MATHEMATICAL ANALYSIS

2ND HOMEWORK

Problem 1. Let

$$f_n(x) = \begin{cases} 0 \quad (x < \frac{1}{n+1}), \\ \sin^2 \frac{\pi}{x} \quad (\frac{1}{n+1} \le x \le \frac{1}{n}), \\ 0 \quad (\frac{1}{n} < x). \end{cases}$$

Show that $\{f_n\}$ converges to a continuous function, but not uniformly.

Problem 2. Suppose that $\{f_n\}, \{g_n\}$ are defined on E, and

(1) $\sum_n f_n$ has uniformly bounded partial sums;
(2) $g_n \to 0$ uniformly on E;
(3) $g_1(x) \ge g_2(x) \ge \cdots$ for every $x \in E$.

Prove that $\sum_n f_n g_n$ converges uniformly on E.

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