Case Study: Probabilities & Decision Trees

Important

Please note that you can download PDF and Microsoft Word versions of this case study using the links on the right.

Case 1

In a hypothetical community, 60% of all people consume at least 6 alcoholic beverages per week and 50% are overweight. The percentage of people who are both overweight and consume this much alcohol is 40%. Construct a 2x2 table to answer (a)-(c) below. For part (d), construct a decision tree.

- *What percentage of people consume at least 6 alcoholic beverages per week, are overweight, or fall into both categories?*

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	Overweight	Not overweight	Total
Drink 6 alcoholic beverages/week	0.4	0.2	0.6
Drink less or non-drinker	0.1	0.3	0.4
Total	0.5	0.5	1

P(Overweight OR Drinker) = 0.4 + 0.1+ 0.2 = 0.7
OR
P(Overweight OR Drinker) = 0.6 + 0.5 - 0.4 = 0.7
OR
1-neither (.03)

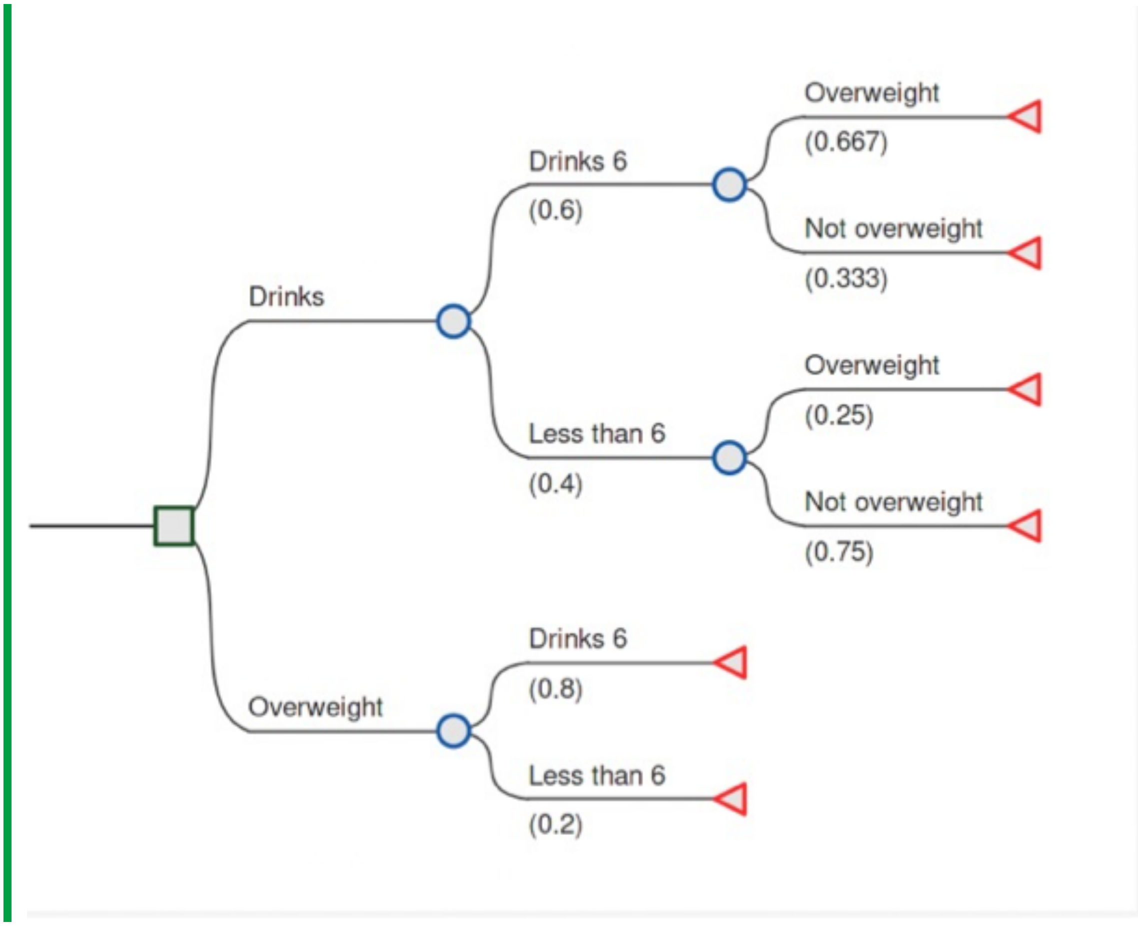
- *You sample at random a person from the community and find that they consume at least 6 alcoholic beverages per week. What is the probability that they are overweight?*

P(Overweight | Drinker) = 0.4 / 0.6 = 0.667

- *What is the probability that someone from this community consumes at least 6 alcoholic beverages per week if they are overweight?*

P(Drinker | Overweight) = 0.4 / 0.5 = 0.8

- *Draw a decision tree to represent this problem*



Case 2

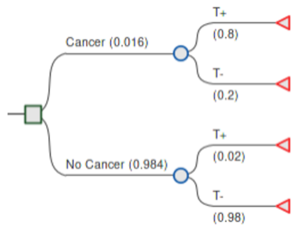
A new screening procedure can detect 80% of women diagnosed with breast cancer but will falsely identify 2% without breast cancer. The prevalence of breast cancer in the population is 1.6 in 100

- *What is the probability that a woman does not have breast cancer if the test is negative?*

*Note: There are two ways to do this – either draw a decision tree OR 2X2 table by creating a hypothetical population

(a)

P (C+ and T+) =. 0128
P(C+ and T-) = .0032
P(C- and T+)= .01968
P(C- and T-)= .96432
P(T+)= .0128+.01968=.03248
P(T-)= .0032+.96432=.96752



Then to solve for prob that woman does not have breast cancer given that test is negative:

P (C- | T-) = P(C- and T-)/P(T-)=.96432/.96752=.9967

- *What is the probability that a woman has breast cancer if the test is positive?*

(b) Prob that a woman has breast cancer given that the test is positive looks like this:

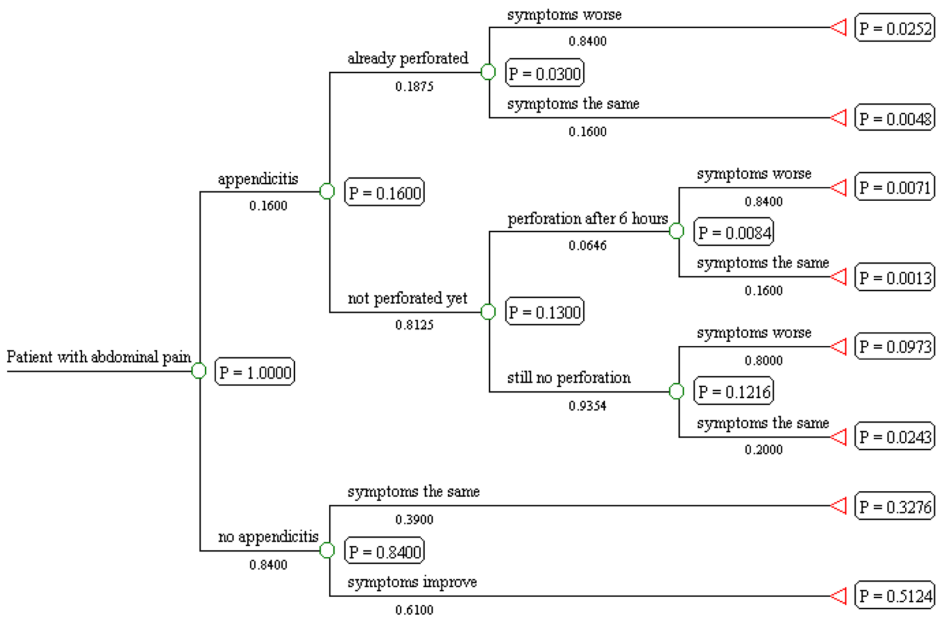
P(C+ | T+) = P(C+ and T+)/P(T+)=.0128/.03248= 0.3941

Another way to do it is to build a 2X2 table: since we know that the incidence of disease is 1.6 in 100, take a hypothetical population of 100,000 women

	Cancer +	Cancer -	
Test +	1,280 (1,600*.8)	1,968 (98,400*.02)	3,248
Test -	320	96,432	96,752
	1,600	98,400	100,000

And now you can get the same probabilities above.

(b) The following chance tree can be drawn for this problem. An easy way to answer the following questions is to calculate the probabilities at the end of each branch of the decision tree.



P(Perforation at the beginning of the six hours) = 0.1600 * 0.1875 = 0.0300. This is the proportion of patients with appendicitis multiplied by the conditional probability of perforation given appendicitis at the time the patient enters the hospital.

- *Calculate the probability that the patient will have a perforated appendix if you wait 6 hours*

P(Perforation after six hours) = 0.0300 + 0.1600 * 0.8125 * 0.0646 = 0.0384 P(Perforation after six hours) = 0.1600 * 0.2400 = 0.0384. This is the proportion of patients with appendicitis times the proportion of patients with a perforated appendix.

- *Calculate the probability that the patient's symptoms will 1) get worse, 2) stay the same, and 3) get better.*

You can find these probabilities by adding up the probabilities at the ends of the branches of the decision tree. P(Symptoms worse) = 0.0252 + 0.0071 + 0.0973 = 0.1296 P(Symptoms the same) = 0.0048 + 0.0013 + 0.0243 + 0.3276 = 0.3580 P(Symptoms improve) = 0.5124

- *Calculate the conditional probability that the patient has a perforated appendix if the symptoms 1) get worse; 2) stay the same or 3) get* better.

By using probability definitions, we can calculate the conditional probabilities. P(E,F) = P(E | F) * P(F) P(E,F): Joint probability of E and F together P(E | F): Conditional probability of E, given F P(F): Probability of F P(E | F) = (E,F) / P(F) P(Perforation | Symptoms worse) = P(Perforation and Symptoms worse) / P(Symptoms worse) = (0.0252 + 0.0071) / 0.1296 = 0.2492 P(Perforation | Symptoms same) = P(Perforation and Symptoms same) / P(Symptoms same) = (0.0048+ 0.0013) / 0.3593 = 0.0170 P(Perforation | Symptoms improve) = P(Perforation and Symptoms improve) / P(Symptoms improve) = 0 / 0.5124 = 0

- *Calculate the conditional probability that the patient has appendicitis if 1) the symptoms get worse, 2) stay the same, or 3) get better*

Using the same probability notations as in question f, we can calculate the following probabilities: P(Appendicitis | Symptoms worse) = (0.0252 + 0.0071 + 0.0973) / 0.1296 = 1 P(Appendicitis | Symptoms same) = (0.0048 + 0.0013 + 0.0243) / 0.3580 = 0.0849 P(Appendicitis | Symptoms improve) = 0 / 0.5124 = 0