VE444: Networks

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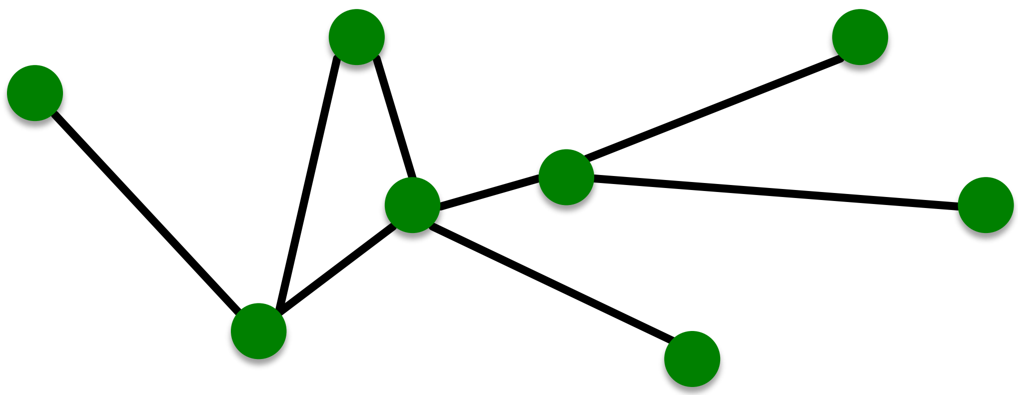
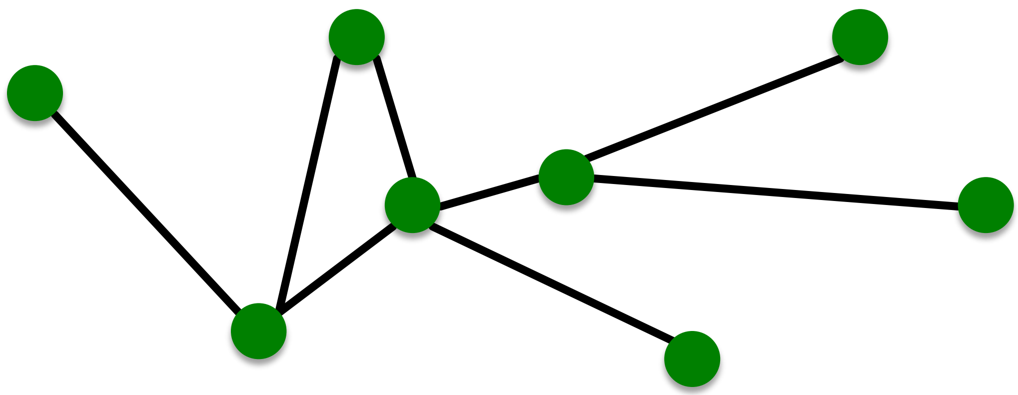
Components of a Network



- **Objects:** nodes, vertices N
- **Interactions:** links, edges E
- **System:** network, graph $G(N,E)$

Components of a Network



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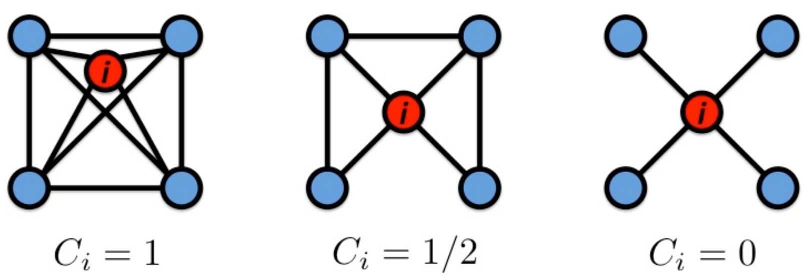
(3) Clustering Coefficient

- **Clustering coefficient (for undirected graphs):**
  - How connected are ***i***'s neighbors to each other?
  - Node $\boldsymbol{i}$ with degree $\boldsymbol{k_i}$
  - $C_i \in [0,1]$
  - $C_i = \dfrac{2e_i}{k_i(k_i-1)}$ where e_i is the number of edges between the neighbors of node i

Note $k_i(k_i-1)/2$ is max number of edges between the k_i neighbors



Clustering coefficient is undefined (or defined to be 0) for nodes with degree 0 or 1

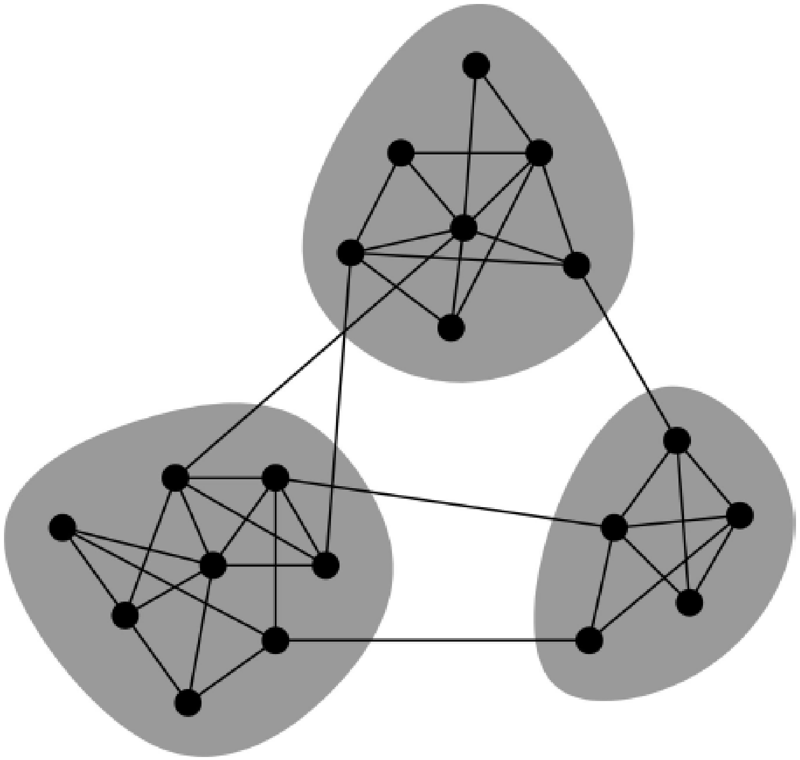
- **Average clustering coefficient:** $C = \dfrac{1}{N}\sum_{i}^{N} C_i$

Strong and weak ties

- If we can categorize all links in the social networks as **strong ties** and **weak ties**
- Relating to the triadic closure, we make the following assumption:
  - If a node A has edges to nodes B and C, then the B-C edge is especially likely to form if A's edges to B and C are both strong ties.
  - More formally, as Granovetter suggested:
- A node A violates the **Strong Triadic Closure property** if it has strong ties to two other nodes B and C, and there is no edge at all(either a strong and weak tie) between B and C.

Communities

- We often think of networks "looking" like this:



- What led to such a conceptual picture?

Motivating example

- **Network could facilitate the flow of information**
- **How do people find out about new jobs?**
  - Mark Granovetter, part of his PhD in 1960s
  - People find the information through personal contacts
- **But:** Contacts were often **acquaintances** rather than close friends
  - **This is surprising:** One would expect your friends to help you out more than casual acquaintances
- **Why is it that acquaintances are most helpful?**

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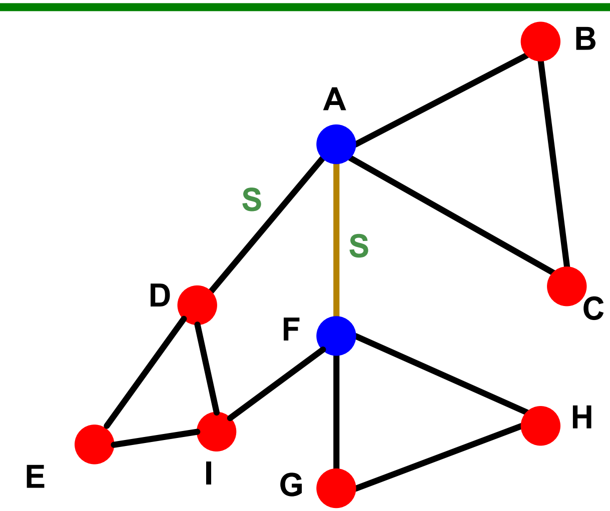
Structural level meets interpersonal level

- **Bridge**: global structural notation
- **Weak/Strong**: local interpersonal distinction
- How they link to each other? Granovetter's Theorem

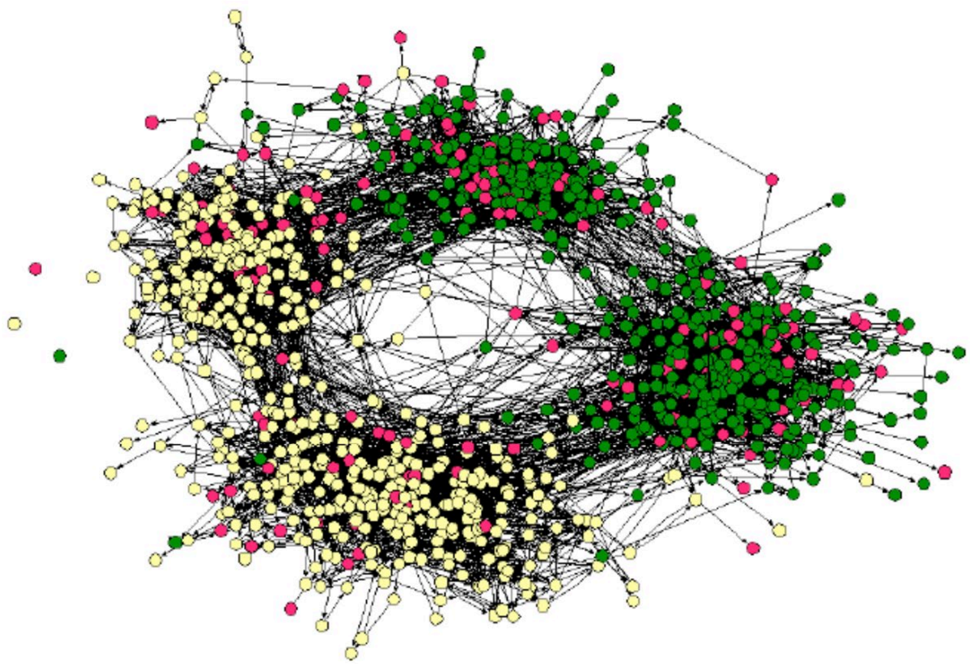
> if a node A in a network satisfies the STC property and is involved in at least two strong ties, then any local bridge it is involved in must be a weak tie



Correlations Exists in Networks

Example:

- Real social network
  - Nodes = people
  - Edges = friendship
  - Node color = race
- People are segregated by race due to homophily



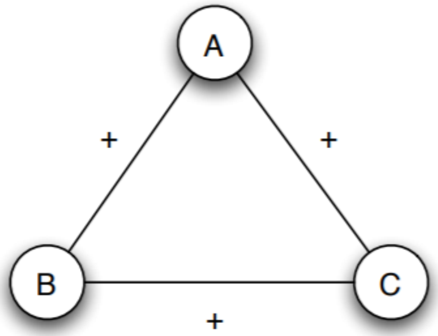
(Easley and Kleinberg, 2010)

Application of homophily

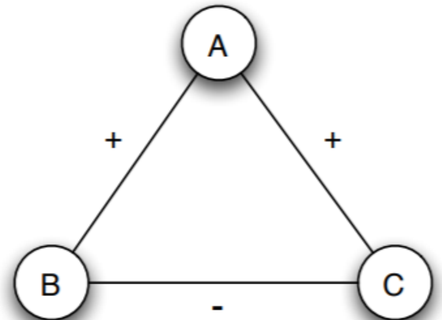
- How do we leverage this correlation observed in networks to help predict node labels?
- Similar nodes are typically close together or directly connected:
  - "**Guilt-by-association**": If I am connected to a node with label X, then I am likely to have label X as well.
  - Example: Malicious/benign web page: Malicious web pages link to one another to increase visibility, look credible, and rank higher in search engines

Structural Balance

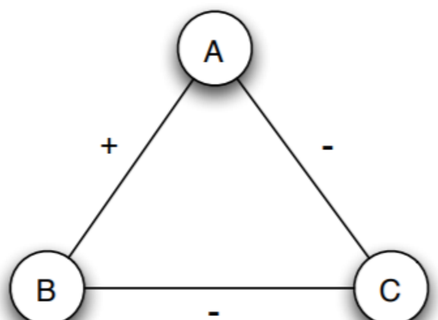
Starting from the local



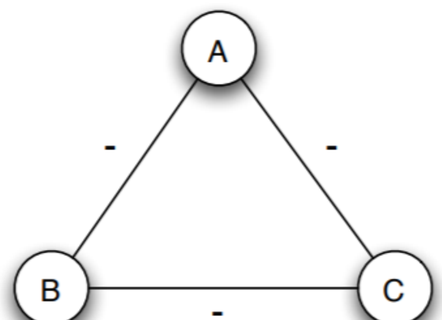
(a) *A, B, and C are mutual friends: balanced.*



(b) *A is friends with B and C, but they don't get along with each other: not balanced.*



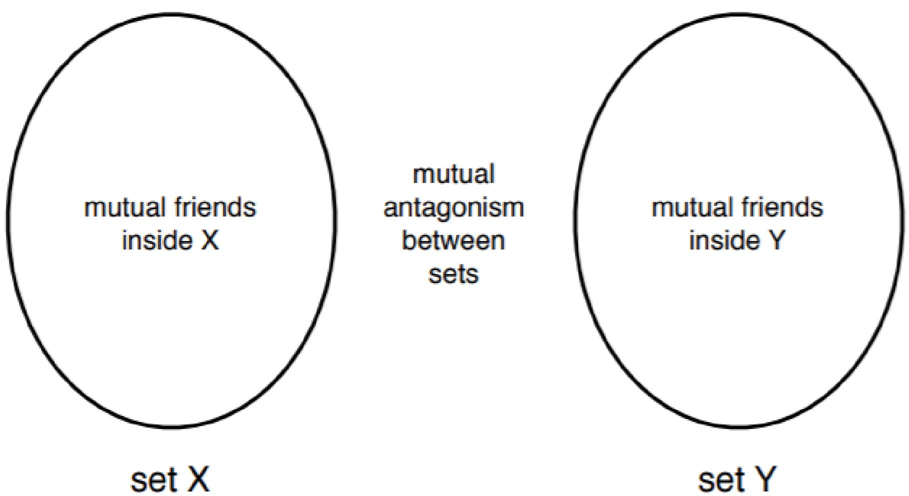
(c) *A and B are friends with C as a mutual enemy: balanced.*



(d) *A, B, and C are mutual enemies: not balanced.*

Structure of a balanced networks

- Balance theorem: If a labeled complete graph is balanced, then either all pairs of nodes are friends, or else the nodes can be divided into two groups, X and Y , such that every pair of nodes in X like each other, every pair of nodes in Y like each other, and everyone in X is the enemy of everyone in Y .



Review

- Basic ingredients
- Equilibrium
- Dominant strategy
- Nash Equilibrium
- Multiple Equilibrium
- Mixed-Strategy Nash Equilibrium
- Nash Equilibrium Existence Theorem
- Pareto Optimal and Social optimal

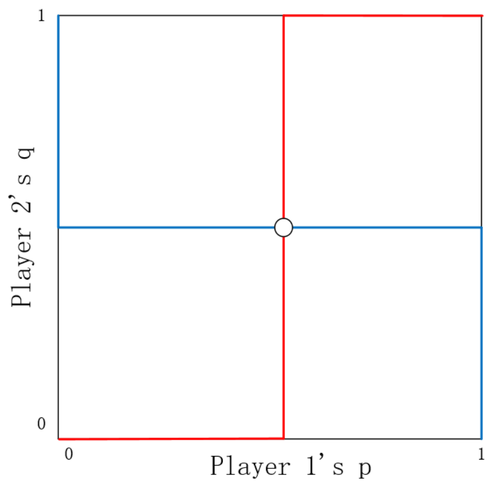
Coming back to the Head-Tail Game

- Best response curve of two players

		Player 2	
		H	T
Player 1	H	$-1, +1$	$+1, -1$
	T	$+1, -1$	$-1, +1$

	H	T	q-mix
H	-1,+1	+1, -1	-q+1(1-q)
T	+1, -1	-1,+1	q – (1-q)

Indifference point is (p=0.5, q=0.5)



Mixed strategy game: summary

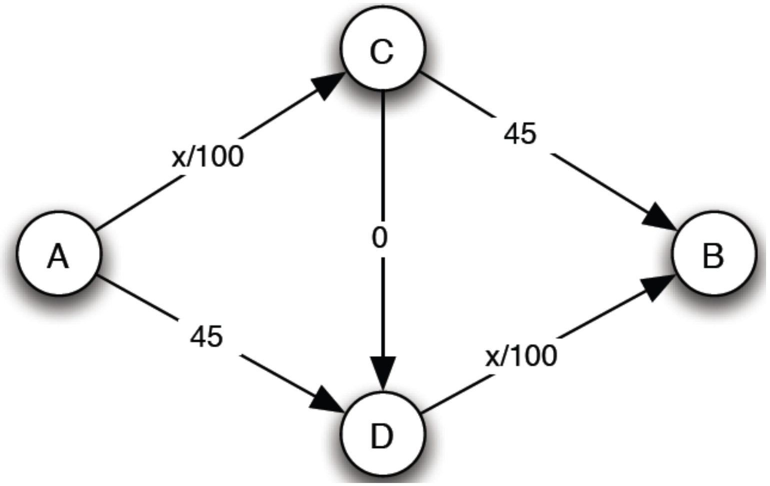
- The necessary condition for a pair of mixed strategies to be best responses of each other is that it makes the other player *indifferent* in payoffs under all his pure strategies
- This is how we compute the mixed strategy: indifference
- Method: assume the mixed strategy for player 1 is p, write the expected payoff for both pure strategies for player 2. The Nash equilibrium is the strategy that make the two functions equal

The best strategy is the one that making the other player doesn't know which of his pure strategy is better

Mechanism Design Basics Review:

- Traffic as a game

- PoA



Auction Basics: Review

- Who get the object
  - Usually the highest or lowest bidder
- What kind of price to pay
  - First price or second price, this influences the strategy of the bidders
- Do the bidders know the price of others
  - Sealed auction, needs to guess about others
  - Unsealed auction, can see other's bids

Auction: Game perspective

- First-price sealed-bid auctions (FPA)
  - Highest bidder wins and pays the value of her bid
- Second-price sealed-bid auctions (SPA)
  - Highest bidder wins and pays the value of the second-highest bid.
- Formulating as a game
  - Players: bidders
  - Strategy: bid
  - Payoff: true value – payment, or 0 (if auction fails)
- A game-oriented thinking
  - Equilibrium! Best response to each other, no one changes

What strategy is optimal?

- From game point of view (dominant strategy): cannot get better payoff by changing to other strategies, regardless of the other players' strategy

- Claim: In a sealed-bid second-price auction, it is a dominant strategy for each bidder i to choose a bid bi = vi.

Proof

- Assume in an auction, you consider the object worth \$100 and you bid \$100, now consider whether you can get better payoff using other strategy
- There are two cases:
  - You win the bid: now you have positive payoff (you only need to pay the second high bid, $v1 – b2 > 0$)
    - Increase the bid won’t change anything
    - Decrease the bid won’t change anything unless less than the second high, then you lose the bid and payoff become 0
  - You lose the bid: now your payoff is 0
    - Decrease the bid won’t change anything
    - Increase the bid won’t change anything unless becoming the highest; note that this means that this is greater than your value, so payoff becomes negative

First-Price Auction

- In second-price sealed-bid auction, bid the true value is a (weakly) dominant strategy
- First-price seal-bid auction does not have such property
  - Truthful bidding zero payoff
  - Shading requests complex trade-off reasoning

Bidding Strategies in First-Price Auction

- What is the optimal strategy in the equilibrium point?
- Simplified case: two bidders, private value independently and uniformly distributed and uniformly distributed between 0 and 1;
- Strategy: A strategy for a bidder is a function $s(v) = b$ that maps her true value v to a non-negative bid b.
- Assumption:
  - $s(\cdot)$ is a strictly increasing, differentiable function;
  - $s(v) \leq v$ for all v
  - E.G., $s(v)=v$; cv ($c<1$);
- The assumption helps us narrow down the strategy set.
  - Rules out non-optimal

Bidding Strategies in First-Price Auction

- If bidder i has a value of vi, the probability that this is higher than the value of i's competitor in the interval [0, 1] is exactly vi
- i's expected payoff: $g(v_i) = v_i(v_i - s(v_i))$.
- Equilibrium strategy: for each bidder i, there is no incentive for i to deviate from strategy $s(\cdot)$ if i's competitor is also using strategy $s(\cdot)$.
- Deviations in the bidding strategy function can instead be viewed as deviations in the "true value" that bidder i supplies to her current strategy $s(\cdot)$. -> Revelation Principle
- Condition that I does not want to deviate from strategy $s(\cdot)$:
  - $v_i(v_i - s(v_i)) \geq v(v_i - s(v))$
- What kind of function satisfies this condition?

Bidding Strategies in First-Price Auction

- What is the optimal strategy in the equilibrium point?
- Equilibrium with many bidders: n bidders, uniform distribution valuation
- For a given bidder i with true value vi, what is the probability that her bid is the highest?

- Expected payoff for bidder i: $G(v_i) = v_i^{n-1}(v_i - \mathrm{s}(v_i))$
- Using Revelation Principle, a deviation from the bidding strategy as supplying a "fake" value v to the function s()
  - $v_i^{n-1}(v_i - \mathrm{s}(v_i)) \geq v^{n-1}(v_i - \mathrm{s}(v))$ for all v between 0 and 1

- Expected payoff function: $G(v) = v^{n-1}(v_i - \mathrm{s}(v))$
  - $s'(v_i) = (n-1)(1 - \frac{s(v_i)}{v_i})$
  - $\mathrm{s}(v_i) = \left(\frac{n-1}{n}\right) v_i$
- As the number of bidders increases, you generally have to bid more aggressively

Seller Revenue Comparision

- How to compare the revenue a seller should expect to make in first-price and second-price auctions?
- Assumption: n bidders with values drawn independently from the uniform distribution on the interval [0, 1].
- Background Math Info: Suppose n numbers are drawn independently from the uniform distribution on the interval [0, 1] and then sorted from smallest to largest. The expected value of the number in the kth position on this list is $\frac{k}{n+1}$.
- Revenue equivalence theorem: given certain conditions, any mechanism that results in the same outcomes (i.e. allocates items to the same bidders) also has the same expected revenue.