

$$\mathbf{d}_{YLD} = \begin{matrix} \text{H} \\ \text{S1} \\ \text{S2} \\ \text{DOC} \\ \text{DS} \end{matrix} \begin{pmatrix} 0 \\ \mathbf{d}\mathbf{w}\mathbf{S1} \frac{1}{r\Delta_t} (1-e^{-r\Delta_t})\Delta_t \\ \mathbf{d}\mathbf{w}\mathbf{S2} \frac{1}{r\Delta_t} (1-e^{-r\Delta_t})\Delta_t \\ 0 \\ 0 \end{pmatrix}$$

$$\mathbf{d}_{YLL}(t) = \begin{matrix} \text{H} \\ \text{S1} \\ \text{S2} \\ \text{D} \\ \text{trDS} \end{matrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{1}{r} (1-e^{-rEx(a(t))}) \end{pmatrix}$$

Let  $S$  be an  $\omega \times k$  matrix,  $d$  be an  $\omega \times k$  matrix, and  $c$  be an  $\omega \times 1$  vector.

The Hadamard product of  $S$  and  $d$  is given by:

$$S \odot d = \begin{bmatrix} s_{11}d_{11} & s_{12}d_{12} & \cdots & s_{1k}d_{1k} \\ s_{21}d_{21} & s_{22}d_{22} & \cdots & s_{2k}d_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ s_{\omega 1}d_{\omega 1} & s_{\omega 2}d_{\omega 2} & \cdots & s_{\omega k}d_{\omega k} \end{bmatrix}$$

Summing across the columns of  $S \odot d$  to obtain a vector:

$$\text{sum}(S \odot d) = \begin{bmatrix} \sum_{j=1}^k s_{1j}d_{1j} \\ \sum_{j=1}^k s_{2j}d_{2j} \\ \vdots \\ \sum_{j=1}^k s_{\omega j}d_{\omega j} \end{bmatrix}$$

Finally, performing the element-wise multiplication with vector  $c$ :

$$\text{sum}(S \odot d) \odot c = \begin{bmatrix} \left( \sum_{j=1}^k s_{1j} d_{1j} \right) c_1 \\ \left( \sum_{j=1}^k s_{2j} d_{2j} \right) c_2 \\ \vdots \\ \left( \sum_{j=1}^k s_{\omega j} d_{\omega j} \right) c_{\omega} \end{bmatrix}$$