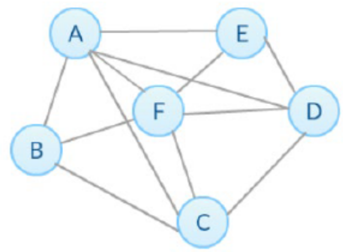
Problem 1. Consider the following network, what is value of node F's local clustering coefficient?



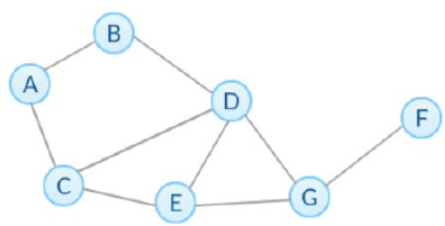
Since there are 5 neighbouring nodes of node F, the maximum number of edges among those neighbours are

$$5 \times 4/2 = 10$$

Since there are 7 edges connected among the neighbors of node F, the local clustering coefficient is

$$7/10 = 0.7$$

Problem 2. Consider the network shown below and either answer the following question



1. What is the diameter of this graph?

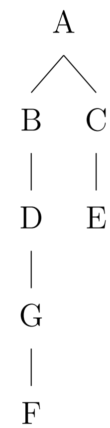
Since diameter is defined as the maximum (shortest path) distance between any pair of nodes in a graph, we then pick pair node A and node F, which has the longest distance of 4 in this graph.

2. The deletion of node G will make the network disconnected. True or False?

True. Because node F will become isolated.

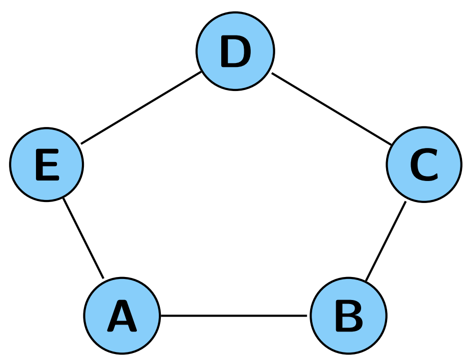
3. Draw the BFS tree from the node A, and determine the depth of this tree.

The BFS tree is drawn below, whose depth is 4 by definition.



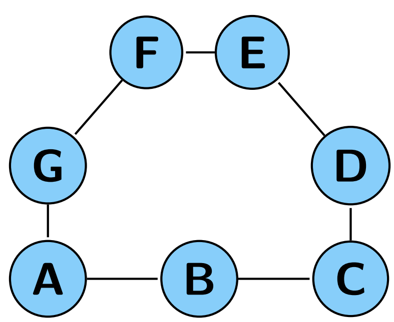
Problem 3 (2.1)**.** Give an example of a graph in which every node is pivotal for at least one pair of nodes. Explain your answer.

Node A is pivotal for the pair (B,E), and vice versa for all other nodes.



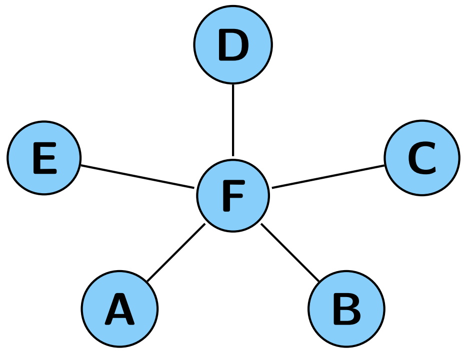
Give an example of a graph in which every node is pivotal for at least two different pairs of nodes. Explain your answer.

Node A is pivotal for the pair (B,G) and (B,F), and vice versa for all other nodes.



Give an example of a graph having at least four nodes in which there is a single node X that is pivotal for every pair of nodes (not counting pairs that include X). Explain your answer.

F is pivotal for the all pairs consisting of two nodes without him.



Problem 4 (2.3)**.** Describe an example of a graph where the diameter is more than three times as large as the average distance.

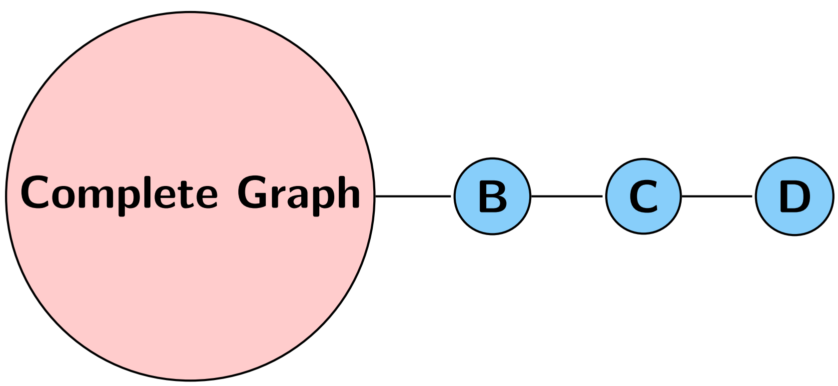
1. Consider a complete graph with n nodes has a span of 3 nodes, the diameter of this graph is 4.

The average distance could be calculated as

$$\frac{\frac{n(n-1)}{2}+1+2+3+4*(n-1)+1+2+3*(n-1)+1+2*(n-1)}{\frac{(n+3)(n+2)}{2}}=\frac{n^2+17n+2}{n^2+5n+6}$$

To satisfy the condition, we let $n \geq 31$ such that

$$\frac{4}{\frac{n^2+17n+2}{n^2+5n+6}}>3$$



2. Suppose we add c extra nodes on a complete graph, the diameter will be c+1.

The average distance will become

$$\frac{\frac{n(n-1)}{2}+\frac{c(c+3)}{2}(n-1)+\frac{c(c+1)(c+2)}{6}}{\frac{(n+c)(n+c-1)}{2}}$$

By changing the value of c, we would have Diameter/Average Distance $> c$ satisfied.

Problem 5 (3.2)**.** Consider the graph in Figure 3.21, in which each edge — except the edge connecting b and c — is labeled as a strong tie (S) or a weak tie (W).

Edge bc must be a weak tie, otherwise edge ec, bf must be connected with some tie.

Problem 6 (5.3)**.**

1. There is no such way. It could at most make two balanced loop, but the last one could not work out.

2. There is no such way, since the edge BD and BC must has one + relation, while this leads to edge AD be + to maintain the balance, however, the other triangle will have two +, which leads to unbalance.

3. No. As is shown above, both two types of unbalanced triangles in a) and b) can not allow extra node added to perform balance. Therefore it is impossible to add one node without being involved in any unbalanced triangles.

Problem 7 (6.5)**.**

(M,M) pair is the nash equilibrium state.

Problem 8 (6.15)**.**

1. According to the setting, we conclude the table as follows.

	A	B	X
A	(-1000,-1000)	(1000,1000)	(1500,0)
B	(1000,1000)	(500,500)	(3000,0)
X	(0,1500)	(0,3000)	(0,0)

2. Yes, consider the utility of (B,any) is always larger than (X,any) where any is the strategy plays by another company.

3. No, (B,B) is not the best response of two companies with each other, when one chooses B, another one could choose A for higher utility.

4. (A,B) and (B,A)

5. Yes. Consider for global optimality. If one company produce B and the other don't enter the market, the overall utiltiy is 3000, where on average each company will have 1500. It is greater than the nash equilibrium where each firm earns 1000.

Problem 9 (9.4)**.**

1. Yes. Because the payoff only depends on bidder b. And bidder a won't pay extra money to win, so he always make the right decision. To prove this, just to consider two cases:

(a) A bids higher than $v_1, b_1 > v_1$

i. Bidder a wins, let bidder b's bid be $b2 \geq v_1$, then payoff of a decrease to $v_1 - b_2$.

ii. Bidder b wins, payoff of a remains 0

(b) A bids lower than $v_1, b_1 < v_1$, a has less chance to win, so payoff decreases to 0.

Hence proved the dominant strategy is still true.

2. Seller's revenue could be calculated the possibility of both a and b bid 1, which is

$$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$$

Problem 10 (9.9)**.**

1. Since only when both bidder bid for 2, the seller will have 2 under a second-price auction, otherwise he will receive 1 for 3 cases: (1,2), (2,1), (1,1). And the possibility of these four conditions are equal, therefore we can calculate the expected revenue as

$$\frac{1+1+1+2}{4} = \frac{5}{4}$$

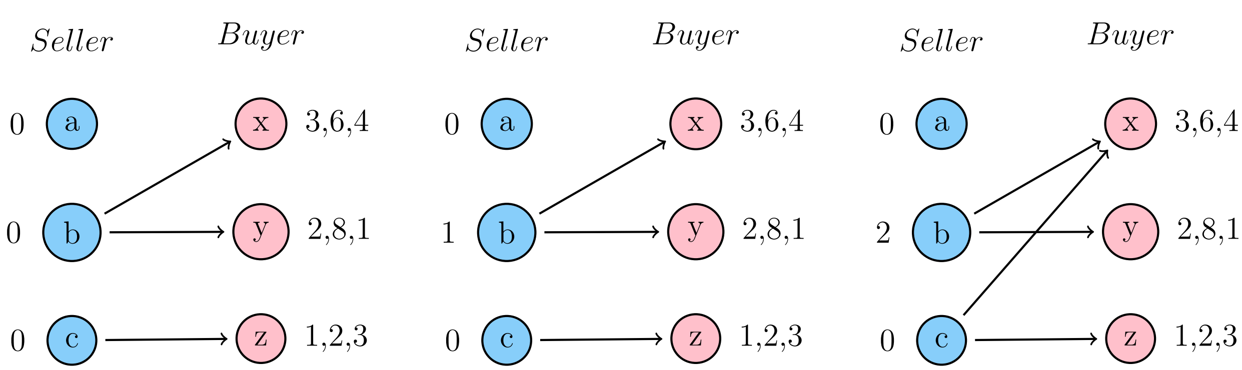
2. Similar as the last one, since when both bidder bid 1, the seller receive 0, and when (1,2) and (2,1), the seller receive R, for (2,2), the seller receive 2. Since these four conditions are both equally likely, we can calculate the expected revenue as

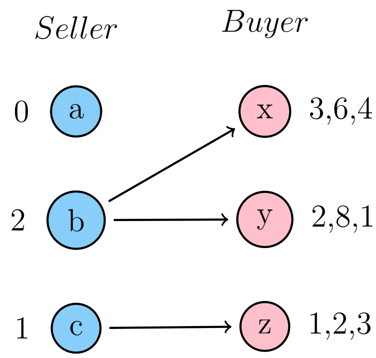
$$\frac{0+R+R+2}{4} = \frac{1+R}{2}$$

3. Conclude on what we have from last two questions. When $\frac{1+R}{2} \geq \frac{5}{4}$, we need $R \geq 1.5$ to have a higher expected revenue.

Problem 11 (10.9)**.**

We could draw the entire auction processes as follows.





Now we have a perfect match, the auction is over. The final market cleaning price is then $(0, 3, 1)$ with pairs (a,x), (b,y), (c,z).