Case Study: Probabilities & Decision Trees

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| Important |
| Please note that you can download PDF and Microsoft Word versions of this case study using the links on the right. |

# Case 1

In a hypothetical community, 60% of all people consume at least 6 alcoholic beverages per week and 50% are overweight. The percentage of people who are both overweight and consume this much alcohol is 40%. Construct a 2x2 table to answer (a)-(c) below. For part (d), construct a decision tree.

* *What percentage of people consume at least 6 alcoholic beverages per week, are overweight, or fall into both categories?*

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* *You sample at random a person from the community and find that they consume at least 6 alcoholic beverages per week. What is the probability that they are overweight?*

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| P(Overweight | Drinker) = 0.4 / 0.6 = 0.667 |

* *What is the probability that someone from this community consumes at least 6 alcoholic beverages per week if they are overweight?*

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| P(Drinker | Overweight) = 0.4 / 0.5 = 0.8 |

* *Draw a decision tree to represent this problem*

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# Case 2

A new screening procedure can detect 80% of women diagnosed with breast cancer but will falsely identify 2% without breast cancer. The prevalence of breast cancer in the population is 1.6 in 100

* *What is the probability that a woman does not have breast cancer if the test is negative?*

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* *What is the probability that a woman has breast cancer if the test is positive?*

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P(Perforation at the beginning of the six hours) = 0.1600 \* 0.1875 = 0.0300. This is the proportion of patients with appendicitis multiplied by the conditional probability of perforation given appendicitis at the time the patient enters the hospital.

* *Calculate the probability that the patient will have a perforated appendix if you wait 6 hours*

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| P(Perforation after six hours) = 0.0300 + 0.1600 \* 0.8125 \* 0.0646 = 0.0384 P(Perforation after six hours) = 0.1600 \* 0.2400 = 0.0384. This is the proportion of patients with appendicitis times the proportion of patients with a perforated appendix. |

* *Calculate the probability that the patient’s symptoms will 1) get worse, 2) stay the same, and 3) get better.*

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| You can find these probabilities by adding up the probabilities at the ends of the branches of the decision tree. P(Symptoms worse) = 0.0252 + 0.0071 + 0.0973 = 0.1296 P(Symptoms the same) = 0.0048 + 0.0013 + 0.0243 + 0.3276 = 0.3580 P(Symptoms improve) = 0.5124 |

* *Calculate the conditional probability that the patient has a perforated appendix if the symptoms 1) get worse; 2) stay the same or 3) get* better.

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| By using probability definitions, we can calculate the conditional probabilities. P(E,F) = P(E | F) \* P(F) P(E,F): Joint probability of E and F together P(E | F): Conditional probability of E, given F P(F): Probability of F P(E | F) = (E,F) / P(F) P(Perforation | Symptoms worse) = P(Perforation and Symptoms worse) / P(Symptoms worse) = (0.0252 + 0.0071) / 0.1296 = 0.2492 P(Perforation | Symptoms same) = P(Perforation and Symptoms same) / P(Symptoms same) = (0.0048+ 0.0013) / 0.3593 = 0.0170 P(Perforation | Symptoms improve) = P(Perforation and Symptoms improve) / P(Symptoms improve) = 0 / 0.5124 = 0 |

* *Calculate the conditional probability that the patient has appendicitis if 1) the symptoms get worse, 2) stay the same, or 3) get better*

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| Using the same probability notations as in question f, we can calculate the following probabilities: P(Appendicitis | Symptoms worse) = (0.0252 + 0.0071 + 0.0973) / 0.1296 = 1 P(Appendicitis | Symptoms same) = (0.0048 + 0.0013 + 0.0243) / 0.3580 = 0.0849 P(Appendicitis | Symptoms improve) = 0 / 0.5124 = 0 |