VE444: Networks

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Review

- Nash Equilibrium Existence Theorem
- If game is zero-sum, we can compute a NE in poly time.
- User optimal v.s. Social optimal

Mechanism Design Basics

Motivating example

- 2012 London Olympics
- Video: 08:00

Motivating example

- 2012 London Olympics
- Phase 1: Round-robin
- 4 teams of 4
- Top 2 teams from each group advance
- Phase 2: Knockout

Motivating example

- Trigger: in group D, Danish PJ upset Chinese team QW
- Next match: Chinese team XY meets Korean team KH to decide who is 1st and 2nd in group A
- Issue: Group A winner would face QW in semis, 2nd-best would only face QW in the final.

- Misalignment between participant and designer's goal.

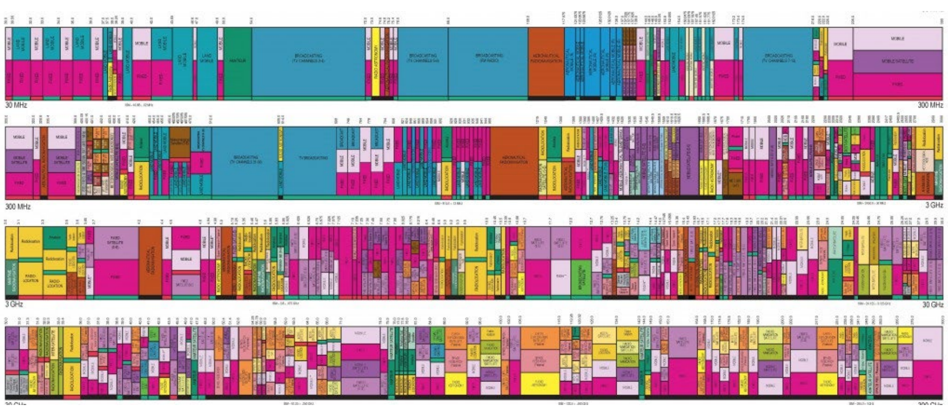
Mechanism design in Practice

- Spectrum auctions since 1994
  - FCC auctions
  - Worldwide innovations in auction design
- Other innovative auctions
  - Electricity
  - Carbon emissions
  - Search auctions
  - Computing resources

FCC completes 3.5 GHz spectrum auction raising $4.5 bn

August 26, 2020

The Federal Communications Commission (FCC) has completed the 3.5 GHz spectrum auction (Auction 105) – raising $4.585 billion.



Mechanism design in Practice

- Kidney exchange
- Resident doctor matching
- Voting

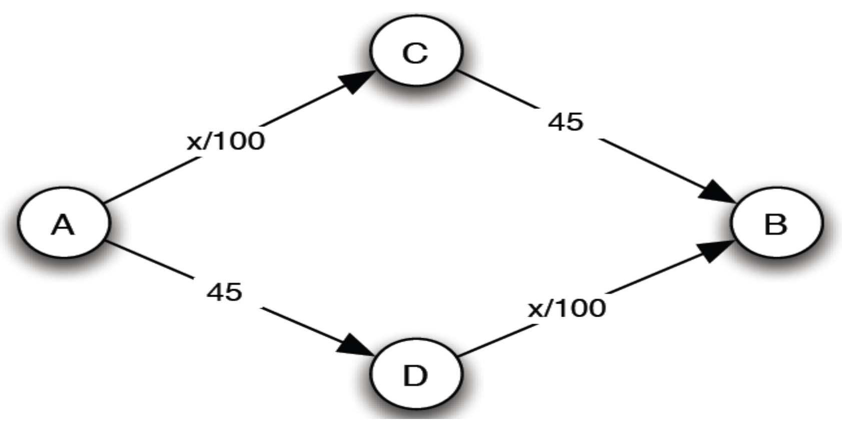
- But first, when is selfish behavior benign?

Traffic as a Game

A game on network structure

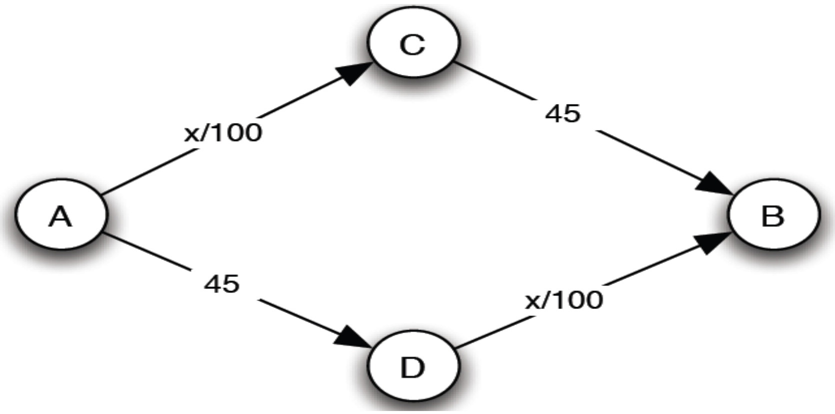
- 4000 vehicles want to travel from A to B

- Players: 4000 drivers
- Strategy set: upper path, lower path
- Payoff: travel time (the less the better, but also depends on other’s choices)
- Equilibrium? Payoff matrix?



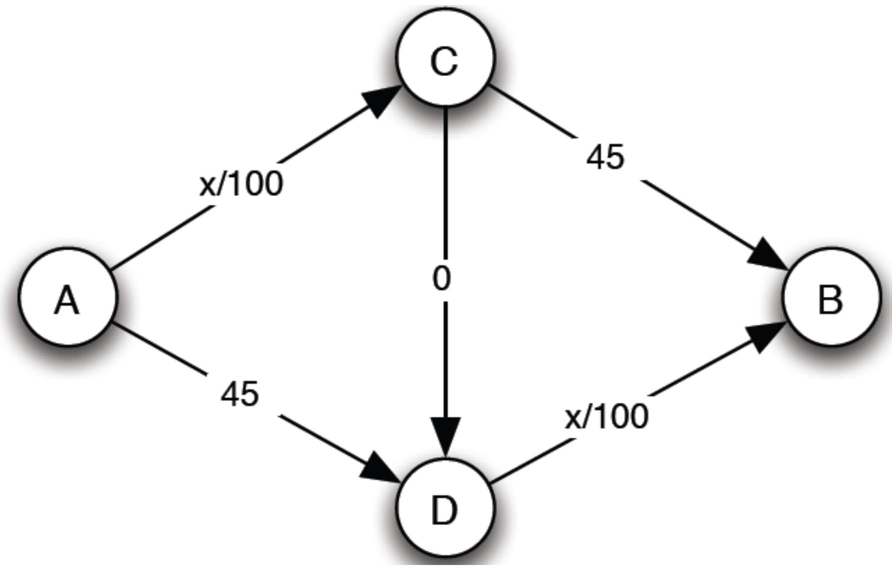
A game on network structure

- Equilibrium: 2000 A-C-B, 2000 A-D-B
- Payoff for each driver: 65
- If anyone deviates, his payoff will be: 2001/100 + 45 > 65



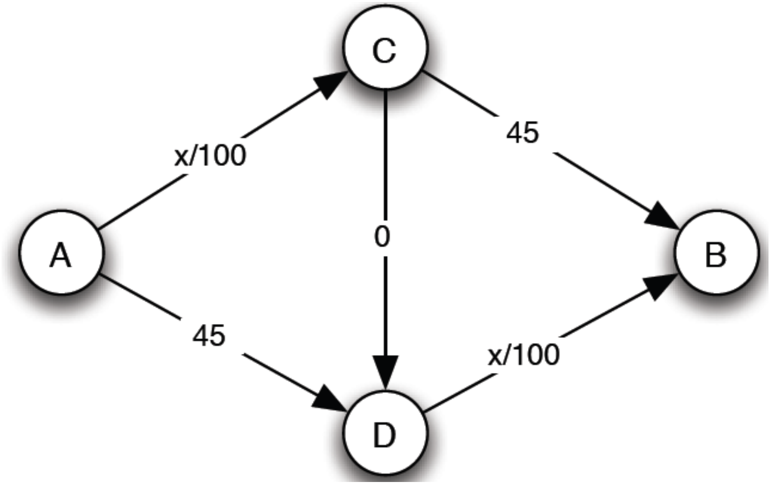
If the government builds a new road

- Assume the government want to do something good: let’s build a new road and it is an express road, e.g., travel time on this road is negligible!
- What will happen?



If the government builds a new road

- Is the previous possible?
  - 2000 A-C-B
  - 2000 A-D-B
- No way, no longer equilibrium
- If you are the one on A-C-B
  - A-C-D-B will be faster, there is incentive to change
- This is called Braess's Paradox



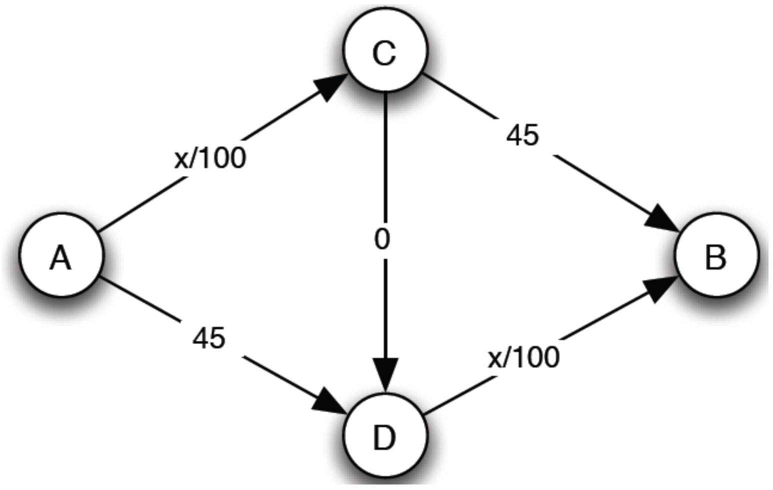
If the government builds a new road

- Equilibrium: every one uses A-C-D-B
  - Travel time is

  4000/100+0+4000/100=80
  - If someone tries to deviate,

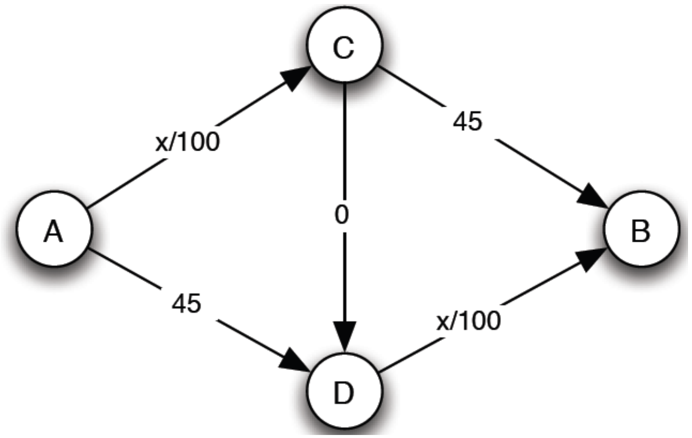
  Travel time is: 45+4000/100=85>80

  It is worse than 65!



Comparing with the social optimal

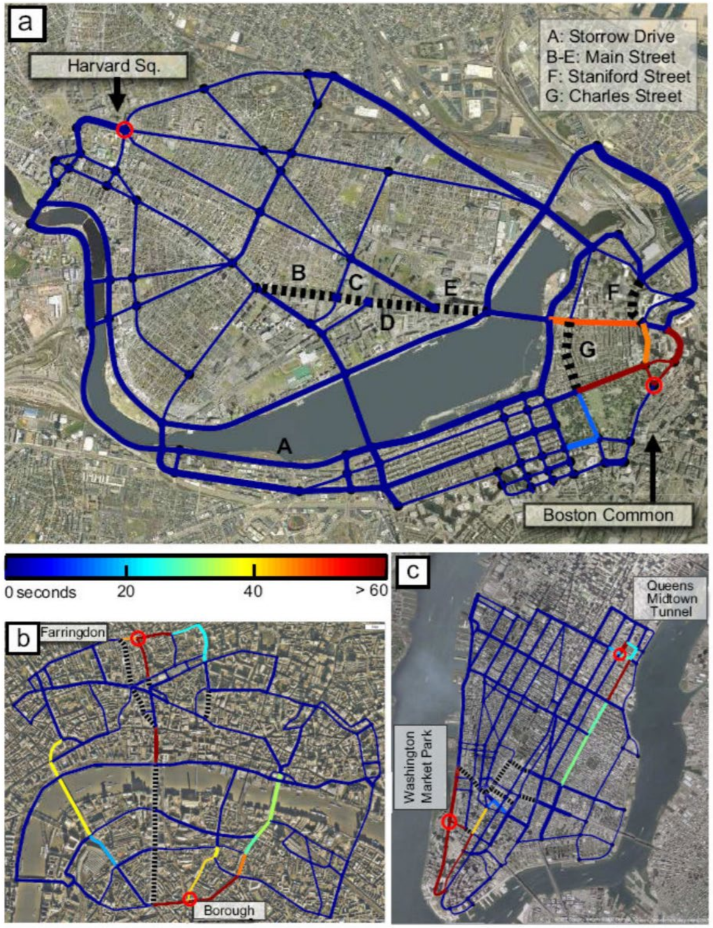
- The outcome of rational behaviors can be inferior to a centrally designed outcome.
- Question is: by how much?
- Price of Anarchy(POA): the ratio between the system performance with strategic players and the best-possible system performance



Real-world practice

- Stuttgart, Germany (1969): The traffic worsened until a newly built road was closed
- New York City (1990): On Earth Day 42nd street was closed and traffic flow improved.
- Seoul, South Korea (2002), replaced a six lane highway with a five mile long park, traffic flow improved.

Real-world practice



Price of Anarchy in Transportation Networks: Efficiency and Optimality Control, Physical Review Letters, 2008

A summary

- A simple example shows a game on network structure
- We see Braess's Paradox
- Invest more resources may not get a good result

- Identify application domains and conditions under which the POA is guaranteed to be close to 1
  - Selfish behavior leads to a near-optimal outcome

Mechanism Design with Money: Auction

An example

- In game and Braess's Paradox, we see the interaction of rational behavior, equilibrium, and network structure
- Can we change the condition and let the players to directly interact (of course, we also need to set up a set of rules for their interaction)?
- Auction can be one of such scenario

Auction is everywhere

- Auctions by Christie and Sotheby's
- Government auctions the land, license, etc
- Electricity
- Carbon emissions
- Search auctions
- Computing resources

The format of auctions

- The equilibrium depends on the format and regulations of the auctions
- The format also influence the strategy choices of the buyers and sellers

Types of Auction

- (Forward) Auction: a seller auctioning one item to a set of buyers
- Procurement auctions: a buyer trying to purchase a single time among a set of sellers.

Types of Auction

- Ascending-bid auctions/English auctions: the seller gradually raises the price, bidders drop out until finally only one bidder remains;
  - This is useful for auctions of art works, antiques, etc.

- Descending-bid auctions/Dutch auctions: the seller gradually lowers the price from some high initial value until the first moment when some bidder accepts and pays the current price;
  - This is useful for auctions of flowers, fresh farm products.

Types of Auction

- First-price sealed-bid auctions: bidders submit simultaneous “sealed bids" to the seller, who would then open them all together. The highest bidder wins the object and pays the value of her bid.
  - This is used in call for bid auctions
- Second-price sealed-bid auctions/Vickrey auctions: Bidders submit simultaneous sealed bids to the sellers; the highest bidder wins the object and pays the value of the second-highest bid
  - This is used in advertisement auction in Internet websites
  - In honor of William Vickrey, first game-theoretic analysis of auctions, Nobel Memorial Prize in Economics in 1996.

When are Auctions Appropriate?

- Known value
  - Seller valuation: x, buyer valuation: y
- Surplus: y-x
- Commit to the mechanism
- Unknown value
  - Independent, private values
  - Common value

Relationship between different formats

- Descending-bid
- Ascending-bid
- First-price
- Second-price

- Descending-bid is analogous to sealed-bid first price auction
- Ascending-bid is analogous to sealed-bid second price auction

A few points in auction formats

- Who get the object
  - Usually the highest or lowest bidder
- What kind of price to pay
  - First price or second price, this influences the strategy of the bidders
- Do the bidders know the price of others
  - Sealed auction, needs to guess about others
  - Unsealed auction, can see other’s bids

Auction: Game perspective

- First-price sealed-bid auctions (FPA)
  - Highest bidder wins and pays the value of her bid
- Second-price sealed-bid auctions (SPA)
  - Highest bidder wins and pays the value of the second-highest bid.
- Formulating as a game
  - Players: bidders
  - Strategy: bid
  - Payoff: true value – payment, or 0 (if auction fails)
- A game-oriented thinking
  - Equilibrium! Best response to each other, no one changes

Second-price sealed-bid auctions

- An object
  - Different people may have different values for it, $v1$, $v2$, …, vk. These are true value/intrinsic value, i.e., player *i* will pay at most *vi*.
  - Players don't know other's true values
  - Every one has a bid, assume $b1 > b2 > \ldots > bk$
  - By the rule of second-price auction, the payoff of the highest bidder is $v1 – b2$ and others are 0; here, $b1$ is the highest bid, $v1$ is the true value

How do you play the game, i.e., how do you bid?

Second-price sealed-bid auctions

- An object
  - Different people may have different values for it, v1, v2, …, vk. These are true value/intrinsic value, i.e., player *i* will pay at most *vi*.
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  - Every one has a bid, assume b1 > b2 > … > bk
  - By the rule of second-price auction, the payoff of the highest bidder is v1 – b2 and others are 0; here, b1 is the highest bid, so v1 is the true value

We assume that people maximize their own profits, i.e., get the object and pay the lowest price possible

What strategy is optimal?

- From game point of view (dominant strategy): cannot get better payoff by changing to other strategies, regardless of the other players' strategy

- Claim: In a sealed-bid second-price auction, it is a dominant strategy for each bidder i to choose a bid bi = vi.

Proof

- Assume in an auction, you consider the object worth \$100 and you bid \$100, now consider whether you can get better payoff using other strategy
- There are two cases:
  - You win the bid: now you have positive payoff (you only need to pay the second high bid, $v1 – b2 > 0$)
    - Increase the bid won't change anything
    - Decrease the bid won't change anything unless less than the second high, then you lose the bid and payoff become 0
  - You lose the bid: now your payoff is 0
    - Decrease the bid won't change anything
    - Increase the bid won't change anything unless becoming the highest; note that this means that this is greater than your value, so payoff becomes negative

First-Price Auction

- In second-price sealed-bid auction, bid the true value is a (weakly) dominant strategy
- First-price seal-bid auction does not have such property
  - Truthful bidding zero payoff
  - Shading requests complex trade-off reasoning

Bidding Strategies in First-Price Auction

- What is the optimal strategy in the equilibrium point?

- Simplified case: two bidders, private value independently and uniformly distributed and uniformly distributed between 0 and 1;
- Strategy: A strategy for a bidder is a function s(v) = b that maps her true value v to a non-negative bid b.
- Assumption:
  - s(·) is a strictly increasing, differentiable function;
  - s(v) ≤ v for all v
- Possible bidding strategy:
  - s(v)=v;
  - s(v)=cv (c<1);
  - s(v)=cv^2, etc.
- The assumption helps us narrow down the strategy set.
  - Rules out non-optimal

Bidding Strategies in First-Price Auction : 2-bidder case

- Equilibrium strategy: for each bidder i, there is no incentive for i to deviate from strategy s(·) if i's competitor is also using strategy s(·).

- What is buyer's best bidding strategy?

- If bidder i has a value of v_i, the probability that this is higher than the value of i's competitor in the interval [0, 1] is exactly vi
- i's expected payoff: g(v_i) = v_i (v_i − s(v_i)) under current strategy function s(·).

- Deviations in the bidding strategy function can instead be viewed as deviations in the "true value" that bidder i supplies to her current strategy function s(·). -> Revelation Principle
- Condition that I do not want to deviate from strategy s(·):
  - $v_i(v_i$ − s(v_i)) ≥ v(v_i− s(v)) for any v in [0,1]
- What kind of function satisfies this condition?

Bidding Strategies in First-Price Auction: n-bidder case

- Equilibrium with many bidders: n bidders, uniform distribution valuation
- For a given bidder i with true value v_i, what is the probability that her bid is the highest?
- Expected payoff for bidder i: $G(v_i) = v_i^{n-1}(v_i - \mathrm{s}(v_i))$ under current bidding strategy
- Using Revelation Principle, a deviation from the bidding strategy is equivalent to supplying a "fake" value v to the function s(·)
  - $v_i^{n-1}(v_i - \mathrm{s}(v_i)) \geq v^{n-1}(v_i - \mathrm{s}(v))$ for all v between 0 and 1
- Expected payoff function: $G(v) = v^{n-1}(v_i - \mathrm{s}(v))$
  - $s'(v_i) = (n-1)(1 - \frac{s(v_i)}{v_i})$
  - Therefore, $\mathrm{s}(v_i) = \left(\frac{n-1}{n}\right) v_i$
- Remarks: As the number of bidders increases, you generally have to bid more aggressively

Seller Revenue Comparision

- How to compare the revenue a seller should expect to make in first-price and second-price auctions?
- Assumption: n bidders with values drawn independently from the uniform distribution on the interval [0, 1].
- Background Math Info: Suppose n numbers are drawn independently from the uniform distribution on the interval [0, 1] and then sorted from smallest to largest. The expected value of the number in the kth position on this list is $\frac{k}{n+1}$.
- Revenue equivalence theorem: given certain conditions, any mechanism that results in the same outcomes (i.e. allocates items to the same bidders) also has the same expected revenue.
  - We can prove that first price and second price auction all generates same expected revenue.