

VE444: Networks

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Game Theory Basics

Interconnectedness at two levels

- **Structural level:**

- What are the properties of a graph
- How graph evolve over time

- **Behaviour level:**

- The outcome for any one in a connected system depends on the combined behaviours of all

Game theory

- Modern game theory started by the works of von Neumann in 1928

Applications of Game Theory

- Theory developed mainly by mathematicians and economists
 - contributions from biologists
- Widely applied in many disciplines
 - from economics to philosophy, including computer science (Systems, Theory and AI)
 - goal is often to understand some phenomena
- “Recently” applied to computer networks
 - Nagle, RFC 970, 1985
 - “datagram networks as a multi-player game”
 - paper in first volume of IEEE/ACM ToN (1993)
 - wider interest starting around 2000

Starting from an example

- Presentation or exam
 - Assume you have two things to do before a deadline tomorrow: study for exam or prepare presentation, and you can do only one
 - Study for exam is predictable, if you study, your grade will be 92, no study, 80
 - Presentation depends on both you and your partner
 - If both prepare presentation, each of you will get 100
 - If only one prepare presentation, each of you will get 92
 - If none of you prepare presentation, each of you will get 84
 - What do you do? (we assume that you and your partner make decisions independently)

Game between Presentation and Exam

- Assume that both of you try to maximize your grades
 - You and your partner all prepare presentation → average grade is $(80 + 100) / 2 = 90$
 - You and your partner all study the exam → average grade is $(92 + 84) / 2 = 88$
 - One studies exam and one prepares presentation →
 - The one prepares presentation $(80 + 92) / 2 = 86$
 - The one studies exam $(92 + 92) / 2 = 92$
- Question: what would you do?

Basic ingredients of a Game

		Your Partner	
		<i>Presentation</i>	<i>Exam</i>
You	<i>Presentation</i>	90, 90	86, 92
	<i>Exam</i>	92, 86	88, 88

- Player: a set of participants
- Strategies: a set of actions each player could choose
- Utility: assigned values to preferences over *goods*
- Payoff: the utility of an outcome
- U_{row}, U_{column}

Underlying assumption

- Utility:
 - Given choices, can tell which one is better
 - Consistent, i.e., $A > B$, $B > C$, and he must say, $A > C$
- Rationality: Utility maximization
- Complete information
 - Know the underlying structure: who, strategy set, payoff
 - John Harsanyi, games with incomplete information, 1994 Nobel Prize in Economics
- Independent
 - Make decision independently, no coalition, etc

Reasoning about behavior

		Your Partner	
		<i>Presentation</i>	<i>Exam</i>
You	<i>Presentation</i>	90, 90	86, 92
	<i>Exam</i>	92, 86	88, 88

- Best response: $U_1(S, T) \geq U_1(S', T)$
- Strictly dominant strategy: a strategy is strictly better than all other options, regardless of what the other player does
- Equilibrium point: a set of strategies we expect players to adopt

Why not presentation?

		Your Partner	
		<i>Presentation</i>	<i>Exam</i>
You	<i>Presentation</i>	90, 90	86, 92
	<i>Exam</i>	92, 86	88, 88

- It is natural if you ask a question, why not (presentation, presentation)
- It cannot stable there, the two players have the incentive to change if it is at (presentation, presentation)
- Our reasoning is rigid, otherwise it violates our assumptions (players are rational)

Prisoner's Dilemma

- Suppose that two suspects have been apprehended by the police and are being interrogated in separate rooms
- The police strongly suspect that these two individuals are responsible for a robbery, but there is not enough evidence to convict either of them of the robbery.
- Each of the suspects is told
 - If you confess, and your partner doesn't, then you will be released and your partner will be charged and sent to prison for 10 years.
 - If you both confess, then you will both be convicted and sent to prison for 4 years
 - Finally, if neither of you confesses, then you will be charged for 1 year of resistance.
- Do you want to confess or not?

Prisoner's Dilemma

		Suspect 2	
		<i>NC</i>	<i>C</i>
Suspect 1	<i>NC</i>	-1, -1	-10, 0
	<i>C</i>	0, -10	-4, -4

- The strictly dominant strategy for both suspect 1 and 2 is confess
- Even though if two NC, they will be sentenced less

Best response vs Dominant strategy

		Suspect 2	
		<i>NC</i>	<i>C</i>
Suspect 1	<i>NC</i>	-1, -1	-10, 0
	<i>C</i>	0, -10	-4, -4

- **Best response** targets at a single strategy of the opponent (T), and is among all strategies of himself
 - For different T , there may be different best response
- **Dominant strategy** targets at all strategies of the opponent, best response targets at one strategy of the opponent
 - If one has strict dominant strategy, we can assume that he will take it (follow our basic assumptions on game)

Optimization v.s. Game Theory

- Optimization v.s. game theory:
 - Optimization: Suppose you are looking for a particular store in an unfamiliar mall.
 - Game: Suppose you are looking for your lost friend in a mall. Should you stay in a central location so that your friend can find you?

Performance enhancing drugs

		Athlete 2
Athlete 1	<i>Don't Use Drugs</i>	<i>Don't Use Drugs</i>
	<i>Use Drugs</i>	<i>Use Drugs</i>

	<i>Don't Use Drugs</i>	<i>Use Drugs</i>
	3, 3	1, 4
	4, 1	2, 2

Performance enhancing drugs

		Athlete 2	
		<i>Don't Use Drugs</i>	<i>Use Drugs</i>
Athlete 1	<i>Don't Use Drugs</i>	3, 3	1, 4
	<i>Use Drugs</i>	4, 1	2, 2

Also called *arms races game*: no good or even harmful for each one internally, but to make sure that each is competitive, have to stay in the competition.

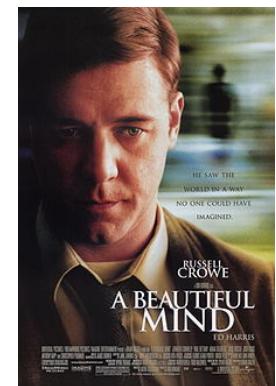
Summary

- If both have strictly dominant strategy, they will both adopt their respective strictly dominant strategy
- If one has strictly dominant strategy, he will choose this one and the opponent will choose the best response for this strictly dominant strategy (there must be one!)
- What if neither has strictly dominant strategy? Where the reasoning starts?

Nash Equilibrium

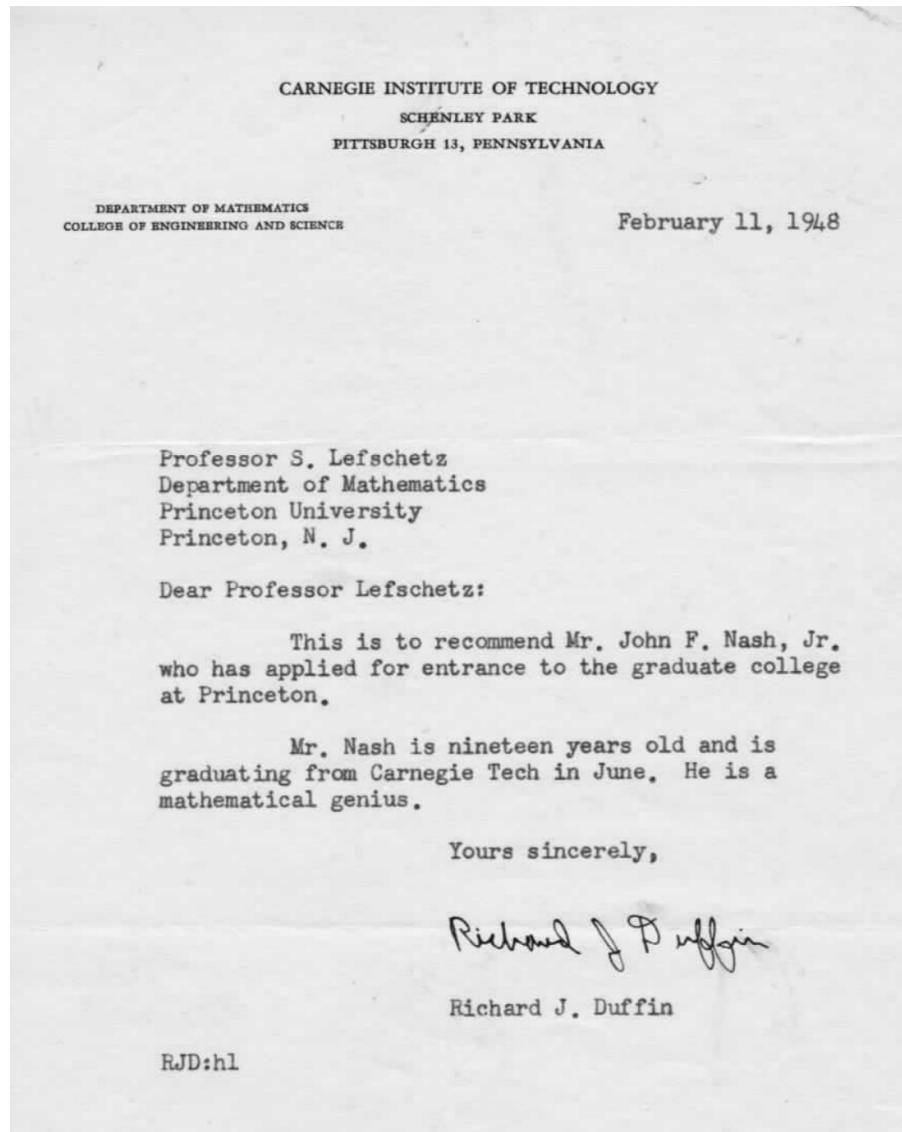
John Nash

- A legendary life of John Nash
 - Born June 13, 1928
 - B.S and M.S in mathematics at 1948
 - Ph.D, 1950, 28 pages dissertation



https://web.archive.org/web/20170607041209if_/https://webspace.princeton.edu/users/mudd/Digitization/AC105/AC105_Nash_John_F Forbes_1950.pdf

John Nash



An example with no dominant strategy

- A three client game
 - Two firms that each hope to do business with one of three clients, A, B, C. Each firm has three possible strategies: whether to approach A, B, or C. The results of their two decisions will work out as follows

An example with no dominant strategy

- If the two firms approach the same client, then the client will give half its business to each.
- Firm 1 is too small to attract business on its own, so if it approaches one client while Firm 2 approaches a different one, then Firm 1 gets a payoff of 0.
- If Firm 2 approaches client B or C on its own, it will get their full business. However, A is a larger client, and will only do business with the firms if both approach A.
- Because A is a larger client, doing business with it is worth 8 (and hence 4 to each firm if it's split), while doing business with B or C is worth 2 (and hence 1 to each firm if it's split).

Reasoning

- The payoff matrix

		Firm 2		
		A	B	C
Firm 1		A	4, 4	0, 2
		B	0, 0	1, 1
		C	0, 0	0, 2
				1, 1

- There is no strictly dominant strategy for any firm
- How should we expect the outcome?

Nash Equilibrium

- Assume player 1 selects strategy S, and player 2 selects strategy T. If S is the best response of T, and T is the best response of S, then we say that strategy group (S, T) is a Nash Equilibrium
 - In Nash Equilibrium, no one has the incentive to change their strategy
- Nash Equilibrium: (**best responses to each other**) No one can become better by unilaterally change his own strategy; though both can become better if both changes.

Reasoning

- The payoff matrix

		Firm 2		
		A	B	C
Firm 1		A	4, 4	0, 2
		B	0, 0	1, 1
		C	0, 0	0, 2
			1, 1	

- Which outcome is the Nash Equilibrium point?
- Find Nash Equilibrium
 - Check every strategy-pair, and see if it is the best response for each other
 - Find best response(s), and find the mutual best responses

Reasoning

- The payoff matrix

		Firm 2		
		A	B	C
Firm 1		A	4, 4	0, 2
		B	0, 0	1, 1
		C	0, 0	0, 2
			1, 1	

- Which outcome is the Nash Equilibrium point?
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Coordination game

- Pick a square

	1	2	3	4	5	6
A	green	blue	green	blue	blue	red
B	purple	purple	blue	green	purple	blue
C	blue	green	purple	blue	blue	green
D	blue	blue	blue	purple	purple	green
E	blue	green	purple	blue	purple	blue
F	purple	blue	blue	blue	green	green

- If we both select the same square, we win \$20

Coordination game

- Pick any day of the year
- If we both select the same day, we win \$20
- Your answer?

Coordination game

- Player's shared goal is to coordinate on the same strategy
- Example
 - You and your partner are each preparing slides for a joint project presentation (assume you can't reach your partner by phone, do things independently)
 - You have to decide whether to prepare your half of the slides in PowerPoint or in Apple's Keynote software.
 - Either would be fine, but it will be much easier to merge your slides with your partner's if you use the same software

Multiple Equilibria: Coordination game

		Your Partner	
		<i>PowerPoint</i>	<i>Keynote</i>
You	<i>PowerPoint</i>	1, 1	0, 0
	<i>Keynote</i>	0, 0	1, 1

- There are two Nash Equilibria (PPT, PPT), (Keynote, Keynote)
- What do players do?
- Schelling focal point model, Social conventions, etc
- Balanced Coordination v.s. unbalanced coordination

Battle of sexes

		Your Partner	
		<i>PowerPoint</i>	<i>Keynote</i>
You	<i>PowerPoint</i>	1, 2	0, 0
	<i>Keynote</i>	0, 0	2, 1

- Assume you and your partner like different software
- It is difficult to predict what will happen purely based on the structure of game
- Additional information may be needed

Mixed strategy

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$