

Case Study: Probabilities & Decision Trees

! Important

Please note that you can download PDF and Microsoft Word versions of this case study using the links on the right.

Case 1

In a hypothetical community, 60% of all people consume at least 6 alcoholic beverages per week and 50% are overweight. The percentage of people who are both overweight and consume this much alcohol is 40%. Construct a 2x2 table to answer (a)-(c) below. For part (d), construct a decision tree.

- *What percentage of people consume at least 6 alcoholic beverages per week, are overweight, or fall into both categories?*
- *You sample at random a person from the community and find that they consume at least 6 alcoholic beverages per week. What is the probability that they are overweight?*

- *What is the probability that someone from this community consumes at least 6 alcoholic beverages per week if they are overweight?*

Case 2

A new screening procedure can detect 80% of women diagnosed with breast cancer but will falsely identify 2% without breast cancer. The prevalence of breast cancer in the population is 1.6 in 100

- *What is the probability that a woman does not have breast cancer if the test is negative?*

- *What is the probability that a woman has breast cancer if the test is positive?*

Case 3

A patient is found to have an abdominal aortic aneurysm (AAA) 5 cm in size. If you operate now and he survives, he will have a life expectancy of 3 additional years. In a series of 100 similar patients from your hospital, 6 died immediately after surgery. If you elect to watch the patient, 60% will rupture their AAA at home (assume at an average of 1 year later). Of those who rupture their AAA, 30% will die, while the other % will undergo emergency surgery and survive, allowing the patient to survive the full 3-years of life expectancy

- *Draw a decision tree for the problem of choosing whether to operate electively. Remember that you must consider the life expectancies as an outcome here. What is the preferred choice?*
- *A 95% confidence interval for the mortality rate of elective surgery at your hospital ranges from 1.4-12.7%. Does this influence your thoughts? Why? Do you need better information about your estimate of mortality? Why? (Note: For the latter, calculate the mortality rate for which you are indifferent between the two choices)*

Case 4

A patient presents to the ER with abdominal pain. As per the ER doc, you estimate that the patient's appendicitis probability is 0.16. If the patient truly has appendicitis, the probability that the appendix was already perforated at the time the patient presents to the ER is 0.1875. You can also observe the patient for 6 hours to be certain that your diagnosis is correct. If your diagnosis ends up being correct, 24% of individuals will have a perforated appendix after 6 hours (Note: this number is not what goes into your tree. You must account for the 18.75% who already had a perforated appendix at the time the patient entered the hospital).

If the appendix is perforated at the time the patient presents to the ER or at the end of 6 hours, there is a 0.84 chance that the symptoms will become worse and a 0.16 chance they will remain the same. If the patient has appendicitis but the appendix does not burst at the end of 6 hours, there is a 0.8 chance the symptoms will worsen and a 0.2 chance that they will remain the same. If the appendix is not diseased, there is a 0.39 chance that the symptoms will remain the same in 6 hours, a 0.61 chance that this will improve, and no chance that they will worsen.

- Calculate the probability that a patient has a perforated appendix by the end of 6 hours given that he had appendicitis but was not perforated at the time he entered the hospital

Draw a decision tree to calculate the probabilities asked in questions 3-6.

- *Calculate the probability that the patient has a perforated appendix at the beginning of 6 hours.*

$P(\text{Perforation at the beginning of the six hours}) = 0.1600 * 0.1875 = 0.0300$. This is the proportion of patients with appendicitis multiplied by the conditional probability of perforation given appendicitis at the time the patient enters the hospital.

- *Calculate the probability that the patient will have a perforated appendix if you wait 6 hours*

- *Calculate the probability that the patient's symptoms will 1) get worse, 2) stay the same, and 3) get better.*

- *Calculate the conditional probability that the patient has a perforated appendix if the symptoms 1) get worse; 2) stay the same or 3) get better.*

- *Calculate the conditional probability that the patient has appendicitis if 1) the symptoms get worse, 2) stay the same, or 3) get better*