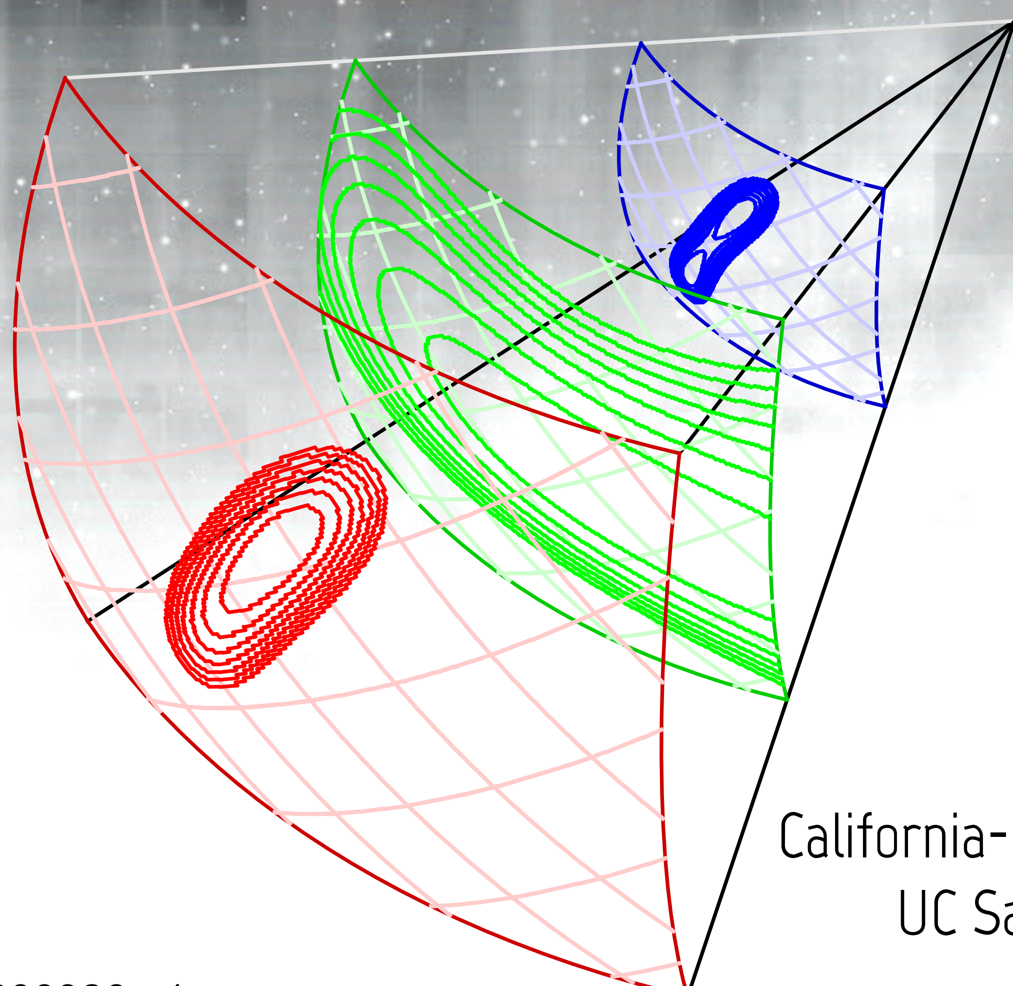
Rapid Bayesian Triangulation

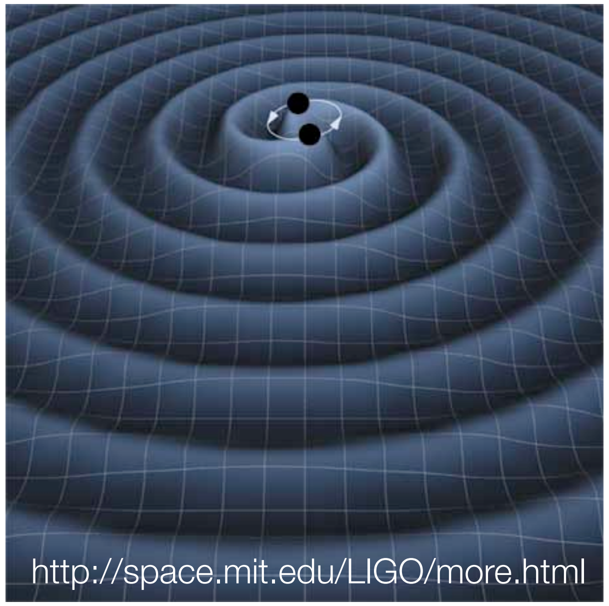
of Compact Binary Mergers using Advanced Gravitational Wave Detectors

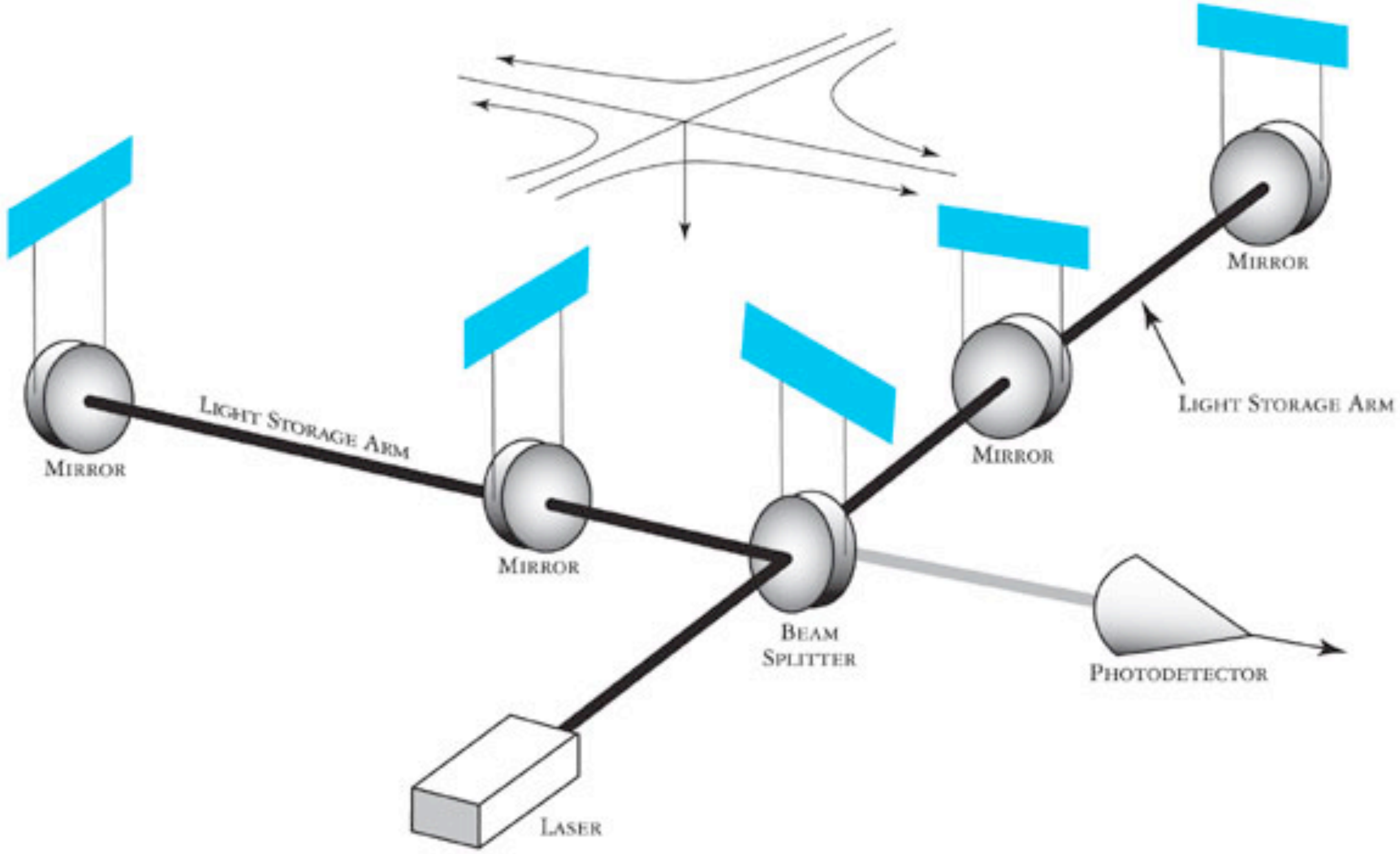


Leo Singer & Larry Price
lsinger@caltech.edu

California-Nevada APS Meeting 2012
UC San Luis Obispo, 2 Nov 2012

LIGO-G1200926-v1





http://www.ligo.caltech.edu/LIGO_web/PR/scripts/facts.html

Possible electromagnetic counterparts

- Two neutron stars merge, form a central compact object and accretion disk
- Accretion disk feeds pair of jets
- Internal shocks in jet produce a prompt γ-ray burst
- Shock between jet and ISM produces optical and radio afterglow

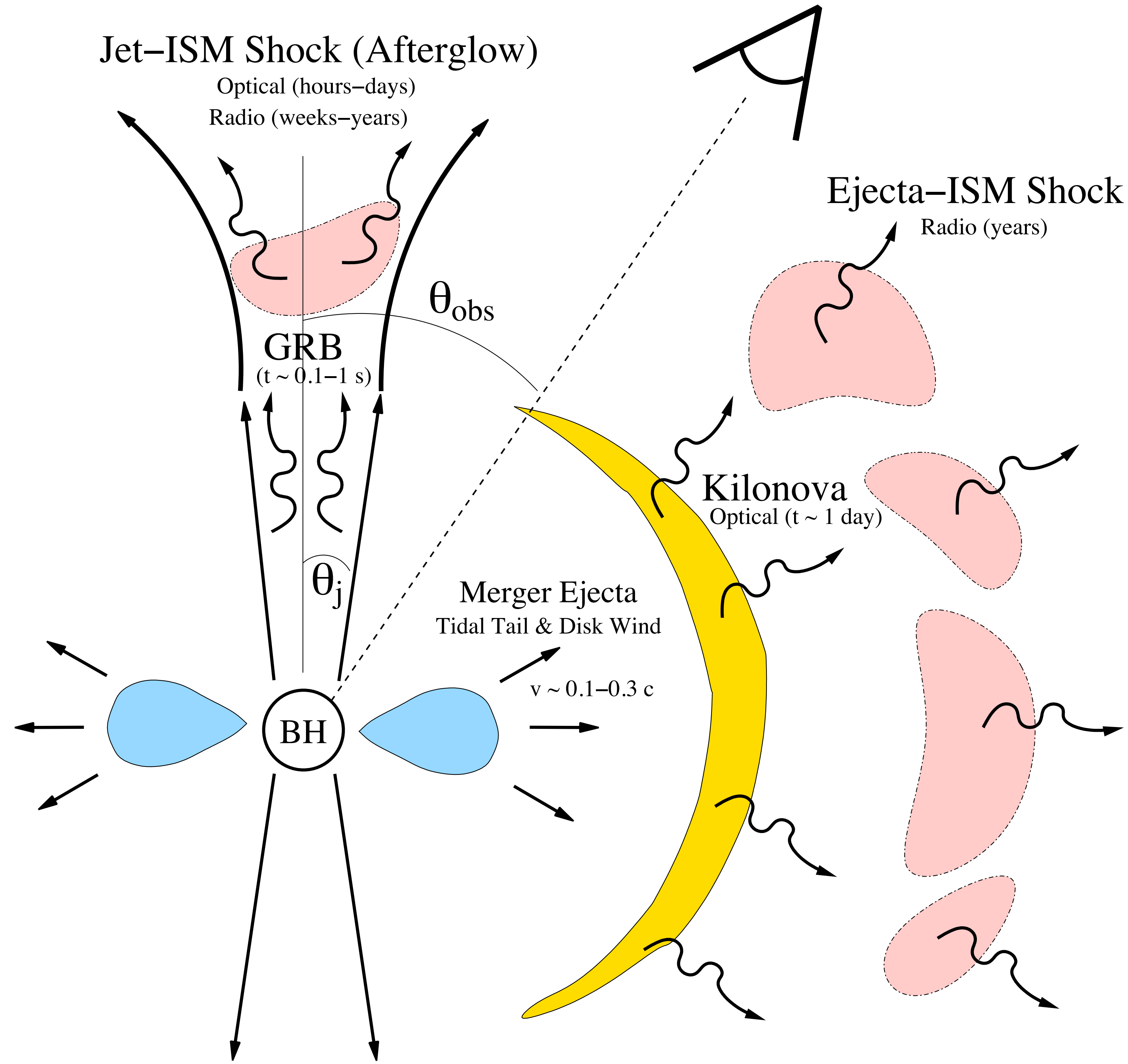


Figure 1 of Meztger & Berger 2012, ApJ, 746, 48

Story so far

- **Global network** of 3 multi-km interferometric observatories: LIGO–Hanford, LIGO–Livingston, Virgo
- **More planned**: KAGRA, LIGO–India
- During **joint LIGO–Virgo science run** in Summer—Fall 2010, sent alerts to astronomers to **point telescopes** see Abadie et al. 2012, A&A 541, A155
- Detectors off-line while they are reconfigured as **advanced** detectors → eventually **10x greater range** for binary neutron stars

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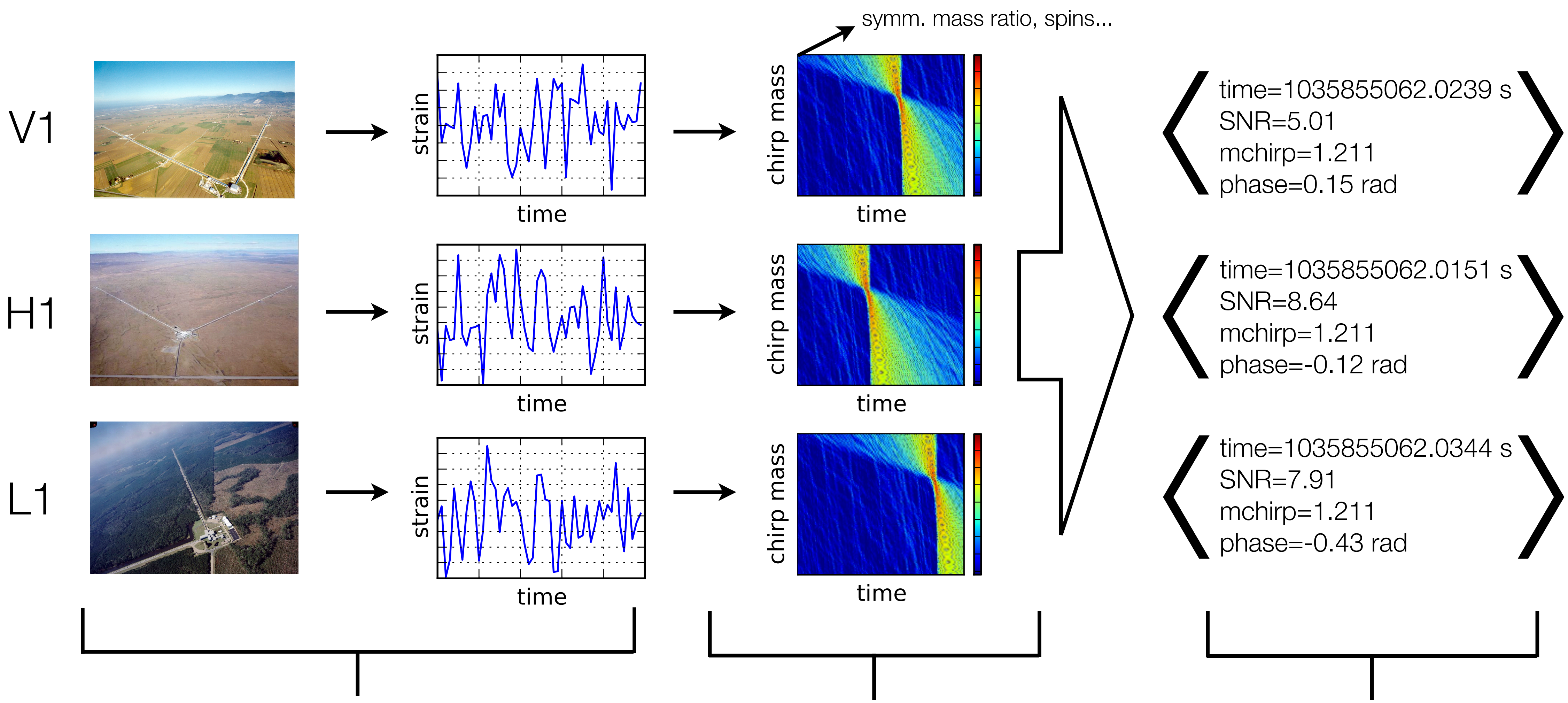
Detection



Strain transduced by detectors

also: data quality, vetoes, aggregate data to analysis clusters

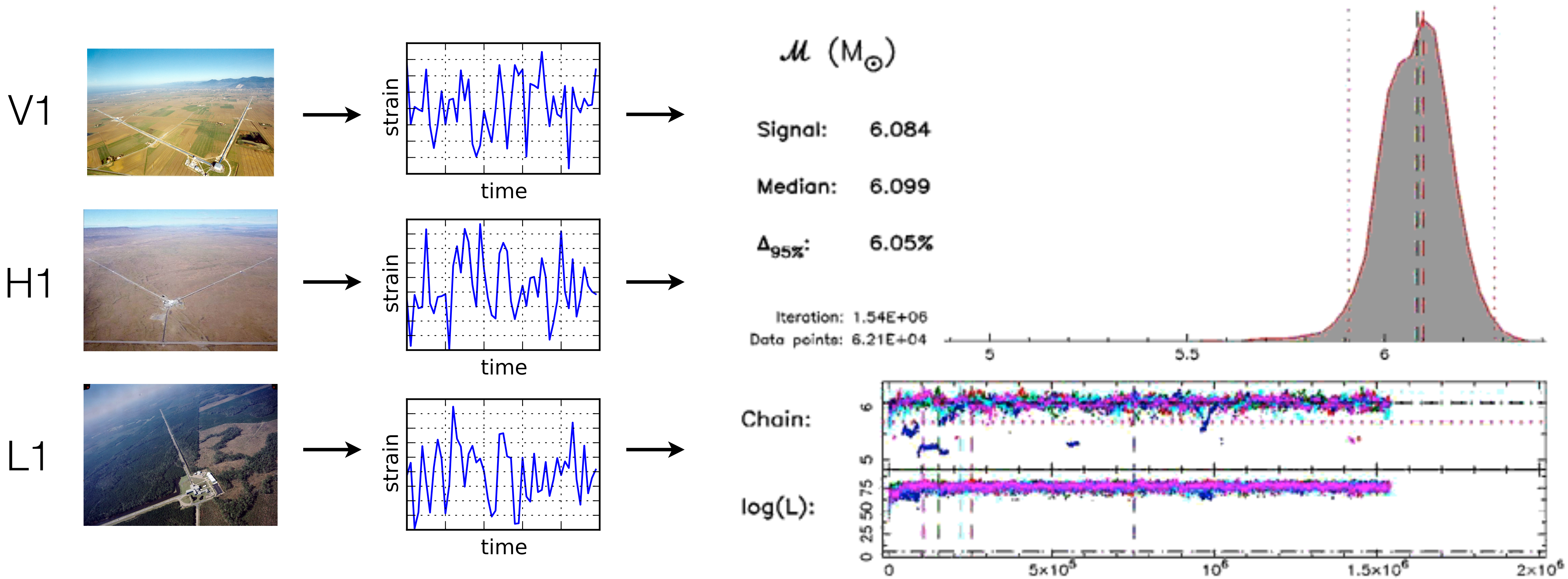
Matched filter

sliding dot product of strain data w/ sampling of all possible inspiral signals

Triggering, coincidence

excursion in matched filter *signal-to-noise ratio (SNR)* at similar times in all detectors

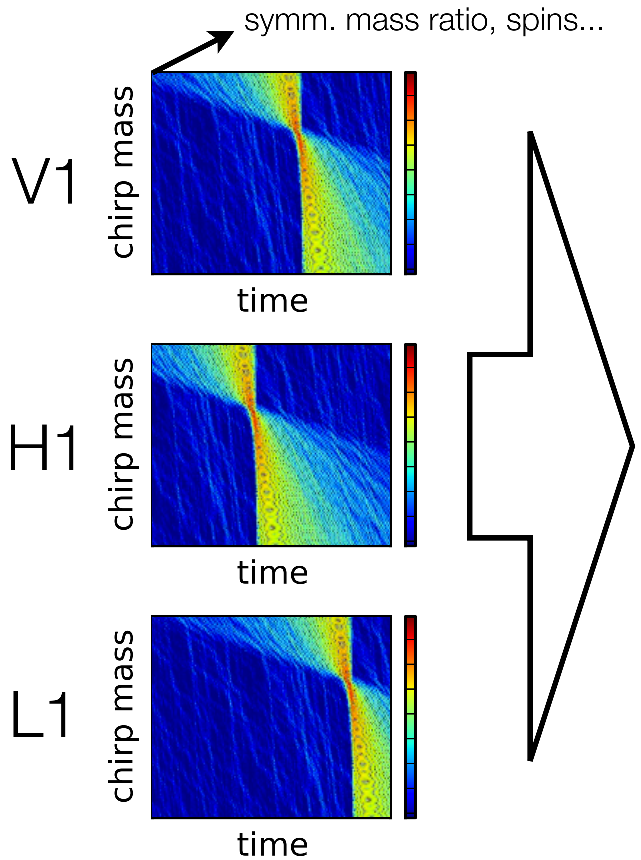
Full Markov-chain Monte Carlo (MCMC) parameter estimation



Vivien Raymond, <http://www.ligo.caltech.edu/~vraymond/>

- Input: the strain time series from all detectors
- Stochastically sample from parameter space, compute overlap of signal with data in each detector
- Sample distribution converges to posterior
- Deals with correlations between all parameters
- Can be computationally expensive

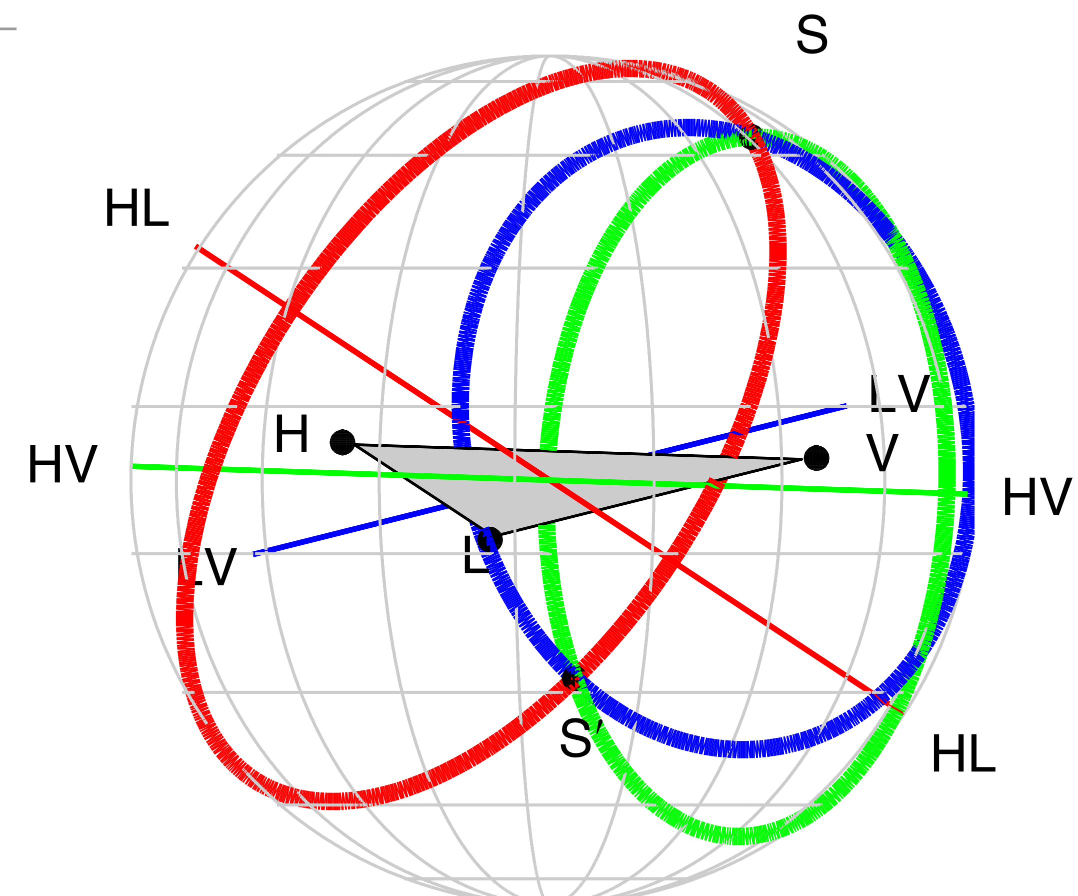
Triangulation



⟨ time=1035855062.0239 s
SNR=5.01
mchirp=1.211
phase=0.15 rad ⟩

⟨ time=1035855062.0151 s
SNR=8.64
mchirp=1.211
phase=-0.12 rad ⟩

⟨ time=1035855062.0344 s
SNR=7.91
mchirp=1.211
phase=-0.43 rad ⟩

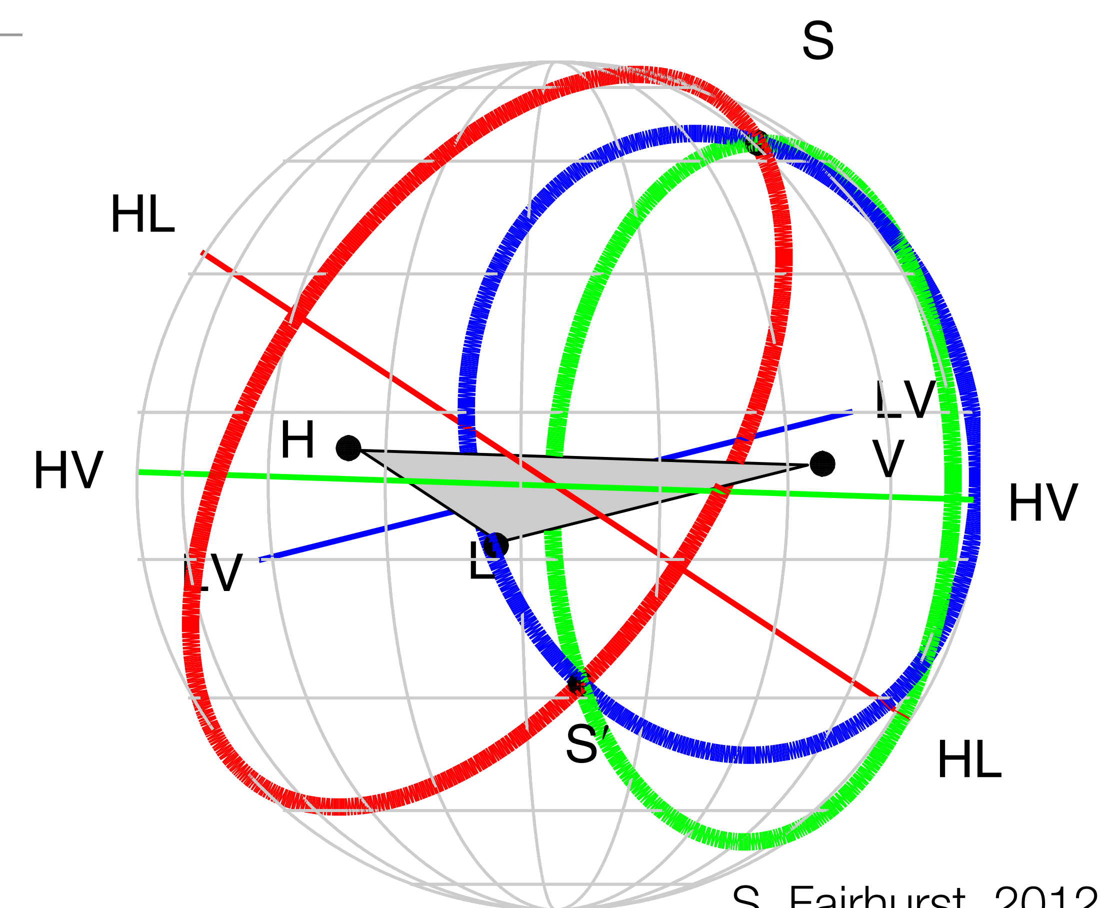


S. Fairhurst, 2012,
http://online.kitp.ucsb.edu/online/chirps_c12/fairhurst/

See also: Fairhurst, 2009, New J. Phys., 11, 123006),
Fairhurst, 2011, Class. Quantum Grav., 28, 105021

Triangulation

- Input: matched-filter **point estimates** of *extrinsic parameters* (**time, phase, amplitude**) in each detector
- Also need **distribution** of point estimates
- Differences in **times of arrival (TOAs)** at different sites constrain source to rings on the sky
- Relative **phases** and **amplitudes** depend on source's sky location and detectors' **antenna patterns**
- Relatively fast



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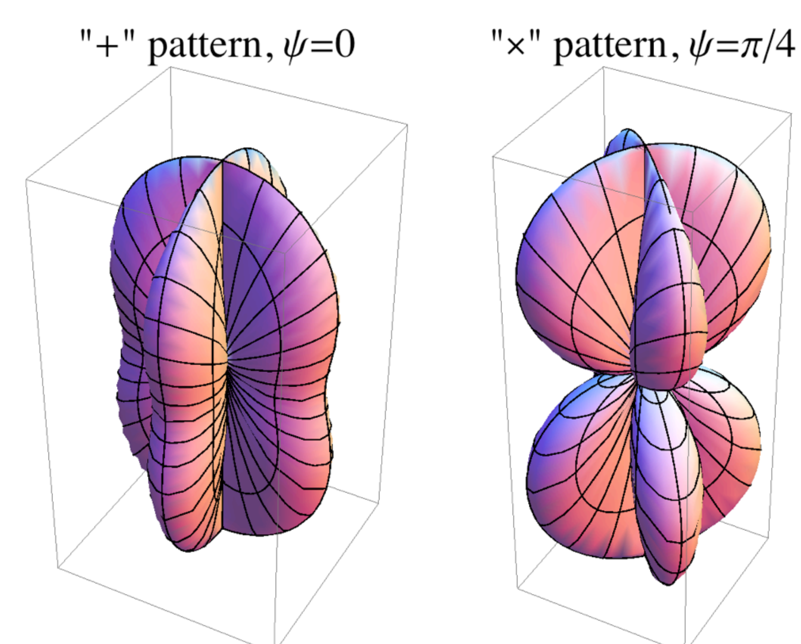
http://online.kitp.ucsb.edu/online/chirps_c12/fairhurst/



What is needed?

- Sky location needs to be available quickly: $\lesssim$10 min
  →full MCMC could use rapid sky map as initial proposal
  →full MCMC may update or supersede this later

- Triangulation based on point estimates of time and amplitude can fill this need
  →and we can impose the same stringent consistency requirements on it that we demand of the full MCMC (more on this soon)

Goals

➡ produce a rapid sky localization algorithm that is ready for doing observations with Advanced LIGO

➡ predict sky localization accuracy in Advanced gravitational wave detector era

Bayes' Rule

- Take some data, X, and form a hypothesis, Θ. How probable is your hypothesis, given the data?

"likelihood" "prior"

"posterior"

$$P(\Theta|X) = \frac{P(X|\Theta) \times P(\Theta)}{P(X)}$$

"evidence"

- **Marginalize** to get rid of nuisance parameters

$$P(\Theta, \lambda|X) = \frac{\sum_\lambda P(X|\Theta, \lambda) P(\Theta, \lambda)}{P(X)}$$

- Or, if hypothesis is **continuously parameterized**,

$$p(\theta|x) = \frac{\int p(x|\theta, \lambda) p(\theta, \lambda) \mathrm{d}\lambda}{p(x)}$$

REV. T. BAYES

Bayes' Rule: problem setup

Data/observation

strain time series

$$\left.x_i(t_j)\right\}_{ij} \quad \begin{matrix} N \text{ detectors} \\ M \text{ samples} \end{matrix}$$

amplitude, SNR — phase — TOA

$$\left.\rho_i, \tau_i, \cancel{\gamma_i}\right\}_i \quad N \text{ detectors}$$

note: not using phase right now

Nuisance variables

component masses

$$m_1, m_2, \mathbf{S}_1, \mathbf{S}_2$$

spins

intrinsic variables (fixed at maximum-likelihood estimates for triangulation)

luminosity distance — polarization angle

$$\tau_\oplus, D_\mathrm{L}, \iota, \psi, \phi_c$$

TOA at geocenter — inclination — coalescence phase

extrinsic variables

Parameters of interest

direction of source $\mathbf{n}$

e.g.,

right ascension, declination α, δ

Outline of calculation

Likelihood: factor into a time of arrival (TOA)-only contribution and an SNR-only contribution, both **Gaussian**

$$\mathcal{L} \propto \mathcal{L}_{\mathrm{SNR}} \times \mathcal{L}_{\mathrm{TOA}}$$

Prior: uniform in

$$\tau_\oplus, \phi_c, \psi, \cos\iota, {D_L}^3$$

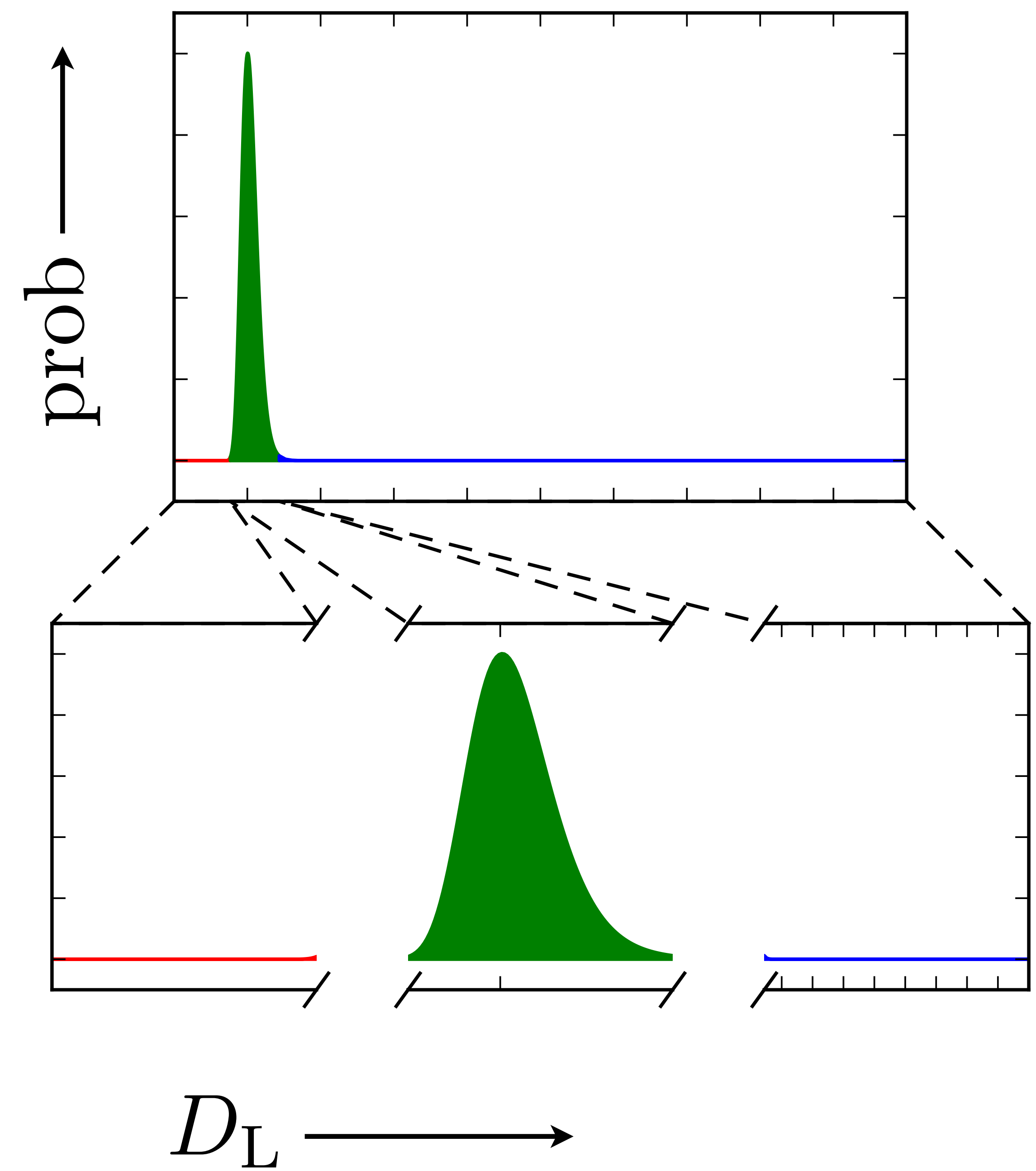
Posterior: factor into an **TOA-only** contribution and an **SNR-only** contribution

$$p(\mathbf{n}|\tau_1, \ldots, \tau_N, \rho_1, \ldots, \rho_N)$$
$$= f_{\mathrm{TOA}}(\mathbf{n}; \tau_1, \ldots, \tau_N) \times f_{\mathrm{SNR}}(\mathbf{n}; \rho_1, \ldots, \rho_N)$$

Evaluate TOA posterior factor first, then evaluate SNR posterior factor for those points that comprise the **99.99th percentile** of the **TOA posterior**.

Distance marginalization

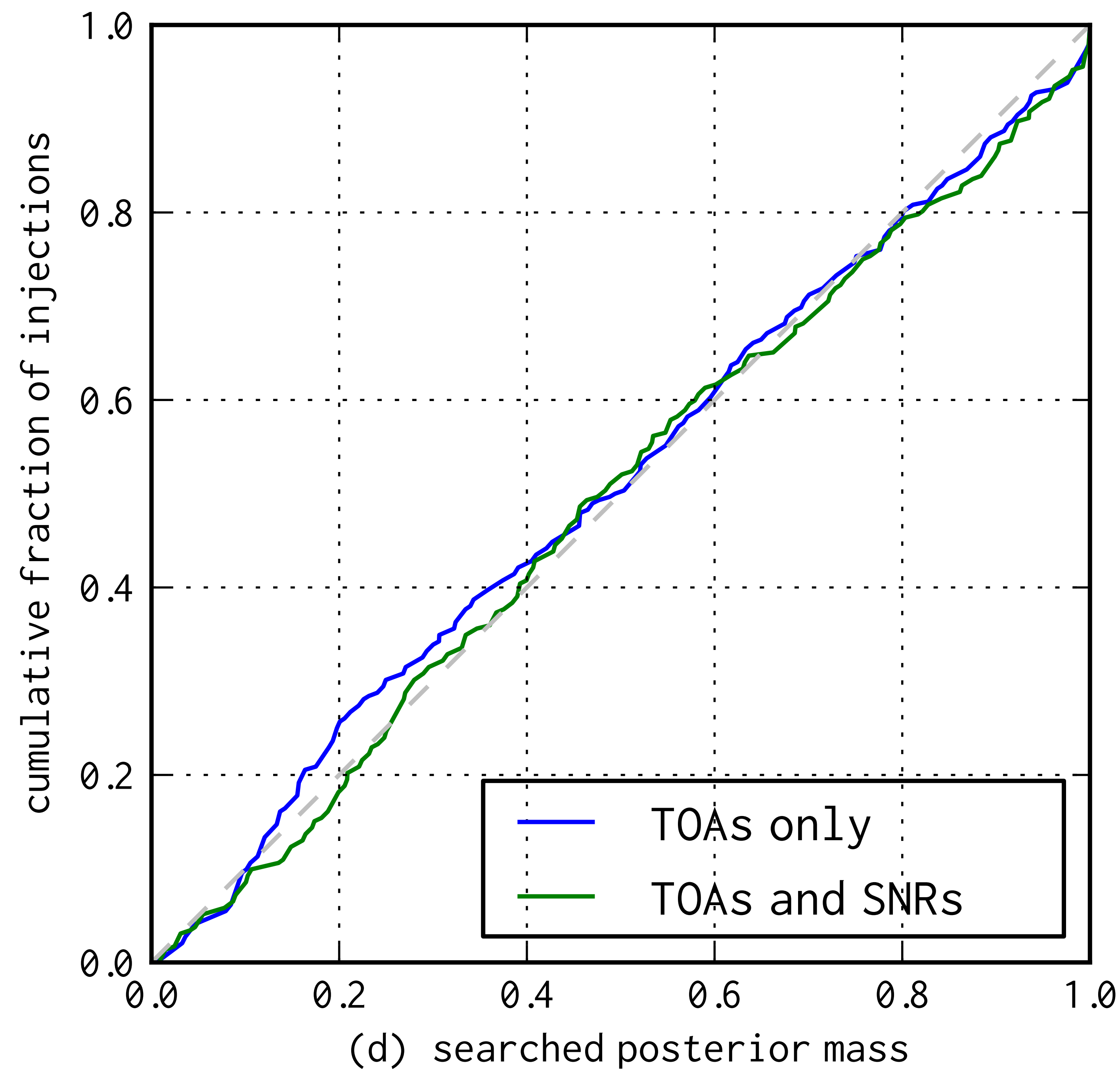
- Radial integrand **peaks sharply** at the distance that is best supported by the data
- Divide integration domain into three sub-domains that enclose the maximum likelihood peak, the small-distance tail, and the large-distance tail
- Use **adaptive Gaussian quadrature** to discover which region dominates



309 injections
3 detectors: H1—L1—V1
Detector configurations: aLIGO—AdVirgo
Component masses: 1.4—1.4 $M_\odot$
Distributed uniformly in volume
from 100 to 300 Mpc,
restricted to SNR≥8 in each detector

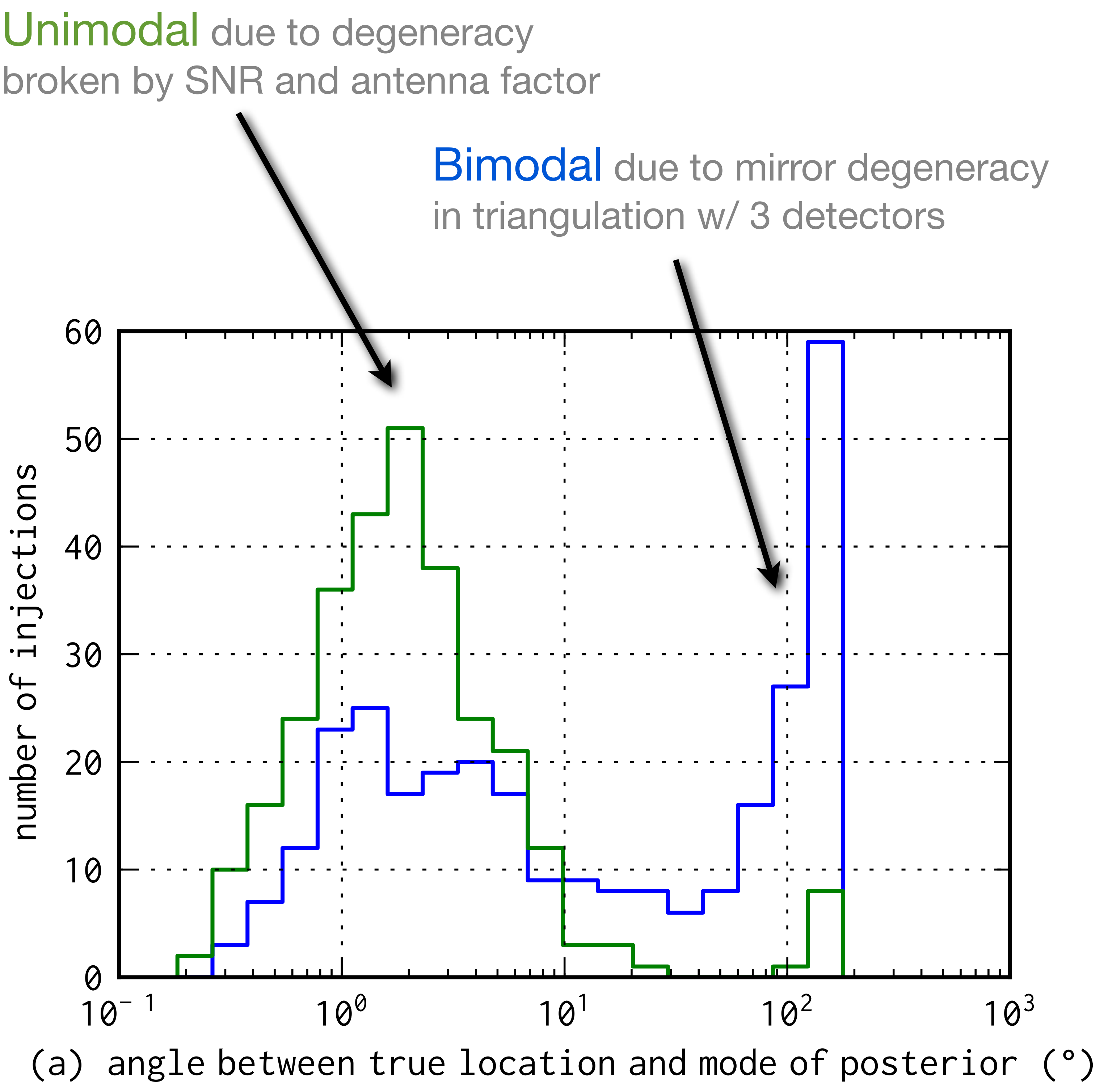
Sanity check

Are a fraction *P* of injections found within the *P*th confidence level? Can the computed distribution represent a valid posterior?



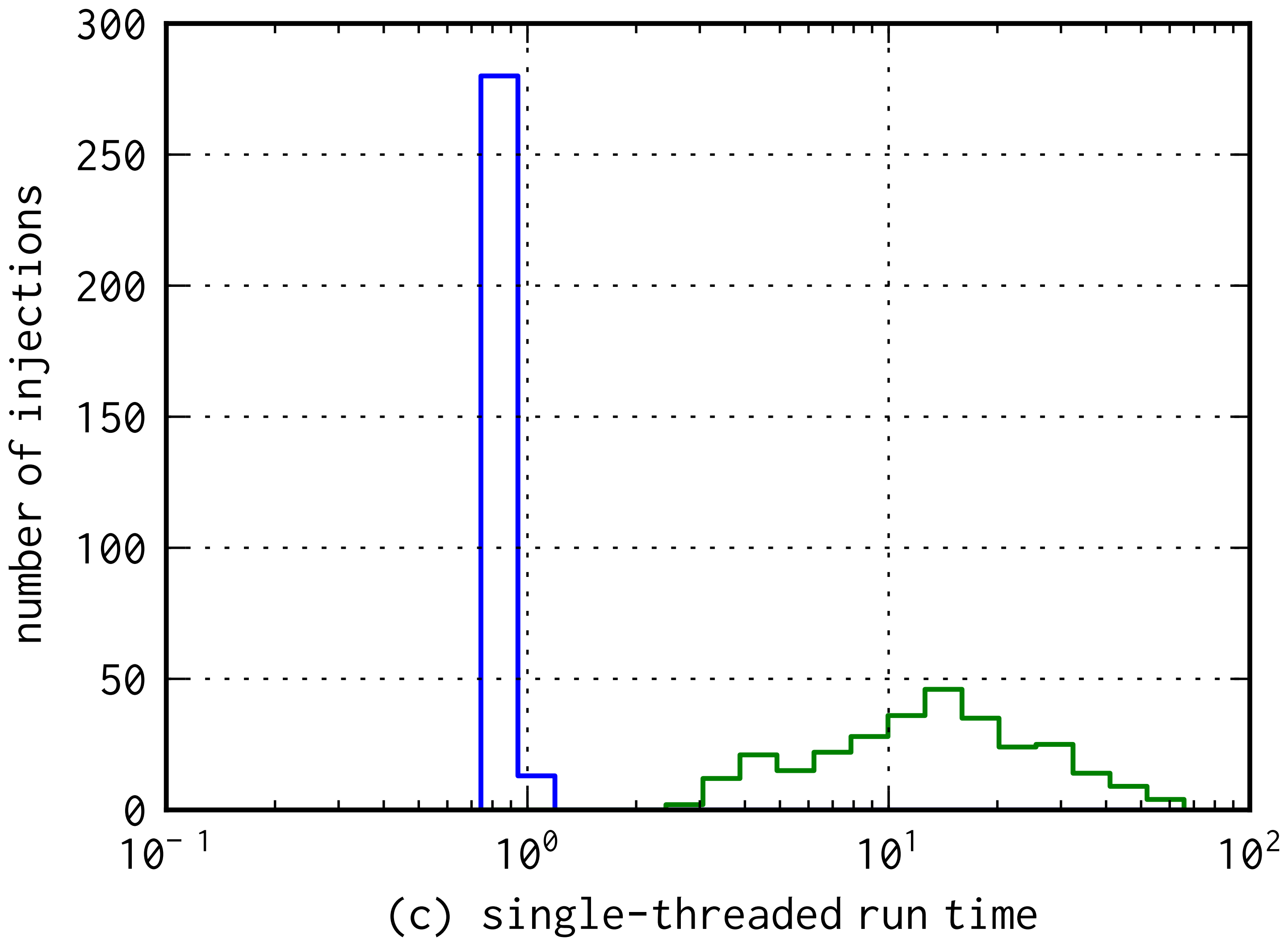
Angular offset

What is the angle between the true location of the source and the *maximum a posteriori* (MAP) estimate?

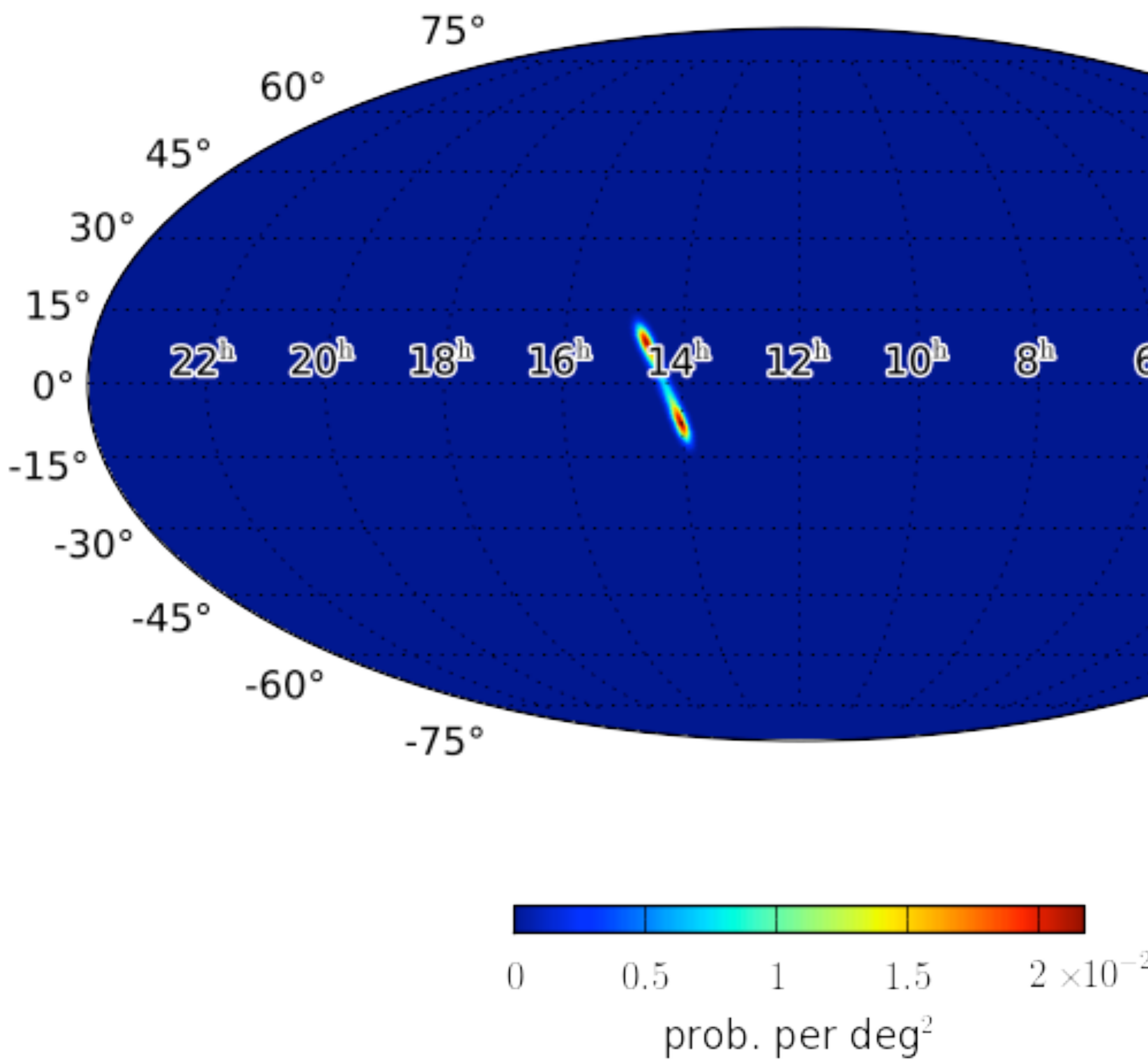


Run time

Working with just a single thread, how long does it take to produce a sky map?



An example



TOA only

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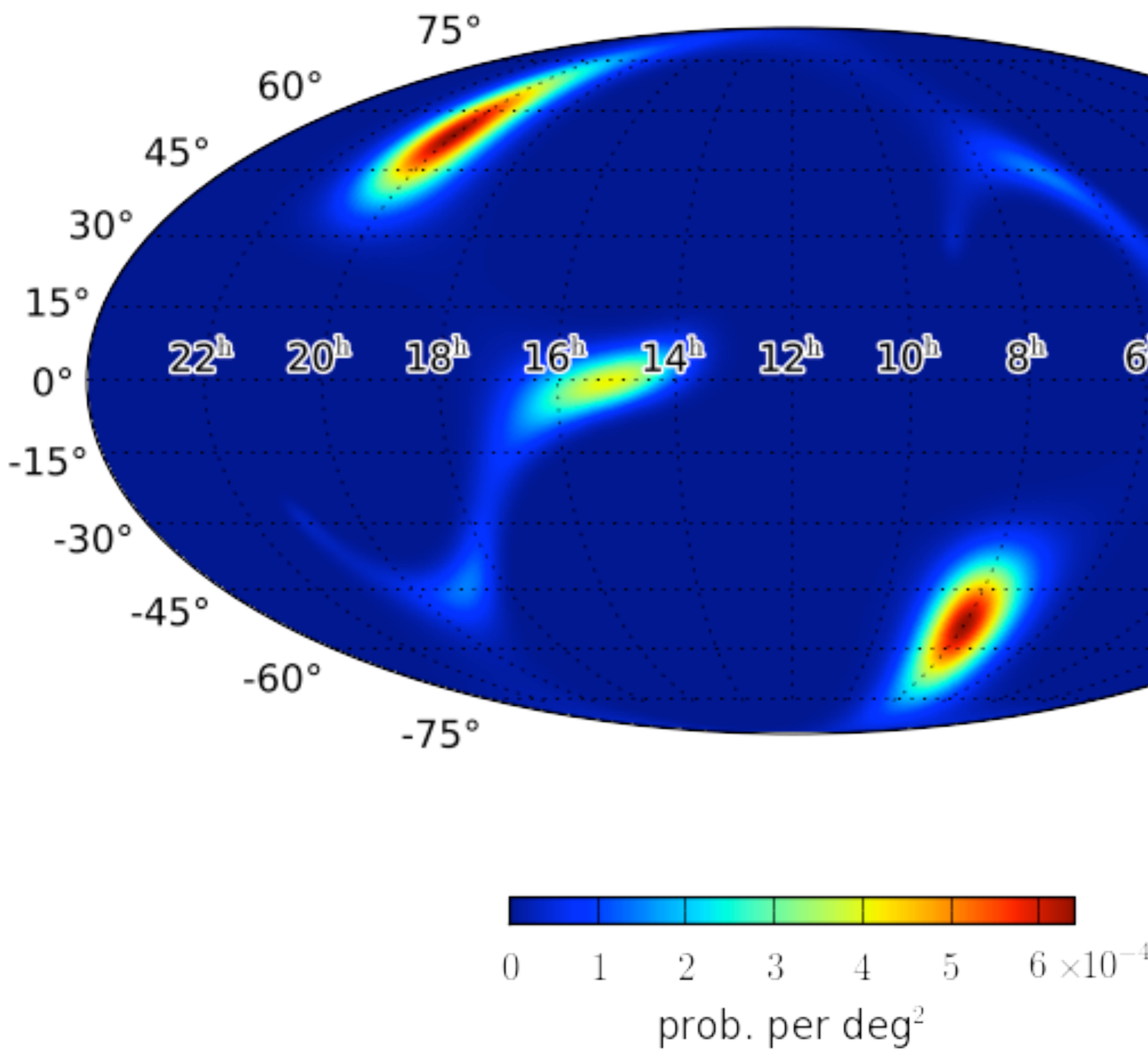
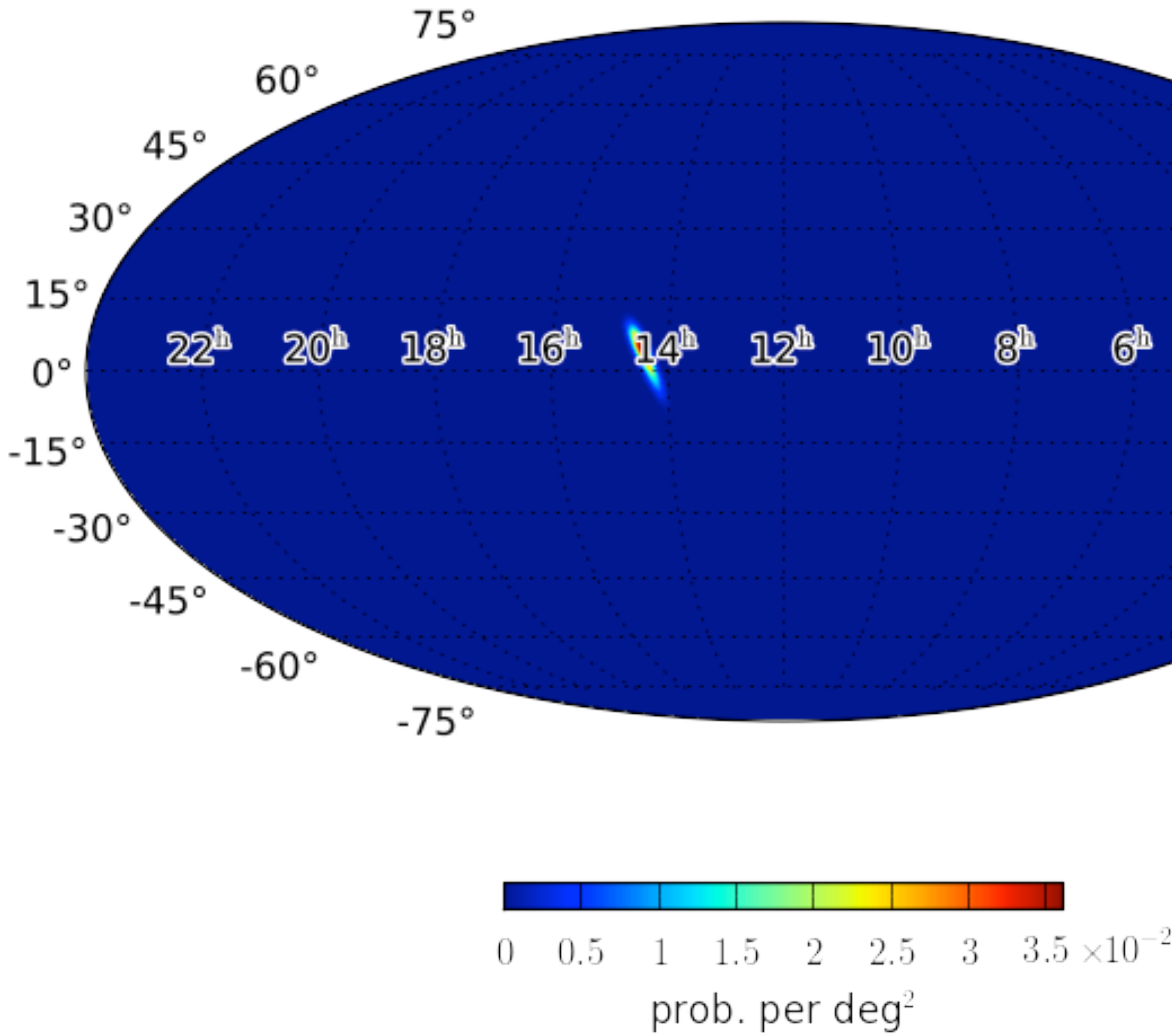

75°
60°
45°
30°
15°
0°
-15°
-30°
-45°
-60°
-75°
22h
20h
18h
16h
14h
12h
10h
8h
6h
4h
2h
0
1
2
3
4
5
6 ×10^-4
prob. per deg²

SNR only



TOA+SNR

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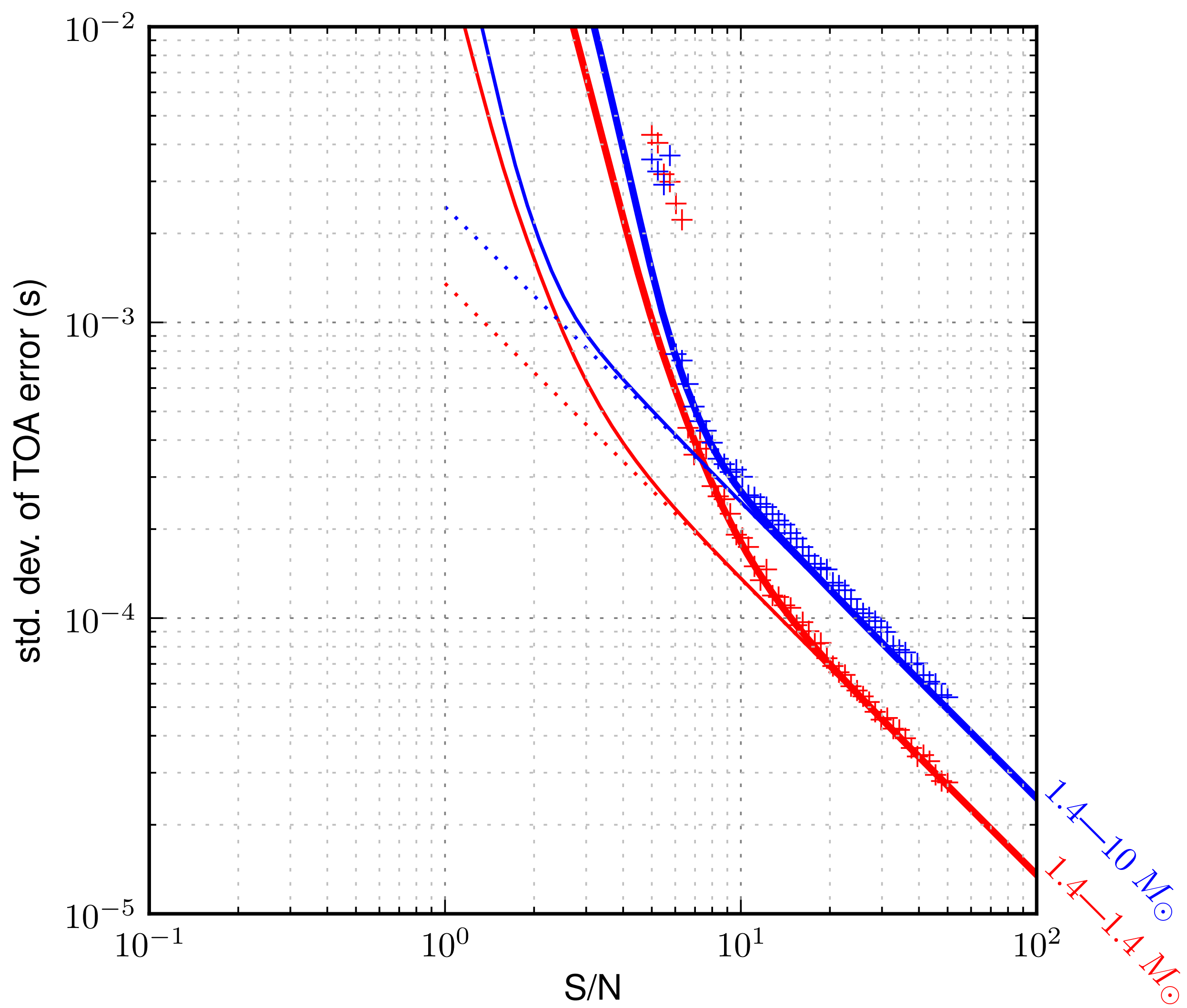
Timing accuracy

- Want to predict TOA accuracy σ_t as a function of SNR instead of tabulate it in advance.
- Cramér-Rao bound→ $\sigma_t \propto 1/\rho$ (see Fairhurst 2009, for example).
- "Threshold effect": breaks down at low SNR, well known in information theory... Barankin bound (Barankin, 1949, Ann. Math. Stat. 20, 477) appears to get SNR scaling right, but not the threshold.
- More modern attacks, particularly in LIGO community:

Nicholson & Vecchio 1998, PRD 57, 4588
Zanolin et al. 2010, PRD, 81, 124048
Vitale & Zanolin 2010, PRD 82, 124065

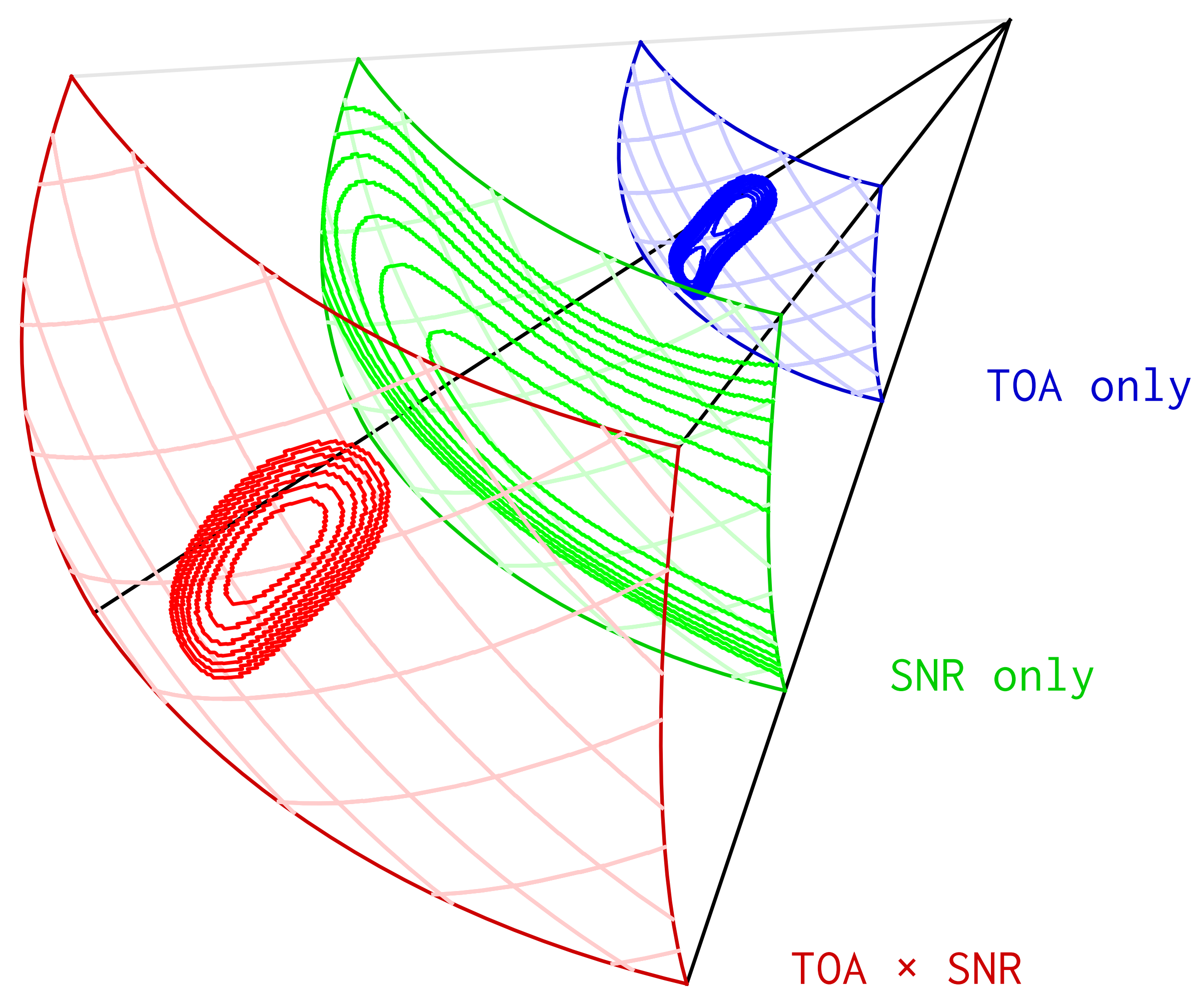


Future work

- Either **calculate** or **tabulate** TOA accuracy as a function of **SNR** and **masses** →need this in order to compute the TOA part of the likelihood

- **Test** on a large astrophysically realistic set of simulated Advanced LIGO events

- **Predict sky localization** areas achievable in the Advanced GW detector era



Backup slides

Likelihood

Likelihood: factor into a time of arrival (TOA)-only contribution and an SNR-only contribution

$$\mathcal{L} \propto \mathcal{L}_{\mathrm{SNR}} \times \mathcal{L}_{\mathrm{TOA}}$$

TOA-only likelihood: Gaussian; depends on sky location and overall event time

$$\mathcal{L}_{\mathrm{TOA}} \propto \left[-\frac{1}{2}\sum_i \frac{(\tau_i - \bar{\tau}_i(\mathbf{n}, \tau_\oplus)^2}{{\sigma_{t_i}}^2}\right]$$

SNR-only likelihood: Gaussian; depends on sky location, distance, inclination, polarization angle, and coalescence phase

$$\mathcal{L}_{\mathrm{SNR}} \propto \exp\left[-\frac{1}{2}\sum_i (\rho_i - \bar{\rho}_i(\mathbf{n}, D_{\mathrm{L}}, \iota, \psi, \phi))^2\right]$$

Fisher information for SNR and TOA estimates

- Fairhurst (2009, New J. Phys., 11, 123006) calculated the Fisher information matrix for the extrinsic parameters associated with

$$
\begin{array}{cc}
 & \begin{array}{ccc} \rho & \gamma & \tau \end{array} \\
\mathcal{I} = \begin{array}{c} \rho \\ \gamma \\ \tau \end{array} & \begin{pmatrix} 1 & 0 & 0 \\ 0 & \rho^2 & -\rho^2\overline{\omega} \\ 0 & \rho^2\overline{\omega} & \rho^2\overline{\omega^2} \end{pmatrix}
\end{array}
$$

where

$$
\overline{\omega^k} = \left[\int \frac{|h(\omega)|^2}{S(\omega)} \omega^k \mathrm{d}\omega\right] \left[\int \frac{|h(\omega)|^2}{S(\omega)} \mathrm{d}\omega\right]^{-1}
$$