node*2+1,mid+1,right,start,end);
}
```

2.2 Fenwick Tree

2.3 Disjoint Set Union (Union - Find)

```
//Union Find with Pass compression
int parent[V]; //initialize with i
int level[V]; //initialize with 1
int Find(int x) // Finding root node of x
{
    if (x==parent[x])
        return x;
    else
        return parent[x] = Find(parent[x]);
}

void Union(int x, int y)
{
    int u = Find(x);
    int v = Find(y);
    if (u == v)
        return;
    if (level[u] > level[v])
    {
        Union(v,u);
        return;
    }
    parent[u] = v;
    if (level[u]==level[v])
        level[v]++;
}
```

3 Mathematics

3.1 Useful Mathematical Formula

- Catalan Number : Number of valid parantheses strings with n pairs

$$C_n = \frac{1}{n+1}\binom{2n}{n}$$

3.2 Number of Integer Partition

```
def partitions(n):
    parts = [1]+[0]*n
    for t in range(1, n+1):
        for i, x in enumerate(range(t, n+1)):
            parts[x] += parts[i]
    return parts[n]
```

3.3 Extended Euclidean Algorithm

```
int Extended_Euclid(int a, int b, int *x, int *y)
{
    if (a == 0)
    {
        *x = 0;
        *y = 1;
        return b;
    }
    int x1, y1;
    int EEd = Extended_Euclid(b%a, a, &x1, &y1);
    *x = y1 - (b/a) * x1;
    *y = x1;
    return EEd;
}
```

3.4 Fast Modulo Exponentiation

Calculating $x^y \bmod p$ in $\mathcal{O}(\log y)$ time.

```
/*
Fast Modulo Exponentiation algorithm
Runs on O(log y) time,
calculate x^y mod p
*/

ll modpow(ll x, ll y, ll p)
{
    ll res = 1;
    x = x % p;
    while (y > 0)
    {
        if (y & 1)
            res = (res*x) % p;
        y = y>>1;
        x = (x*x) % p;
    }
    return res;
}
```

3.5 Miller-Rabin Primality Testing

Base values of a chosen so that results are tested to be correct up to 10^14.

```
bool MRwitness(ll n, ll s, ll d, ll a)
{
    ll x = modpow(a, d, n);
    ll y = -1;

    while (s)
    {
        y = (x * x) % n;
        if (y == 1 && x != 1 && x != n-1)
            return false;
        x = y;
        s--;
    }
    return (y==1);
}

bool Miller_Rabin(ll n)
{
    if (n<2)
        return false;
    if (n == 2 || n == 3 || n == 5 || n == 7 || n == 11 || n == 13 || n == 17)
        return true;
    if (n%2 == 0 || n%3 == 0 || n%5 == 0)
        return false;
    ll d = (n-1) / 2;
    ll s = 1;
    while (d%2==0)
```

```
    {
        d /= 2;
        s++;
    }
    int candidate[7] = {2,3,5,7,11,13,17};
    bool result = true;
    for (auto i : candidate)
    {
        result = result & MRwitness(n,s,d,i);
        if (!result)
            break;
    }
    return result;
}
```

3.6 Pollard-Rho Factorization

```
ll PollardRho(ll n)
{
    srand (time(NULL));
    if (n==1)
        return n;
    if (n % 2 == 0)
        return 2;
    ll x = (rand()%(n-2))+2;
    ll y = x;
    ll c = (rand()%(n-1))+1;
    ll d = 1;
    while (d==1)
    {
        x = (modpow(x, 2, n) + c + n)%n;
        y = (modpow(y, 2, n) + c + n)%n;
        y = (modpow(y, 2, n) + c + n)%n;
        d = gcd(abs(x-y), n);
        if (d==n)
            return PollardRho(n);
    }
    return d;
}
```

3.7 Euler Totient

Calculating number of integers below n which is coprime with n.

```
#define ll long long
ll euler_phi(ll n)
{
    ll p=2;
    ll ephi = n;
    while(p*p<=n)
    {
        if (n%p == 0)
            ephi = ephi/p * (p-1);
        while(n%p==0)
            n/=p;
        p++;
    }
    if (n!=1)
    {
        ephi /= n;
        ephi *= (n-1);
    }
    return ephi;
}
```

3.8 Modular Multiplicative Inverse

```
ll modinv(ll x, ll p)
{
    return modpow(x,p-2,p);
}
```

3.9 Fast Fourier Transform

4 Geometry

4.1 CCW

```
//Is 3 points Counterclockwise? 1 : -1
//0 : on same line
int CCW(Point a, Point b, Point c)
{
    int op = a.x*b.y + b.x*c.y + c.x*a.y;
    op -= (a.y*b.x + b.y*c.x + c.y*a.s);
    if (op > 0)
        return 1;
    else if (op == 0)
        return 0;
    else
        return -1;
}
```

4.2 Point in polygon

Returns boolean, if point is in the polygon (represented as vector of points).

```
// point in polygon test
inline double is_left(Point p0, Point p1, Point p2)
{
    return (p1.x - p0.x) * (p2.y - p0.y) - (p2.x - p0.x) * (p1.y - p0.y);
}
bool is_in_polygon(Point p, vector<Point>& poly)
{
    int wn = 0;
    for (int i = 0; i < poly.size(); ++i)
    {
        int ni = (i + 1 == poly.size()) ? 0 : i + 1;
        if (poly[i].y <= p.y)
        {
            if (poly[ni].y > p.y)
                if (is_left(poly[i], poly[ni], p) > 0)
                    ++wn;
        }
        else
        {
            if (poly[ni].y <= p.y)
                if (is_left(poly[i], poly[ni], p) < 0)
                    --wn;
        }
    }
    return wn != 0;
}
```

4.3 Closest Pair Problem

4.4 Smallest Enclosing Circle

4.5 Convex Hull (Graham Scan)

4.6 Intersection of Line Segment

5 Graphs

5.1 Topological Sorting

Topological sorting with dfs

```
vector <int> graph[V];
bool visited[V];
vector <int> sorted;

void dfs(int root)
{
    visited[root] = 1;
    for (auto it:graph[root])
    {
        if (!visited[it])
            dfs(it);
    }
    sorted.push_back(root);
}

int main()
{
    int n, m;
    scanf("%d%d",&n,&m);
    for (int i = 0; i<m; i++)
    {
        int small,big;
        scanf("%d%d",&small,&big);
        graph[small].push_back(big);
    }
    for (int i = 1; i<=n; i++)
        if (!visited[i])
            dfs(i);
    reverse(sorted.begin(),sorted.end()); // must reverse!
}
```

5.2 Bipartite Checking

```
vector <int> graph[20200];
vector <pair <int, int>> edge;
bool visited[202020];
int color[202020];
void dfs(int root)
{
    for (auto it:graph[root])
    {
        if (!visited[it])
        {
            visited[it] = true;
            color[it] = color[root]%2+1;
            dfs(it);
        }
    }
}
bool is_bipartite(vector <int> &mygraph, int v, int e)
{
    for (int i = 1; i<=v; i++)
    {
        if (!visited[i])
        {
            visited[i] = 1;
            color[i] = 1;
            dfs(i);
        }
    }
    for (int i = 0; i<e; i++)
        if (color[edge[i].first]==color[edge[i].second])
            return false;
    return true;
}
```

5.3 MST Kruskal Algorithm

```
//Based on Union-Find implementation
//O(E log E) if path-compressed Union Find.
int Kruskal()
{
    int mstlen = 0;
    sort(edgelist.begin(),edgelist.end());
    for (auto it:edgelist)
    {
        if (Find(it.s)==Find(it.e)) // Cycle Detection
            continue;
        else
        {
```

```
            Union(it.s,it.e);
            mstlen += it.w;
        }
    }
    return mstlen;
}
```

5.4 MST Prim Algorithm

```
vector <pii> Tree[101010];
// Note that we use {weight, destination} pair here.
// This is to use priority_queue!
bool visit[101010];
priority_queue <pii, vector<pii>, greater<pii>> pq;
void add(int i)
{
    visit[i] = true;
    for (auto it:Tree[i])
        pq.push(it);
}

int Prim(int start)
{
    int mstlen = 0;
    add(start);
    while(!pq.empty())
    {
        int cur = pq.top().second;
        int weight = pq.top().first;
        pq.pop();
        if (visit[cur])
            continue;
        else
        {
            mstlen+=weight;
            add(cur);
        }
    }
    return mstlen;
}
```

5.5 Ford-Fulkerson Algorithm

6 Shortest Path

6.1 Dijkstra

$\mathcal{O}(E \log V)$ Single-Start-Shortest-Path.
Not working for graph with minus weight.

```
const int INF = 987654321;
const int MX = V+something;
struct Edgeout
{
    int dest, w;
    bool operator<(const Edgeout &p) const
    {
        return w > p.w;
    }
};

vector <Edgeout> edgelist[MX];
int V, E, start;
int dist[MX];

bool relax(Edgeout edge, int u)
{
    bool flag = 0;
    int v = edge.dest, w = edge.w;
    if (dist[v] > dist[u] + w && (dist[u]!=INF))
    {
        flag = true;
        dist[v] = dist[u]+w;
    }
    return flag;
}

int dijkstra()
{
    fill(dist,dist+MX,INF);
    dist[start] = 0;
    priority_queue<Edgeout> pq;
    pq.push({start,0});
    while(!pq.empty())
    {
        Edgeout x = pq.top();
        int v = x.dest, w = x.w;
```

```
        pq.pop();
        if (w>dist[v])
            continue;
        for (auto ed : edgelist[v])
            if (relax(ed,v))
                pq.push({ed.dest,dist[ed.dest]});
    }
}
```

6.2 Bellman Ford

$\mathcal{O}(EV)$ Single-Start-Shortest-Path.
Not working for graph with minus cycle → must detect.

```
struct Edge
{
    int u, v, w;
};

vector <Edge> edgelist;
int V, E;
int dist[V+1];

bool relax_all_edge()
{
    bool flag = false;
    for (auto it:edgelist)
    {
        int u = it.u, v = it.v, w = it.w;
        if (dist[v] > dist[u] + w && (dist[u]!=INF))
        {
            flag = true;
            dist[v] = dist[u]+w;
        }
    }
    return flag;
}

int bellman_ford()
{
    fill(dist,dist+V+2,INF);
    dist[1] = 0;
    for (int i = 0; i<V-1; i++)
    {
        relax_all_edge();
    }
    if (relax_all_edge())
        return -1;
    else
        return 0;
}
```

6.3 Floyd-Warshall

Works on adjacency matrix, in $\mathcal{O}(V^3)$.

```
int d[120][120];
int n;
void Floyd_Warshall()
{
    for (int i = 1; i<=n; i++)
        for (int j = 1; j<=n; j++)
                d[j][k] = MIN(d[j][k],d[j][i]+d[i][k]);
}
```

7 Dynamic

7.1 Longest Increasing Subsequence

Find LIS in $\mathcal{O}(n \log n)$ time.

7.2 Largest Sum Subarray

Computes sum of largest sum subarray in $\mathcal{O}(N)$

```
void consecsum(int n)
{
    dp[0] = number[0];
    for (int i = 1; i<n; i++)
        dp[i] = MAX(dp[i-1]+number[i],number[i]);
}

int maxsum(int n)
{
    consecsum(n);
    int max_sum=-INF;
    for (int i = 0; i<n; i++)
        dp[i] > max_sum ? max_sum = dp[i] : 0;
    return max_sum;
}
```

7.3 Knapsack

7.4 Longest Common Subsequence

```
//input : two const char*
//output : their LCS, in c++ std::string type
string lcsf(const char *X,const char *Y)
{
    int m = (int)strlen(X);
    int n = (int)strlen(Y);
    int L[m+1][n+1];
    for (int i=0; i<=m; i++)
    {
        for (int j=0; j<=n; j++)
        {
            if (i == 0 || j == 0)
                L[i][j] = 0;
            else if (X[i-1] == Y[j-1])
                L[i][j] = L[i-1][j-1] + 1;
            else
                L[i][j] = max(L[i-1][j], L[i][j-1]);
        }
    }
    int index = L[m][n];

    char lcsstring[index+1];
    lcsstring[index] = 0;

    int i = m, j = n;
    while (i > 0 && j > 0)
    {
        if (X[i-1] == Y[j-1])
        {
            lcsstring[index-1] = X[i-1];
            i--; j--; index--;
        }
        else if (L[i-1][j] > L[i][j-1])
            i--;
        else
            j--;
    }
    string lcsstr = lcsstring;
    return lcsstr;
}
```

7.5 Edit Distance

8 String

8.1 KMP Algorithm

8.2 Manacher's Algorithm

8.3 Trie

8.4 Rabin-Karp Hashing

8.5 Aho-Corasick Algorithm

9 Miscellaneous

9.1 Useful Bitwise Functions in C++

```
int __builtin_clz(int x);// number of leading zero
int __builtin_ctz(int x);// number of trailing zero
int __builtin_clzll(ll x);// number of leading zero
int __builtin_ctzll(ll x);// number of trailing zero
int __builtin_popcount(int x);// number of 1-bits in x
int __builtin_popcountll(ll x);// number of 1-bits in x

lsb(n): (n & -n); // last bit (smallest)
floor(log2(n)): 31 - __builtin_clz(n | 1);
floor(log2(n)): 63 - __builtin_clzll(n | 1);

// compute next perm. ex) 00111, 01011, 01101, 01110, 10011, 10101..
ll next_perm(ll v)
{
  ll t = v | (v-1);
  return (t + 1) | (((~t & -~t) - 1) >> (__builtin_ctz(v) + 1));
}
```

9.2 List of Useful Numbers

```
< 10^k    prime  # of prime        < 10^k             prime
-------------------------------------------------------------
1             7           4         10           9999999967
2            97          25         11          99999999977
3           997         168         12         999999999989
4          9973        1229         13        9999999999971
5         99991        9592         14       99999999999973
6        999983       78498         15      999999999999989
7       9999991      664579         16     9999999999999937
8      99999989     5761455         17    99999999999999997
9     999999937    50847534         18   999999999999999989
```

10 Debugging Checkpoints