Problem 1. Construct a balanced outcome of network exchange for the graph of Figure 12.10 in NCM?

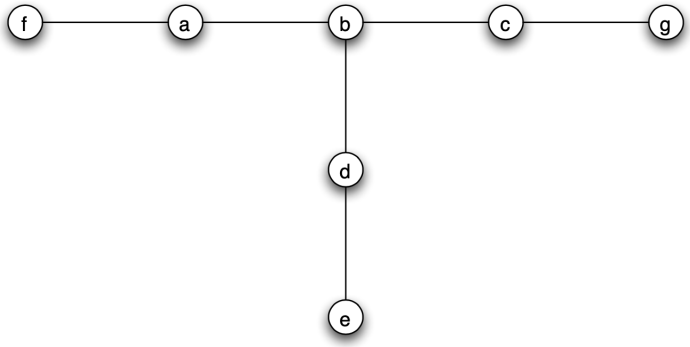


Figure 1: Figure 12.10 in NCM

Solution. Since this network is symmetric in structure, we only have to consider 3 nodes, for example, let node f, a, b have value x, y, z respectively.

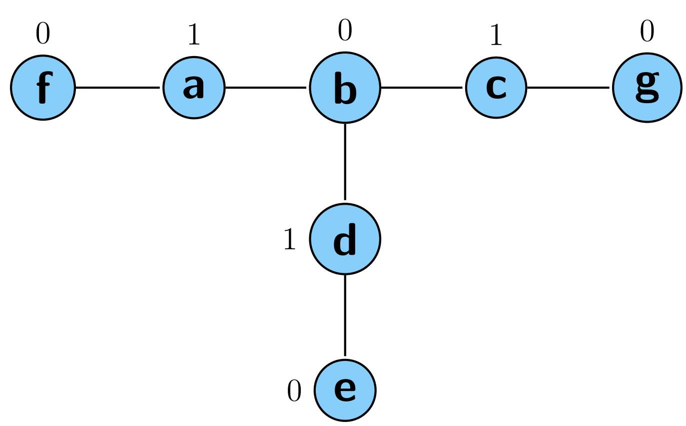
For pair $(f - a)$, f node has outside option 0, whereas a has outside option $1 - z$. The surplus become $s = 1 - (1 - z) = z$, therefore, we have

$$V(f) = 0 + \frac{1}{2}s = \frac{z}{2}$$
$$V(a) = 1 - z + \frac{1}{2}s = 1 - \frac{z}{2}$$

For pair $(a - b)$, a node has outside option $1 - x$, whereas b has outside option $1 - y$. The surplus become $s = 1 - (1 - x) - (1 - y) = x + y - 1$, therefore, we have

$$V(a) = 1 - x + \frac{1}{2}s = \frac{1 - x + y}{2}$$
$$V(b) = 1 - y + \frac{1}{2}s = \frac{1 + x - y}{2}$$

Now that we represent all x, y values using one notation z, let $z = 0$, and the resulting balanced graph is listed as below.



Problem 2. At the beginning of the cascading, we introduced the herding experiment, guessing which box. Read the Section 16.5 A simple, general Cascade Model in NCM, and finish exercise 16.4(Problem 4 in Chapter 16.)

Solution.

1. Given $P[G] = P[B] = \frac{1}{2}, P[H|G] = \frac{2}{3}, P[L|B] = \frac{2}{3}$. Since there is a false rejection cascading, we must find $P[G|9R]$, which is same as $P[G|2L]$ because the first two people start the cascading by receiving low signal. Applying the Bayes rule, we have

$$P[G \mid 2L] = \frac{P[G] \cdot P[2L \mid G]}{P[2L]}$$

We know that $P[2L \mid G] = P[L \mid G]^2 = \frac{1}{3}^2 = \frac{1}{9}$, for $P[2L]$, we have to consider two scenarios.

$$P[2L] = P[G] \cdot P[2L|G] + P[B] \cdot P[2L|B] = \frac{1}{2} \cdot \frac{1}{9} + \frac{1}{2} \cdot \frac{4}{9} = \frac{5}{18}$$

Finally, substitute all results back to the equation and we have

$$P[G|2L] = \frac{\frac{1}{2} \cdot \frac{1}{9}}{\frac{5}{18}} = \frac{1}{5}$$

2. Yes, it depends on our signal. Because now there are 4 genuine signal in the signal sequence S.

   - $S = [LLHL]$.

   $$P[G|S] = \frac{P[G] \cdot P[S|G]}{P[S]} = \frac{\frac{1}{2} \cdot (\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{1}{3})}{P[G] \cdot P[S|G] + P[B] \cdot P[S|B]} = \frac{\frac{1}{2} \cdot \frac{2}{81}}{\frac{5}{81}} = \frac{1}{5}$$

   Since $\frac{1}{2} > \frac{1}{5}$, we should choose Reject.

   - $S = [LLHH]$.

   $$P[G|S] = \frac{P[G] \cdot P[S|G]}{P[S]} = \frac{\frac{1}{2} \cdot (\frac{1}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3})}{P[G] \cdot P[S|G] + P[B] \cdot P[S|B]} = \frac{\frac{1}{2} \cdot \frac{4}{81}}{\frac{4}{81}} = \frac{1}{2}$$

   Now there is a tie, so we assume we will choose our observation.

3. There are two cases to be discussed.

   - I choose Reject. In this case, the 11^{th} person cannot tell what is genuine observation of me, thus the cascading continues, and he will only choose Reject.
   - I choose Accept. According to the last question, it only happens when the genuine signal sequence goes as $S = [LLHH]$, thus when 11^{th} person receives LOW, he will choose Reject. When he receives HIGH, he will choose Accept.

Problem 3. Consider a trading network with intermediaries in which there are two sellers S1,S2, three buyers B1, B2, B3, and two traders (intermediaries) T 1, T 2. Each seller can trade with either trader. Buyer B1 can only trade with trader T1. Buyer B2 can trade with either trader. Buyer B3 can only trade with trader T2. The sellers each have one unit of the object and value it at 0; the buyers are not endowed with the object. Buyer B1 values a unit at 1, buyer B2 values a unit at 2 and buyer B3 values a unit at 3.

(a) Draw the trading network, with the traders as squares, the buyers and the seller as circles, and with edges connecting nodes who are able to trade with each other. Label each node as S1, S2, B1, B2, B3, T1 or T2.

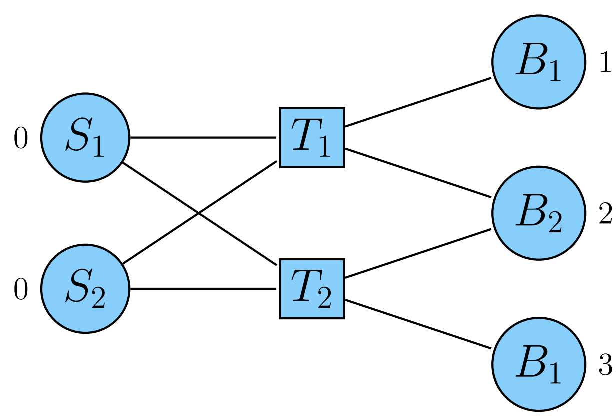
(b) Suppose the prices and the flow of goods are as follows.

- Trader T1 offers a bid price of 1 to each seller, an ask price of 1 to B1, and an ask price of 2 to B2.
- Trader T2 offers a bid price of 1 to each seller, an ask price of 2 to B2, and an ask price of 3 to B3.
- One unit of the good flows from seller S1 to buyer B2 through trader T1 and one unit of the good flows from seller S2 to buyer B3 through trader T2.

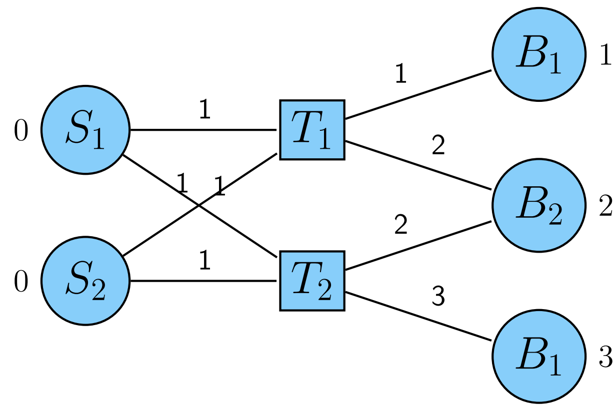
(If it is useful, it is okay to write these prices and this flow of goods on the picture you drew for part (a), provided the picture itself is still clear.) Do these prices and this flow of goods form a Nash equilibrium? If you think the answer is yes, give a brief (1-3 sentence) explanation why. If you think the answer is no, describe a way in which one of the traders could changes its prices so as to increase its profit.

Solution.

1. The trading network is drawn as follows.



2. We could represent the bid and ask price on the edges.



This network is not balanced, because the trader T_2 could raise the bid price for S_1 to 1.1, then S_1 will sell to T_2 and T_2's profit increases from $3-1=2$ to $3+2-1-1.1=2.9$

Problem 4. Let's consider the limiting values that result from the Basic PageRank Update Rule (i.e. the version where we don't introduce a scaling factor s). In Chapter 14, these limiting values are described as capturing "a kind of equilibrium based on direct endorsement: they are values that remain unchanged when everyone divides up their PageRank and passes it forward across their out-going links."

This description gives a way to check whether an assignment of numbers to a set of Web pages forms an equilibrium set of PageRank values: the numbers should add up to 1, and they should remain unchanged when we apply the Basic PageRank Update Rule. For example, this is illustrated in Chapter 14 via Figure 14.6: you can check that if we assign a PageRank of 4/13 to page A, 2/13 to each of B and C, and 1/13 to the five other pages, then these numbers add up to 1 and they remain unchanged when we apply the Basic PageRank Update Rule. Hence they form an equilibrium set of PageRank values.

For each of the following two networks, use this approach to check whether the numbers indicated in the figure form an equilibrium set of PageRank values. (In cases where the numbers do not form an equilibrium set of PageRank values, you do not need to give numbers that do; you simply need to explain why the given numbers do not.)

1. Does the assignment of numbers to the nodes in Figure 14.19 form an equilibrium set of PageRank values for this network of Web pages? Give an explanation for your answer.

2. Does the assignment of numbers to the nodes in Figure 14.20 form an equilibrium set of PageRank values for this network of Web pages? Give an explanation for your answer.

Solution.

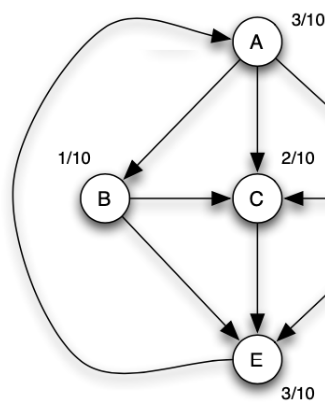


Figure 14.19: A network of Web pages.

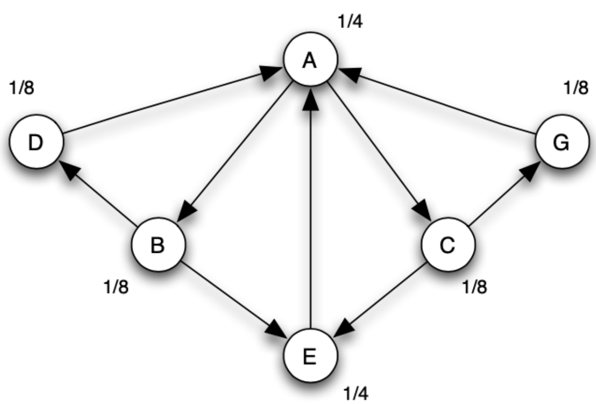


Figure 14.20: A network of Web pages.

1. According to the page rank updating rule, we apply one round update.

$$
\begin{aligned}
A' &= \frac{\frac{3}{10}}{1} = \frac{3}{10} \\
B' &= \frac{\frac{3}{10}}{3} = \frac{1}{10} \\
C' &= \frac{\frac{3}{10}}{3} + \frac{\frac{2}{10}}{2} + \frac{\frac{2}{10}}{2} = \frac{2}{10} \\
D' &= B = \frac{1}{10} \\
E' &= \frac{\frac{2}{10}}{1} + \frac{\frac{1}{10}}{2} + \frac{\frac{1}{10}}{2} = \frac{3}{10}
\end{aligned}
$$

   Since $A + B + C + D + E = 1$ and $A' = A, B' = B, C' = C, D' = D, E' = E$, we conclude the state is at equilibrium.

2. Similar as above, we update node A first.

$$
A' = 2 \times (\frac{1}{8}) + \frac{1}{4} = \frac{1}{2} \neq A
$$

   Here we notice the the vale of node A changes during the rank update, therefore the state is not at equilibrium.

Problem 5. Suppose a search engine has two ad slots that it can sell. Slot a has a clickthrough rate of 4 and slot b has a clickthrough rate of 3. There are three advertisers who are interested in these slots. Advertiser x values clicks at 4 per click, advertiser y values clicks at 3 per click, and advertiser z values clicks at 1 per click.

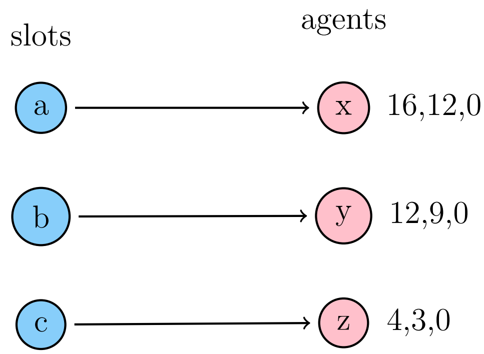
(a) Suppose that the search engine runs the VCG Procedure to allocate slots. What assignment of slots will occur and what prices will the advertisers pay? Give an explanation for your answer.

(b) Now the search engine is considering the creation of a third ad slot which will have a clickthrough rate of 2. Let's call this new ad slot c. Suppose that search engine does create this slot and again uses the VCG Procedure to allocate slots. What assignment of slots will occur and what prices will the advertisers pay? Give an explanation for your answer.

(c) What revenue will the search engine receive from the VCG Procedure in parts (a) and (b)? If you were running the search engine, given this set of advertisers and slots, and could choose whether to create slot c or not, what would you do? Why? (In answering this question assume that you have to use the VCG Procedure to allocate any slots you create.)

Solution.

1. First we can draw the entire trading network of the ads.



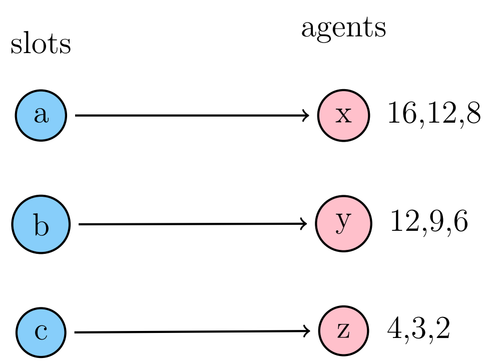
The prices of x can be calculated as $12 - 9 + 3 - 0 = 6$

The prices of y can be calculated as $0 + 3 - 0 = 3$

The prices of z can be calculated as $0 + 0 = 0$

Therefore, slots a goes to agent x, slots b goes to agent y. Agent z has no ad slot.

2. Now we have a new ad net, shown as follows.



The prices of x can be calculated as $12 - 9 + 3 - 2 = 4$

The prices of y can be calculated as $3 - 2 = 1$

The prices of z can be calculated as $0 + 0 = 0$

Therefore, slots a goes to agent x, slots b goes to agent y, slot c goes to agent z.

3. Comparing last two problem, the first case has revenue as 9 and the second is 5. We notice that as slot c is created, the total revenue decreases, therefore, a reasonable choice is to not create ad slot c.

Problem 6. Consider the model from Chapter 19 for the diffusion of a new behavior through a social network. Recall that for this we have a network, a behavior B that everyone starts with, and a threshold q for switching to a new behavior A — that is, any node will switch to A if at least a q fraction of its neighbors have adopted A. Consider the network depicted in Figure 19.29; suppose that each node starts with the behavior B, and each node has a threshold of q = $\frac{2}{5}$ for switching to behavior A. Now, let e and f form a two-node set S of initial adopters of behavior A. If other nodes follow the threshold rule for choosing behaviors, which nodes will eventually switch to A?

1. Find a cluster of density $1 - q = \frac{3}{5}$ in in the part of the graph outside S that blocks behavior A from spreading to all nodes, starting from S, at threshold q.

2. Suppose you're allowed to add one node to the set S of initial adopters, which currently consists of e and f. Can you do this in such a way that the new 3-node set causes a cascade at threshold q = $\frac{2}{5}$?

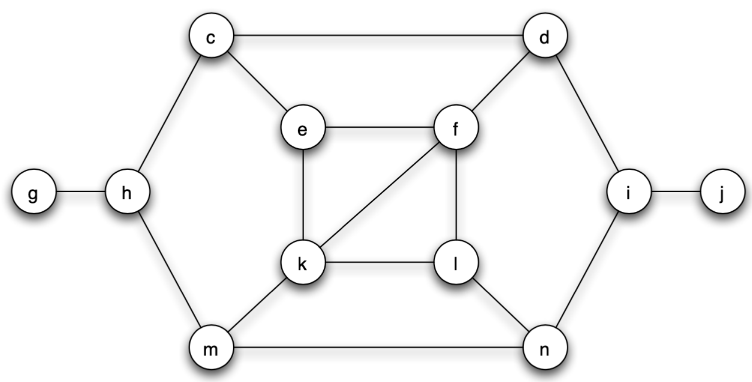


Figure 19.29: A social network in which a new behavior is spreading.

Provide an explanation for your answer, either by giving the name of a third node that can be added, together with an explanation for what will happen, or by explaining why there is no choice for a third node that will work to cause a cascade.

Solution.

1. Given $A = \{e, f\}$ and other nodes belong to B, since $q = \frac{2}{5}$, we first have node k, then node l goes to A, and then it terminates. Thus $S = \{e, f, k, l\}$.

2. We have found that cluster $\{g, h, c, m, d, n, i, q\}$ satisfies the requirement.

3. Yes, node h or node i both satisfies.