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# %% Imports
# types
from typing import (
   Tuple,
    List,
    TypeVar,
    Optional,
    Dict,
    Any,
    Union,
    Sequence,
    Generic,
    Iterable,
from mixturedata import MixtureParameters
from gaussparams import GaussParams
#from estimatorduck import StateEstimator
from mixturereduction import gaussian_mixture_moments #done
# packages
from dataclasses import dataclass
from singledispatchmethod import singledispatchmethod
import numpy as np
from scipy import linalg
from scipy.special import logsumexp
from ekf import EKF as StateEstimator
# Local
import discretebayes
# %% TypeVar and aliases
MT = TypeVar("MT") # a type variable to be the mode type
# %% IMM
@dataclass
class IMM(Generic[MT]):
    # The M filters the IMM relies on
    filters: List[StateEstimator[MT]]
    # the transition matrix. PI[i, j] = probability of going from model i to j: shape (M, M)
    PI: np.ndarray
    # init mode probabilities if none is given
    initial_mode_probabilities: Optional[np.ndarray] = None
    def __post_init__(self):
        # This have to be satisfied!
        if not np.allclose(self.PI.sum(axis=1), 1):
            raise ValueError("The rows of the transition matrix PI must sum to 1.")
        # Nice to have a reasonable initial mode probability
        if self.initial_mode_probabilities is None:
            eigvals, eigvecs = linalg.eig(self.PI)
            self.initial_mode_probabilities = eigvecs[:, eigvals.argmax()]
            self.initial_mode_probabilities = (
                self.initial_mode_probabilities / self.initial_mode_probabilities.sum()
            )
    def mix probabilities(
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self,
    immstate: MixtureParameters[MT],
    # sampling time
    Ts: float,
) -> Tuple[
   np.ndarray, np.ndarray
]: # predicted_mode_probabilities, mix_probabilities: shapes = ((M, (M ,M))).
    # mix_probabilities[s] is the mixture weights for mode s
    """Calculate the predicted mode probability and the mixing probabilities."""
    # My comment: this step should implement step 1., 6.27
    predicted_mode_probabilities, mix_probabilities = discretebayes.discrete_bayes(immsta
    # TODO hint: discretebayes.discrete bayes
    # Optional assertions for debugging
    assert np.all(np.isfinite(predicted_mode_probabilities))
    assert np.all(np.isfinite(mix probabilities))
    assert np.allclose(mix probabilities.sum(axis=1), 1)
    return predicted mode probabilities, mix probabilities
def mix_states(
    self,
    immstate: MixtureParameters[MT],
    # the mixing probabilities: shape=(M, M)
    mix_probabilities: np.ndarray,
) -> List[MT]:
    #My comment: Here we are implementing the step 2 of the algorithm, 6.29 and 6.30
    means = np.array([component.mean for component in immstate.components])
    covs = np.array([component.cov for component in immstate.components])
    mixed states = gaussian mixture moments(mix probabilities, immstate.components[:].mea
    mixed states = []
    for i in range(len(means)):
        mixed states.append(gaussian mixture moments(mix probabilities[i], means[i], covs
    mixed states = np.array(mixed states)
    return mixed_states
def mode matched prediction(
    self,
    mode states: List[MT],
    # The sampling time
    Ts: float,
) -> List[MT]:
    #My comment, here we are doing step 3, mode mathed prediction
    modestates_pred = []
    for i in range(len(self.filters)):
        modestates_pred.append(self.filters[i].predict(mode_states[i], Ts))
    return np.array(modestates_pred)
def predict(
    immstate: MixtureParameters[MT],
    # sampling time
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Ts: float,
) -> MixtureParameters[MT]:
   Predict the immstate Ts time units ahead approximating the mixture step.
    Ie. Predict mode probabilities, condition states on predicted mode,
    appoximate resulting state distribution as Gaussian for each mode, then predict each I
    predicted_mode_probability, mixing_probability = self.mix_probabilities(immstate, Ts)
   mixed_mode_states: List[MT] = self.mix_states(immstate, mixing_probability) #Done
   predicted mode states = self.mode matched prediction(mixed mode states, Ts) #Done
    predicted immstate = MixtureParameters(
        predicted mode probability, predicted mode states
    return predicted immstate
def mode_matched_update(
   self,
    z: np.ndarray,
    immstate: MixtureParameters[MT],
    sensor state: Optional[Dict[str, Any]] = None,
) -> List[MT]:
    """Update each mode in immstate with z in sensor_state."""
    #MY comment: This implements step 3 update
   updated state = []
    for filt, mode state in zip(self.filters, immstate.components):
        updated state.append(filt.update(z, mode state, sensor state))
    return np.arrray(updated_state)
def update_mode_probabilities(
   self,
    z: np.ndarray,
    immstate: MixtureParameters[MT],
    sensor_state: Dict[str, Any] = None,
) -> np.ndarray:
    """Calculate the mode probabilities in immstate updated with z in sensor_state"""
    # This function will update the mode probabilities pk(sk). Given in equation 6.33
    #mode Loglikelihood = (z-h(immstate[:].mean))@np.inv(immstate[:].cov)@(z-h(immstate[
   mode loglikelihood = np.array([ekf filter.loglikelihood(z, comp, sensor state) for ek
    # potential intermediate step logjoint =
   predicted_mode_probabilities = self.PI*immstate.weights #Done
    denominator = np.sum(np.exp(mode_loglikelihood)*predicted_mode_probabilities) #done
    log pred mode probs = np.log(predicted mode probabilities)
    log denominator = np.log(denominator)
    log updated mode probs = mode loglikelihood + log pred mode probs - log denominator
   updated_mode_probabilities = np.exp(log_updated_mode_probs)
    assert np.all(np.isfinite(updated_mode_probabilities))
    assert np.allclose(np.sum(updated_mode_probabilities), 1)
    return updated mode probabilities
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def update(
    self,
    z: np.ndarray,
    immstate: MixtureParameters[MT],
    sensor_state: Dict[str, Any] = None,
) -> MixtureParameters[MT]:
    """Update the immstate with z in sensor_state."""
    updated_weights = self.updated_mode_probabilities(z, immstate, sensor_state) #Done
    updated states = self.mode matched update(z, immstate, sensor state)
    updated immstate = MixtureParameters(updated weights, updated states)
    return updated immstate
def step(
    self,
    z,
    immstate: MixtureParameters[MT],
    Ts: float,
    sensor_state: Dict[str, Any] = None,
) -> MixtureParameters[MT]:
    """Predict immstate with Ts time units followed by updating it with z in sensor_state
    predicted immstate = self.predict(immstate, Ts) # Done
    updated_immstate = self.update(z, predicted_immstate, sensor_state) # Done
    return updated_immstate
def loglikelihood(
    self,
    z: np.ndarray,
    immstate: MixtureParameters,
    sensor_state: Dict[str, Any] = None,
) -> float:
    # THIS IS ONLY NEEDED FOR IMM-PDA. You can therefore wait if you prefer.
    mode conditioned 11 = None # TODO in for IMM-PDA
    11 = None # TODO
    return 11
def reduce mixture(
    self, immstate mixture: MixtureParameters[MIXtureParameters[MI]]
) -> MixtureParameters[MT]:
    """Approximate a mixture of immstates as a single immstate"""
    # extract probabilities as array
    weights = immstate_mixture.weights
    component_conditioned_mode_prob = np.array(
        [c.weights.ravel() for c in immstate_mixture.components]
    )
    # flip conditioning order with Bayes
    #components, is this p(X|SK), BUT WE WANT P(SK|X). Is mode_prob p(x)
    mode_prob, mode_conditioned_component_prob = discretebayes.discrete_bayes(weights, coll)
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# Hint list_a of lists_b to list_b of lists_a: zip(*immstate_mixture.components)
    mode states = None # TODO:
    immstate reduced = MixtureParameters(mode prob, mode states)
    return immstate reduced
def estimate(self, immstate: MixtureParameters[MT]) -> GaussParams:
    """Calculate a state estimate with its covariance from immstate"""
    # ! You can assume all the modes have the same reduce and estimate function
    # ! and use eg. self.filters[0] functionality
    means = []
    covs = []
    for mean, cov in immstate.components:
        means.append(means)
        covs.append(cov)
    means = np.array(means)
    covs = np.array(cov)
    means_reduced, covs_reduced = mixturereduction.GaussianMixtureMoments(immstate.weight)
    estimate = GaussParams(means_reduced, covs_reduced)
    return estimate
def gate(
   self,
    z: np.ndarray,
    immstate: MixtureParameters[MT],
    gate size: float,
   sensor state: Dict[str, Any] = None,
) -> bool:
    """Check if z is within the gate of any mode in immstate in sensor state"""
    # THIS IS ONLY NEEDED FOR PDA. You can wait with implementation if you want
    gated_per_mode = None # TODO
    gated = None # TODO
    return gated
def NISes(
   self,
    z: np.ndarray,
    immstate: MixtureParameters[MT],
    sensor state: Optional[Dict[str, Any]] = None,
) -> Tuple[float, np.ndarray]:
    """Calculate NIS per mode and the average"""
    NISes = np.array(
        fs.NIS(z, ms, sensor_state=sensor_state)
            for fs, ms in zip(self.filters, immstate.components)
        ]
    )
    innovs = [
        fs.innovation(z, ms, sensor_state=sensor_state)
        for fs, ms in zip(self.filters, immstate.components)
    1
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v_ave = np.average([gp.mean for gp in innovs], axis=0, weights=immstate.weights)
   S_ave = np.average([gp.cov for gp in innovs], axis=0, weights=immstate.weights)
   NIS = (v_ave * np.linalg.solve(S_ave, v_ave)).sum()
   return NIS, NISes
def NEESes(
   self,
    immstate: MixtureParameters,
   x_true: np.ndarray,
   idx: Optional[Sequence[int]] = None,
):
   NEESes = np.array(
            fs.NEES(ms, x true, idx=idx)
            for fs, ms in zip(self.filters, immstate.components)
    )
   est = self.estimate(immstate)
   NEES = self.filters[0].NEES(est, x_true, idx=idx) # HACK?
    return NEES, NEESes
@singledispatchmethod
def init_filter_state(
   self,
    init, # Union[
         MixtureParameters, Dict[str, Any], Tuple[Sequence, Sequence], Sequence
    # 1,
) -> MixtureParameters:
   Initialize the imm state to MixtureParameters.
    - If mode probabilities are not found they are initialized from self.initial mode pro
    - If only one mode state is found, it is broadcasted to all modes.
   MixtureParameters: goes unaltered
   dict:
        ["weights", "probs", "probabilities", "mode_probs"]
            in this order can signify mode probabilities
        ["components", "modes"] signify the modes
    tuple: first element is mode probabilities and second is mode states
    Sequence: assumed to be only the mode states
   mode probabilities: array like
   components:
    """ # TODO there are cases where MP unaltered can lead to trouble
    raise NotImplementedError(
        f"IMM do not know how to initialize a immstate from: {init}"
@init_filter_state.register
def _(self, init: MixtureParameters[MT]) -> MixtureParameters[MT]:
    return init
@init filter state.register(dict)
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def _(self, init: dict) -> MixtureParameters[MT]:
    # extract weights
    got_weights = False
    got components = False
    for key in init:
        if not got_weights and key in [
            "weights",
            "probs",
            "probabilities",
            "mode_probs",
        1:
            weights = np.asfarray([key])
            got weights = True
        elif not got_components and key in ["components", "modes"]:
            components = self.init components(init[key])
            got_components = True
    if not got weights:
        weights = self.initial_mode_probabilities
    if not got_components:
        components = self.init_components(init)
    assert np.allclose(weights.sum(), 1), "Mode probabilities must sum to 1 for"
    return MixtureParameters(weights, components)
@init_filter_state.register(tuple)
def _(self, init: tuple) -> MixtureParameters[MT]:
    assert isinstance(init[0], Sized) and len(init[0]) == len(
        self.filters
    ), f"To initialize from tuple the first element must be of len(self.filters)={len(self.
    weights = np.asfarray(init[0])
    components = self.init_compontents(init[1])
    return MixtureParameters(weights, components)
@init_filter_state.register(Sequence)
def _(self, init: Sequence) -> MixtureParameters[MT]:
    weights = self.initial_mode_probabilities
    components = self.init_components(init)
    return MixtureParameters(weights, components)
@singledispatchmethod
def init components(self, init: "Union[Iterable, MT like]") -> List[MT]:
    """ Make an instance or Iterable of the Mode Parameters into a list of mode parameter:
    return [fs.init filter state(init) for fs in self.filters]
@init_components.register(dict)
def _(self, init: dict):
    return [fs.init filter state(init) for fs in self.filters]
@init_components.register(Iterable)
def _(self, init: Iterable) -> List[MT]:
    if isinstance(init[0], (np.ndarray, list)):
            fs.init_filter_state(init_s) for fs, init_s in zip(self.filters, init)
    else:
        return [fs.init_filter_state(init) for fs in self.filters]
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def estimate sequence(
   self,
    # A sequence of measurements
   Z: Sequence[np.ndarray],
    # the initial KF state to use for either prediction or update (see start with predict
    init_immstate: MixtureParameters,
    # Time difference between Z's. If start_with_prediction: also diff before the first Z
   Ts: Union[float, Sequence[float]],
   # An optional sequence of the sensor states for when Z was recorded
    sensor_state: Optional[Iterable[Optional[Dict[str, Any]]]] = None,
    # sets if Ts should be used for predicting before the first measurement in Z
   start_with_prediction: bool = False,
) -> Tuple[List[MixtureParameters], List[MixtureParameters], List[GaussParams]]:
    """Create estimates for the whole time series of measurements. """
    # sequence Length
    K = len(Z)
    # Create and amend the sampling array
   Ts_start_idx = int(not start_with_prediction)
    Ts_arr = np.empty(K)
   Ts_arr[Ts_start_idx:] = Ts
    # Insert a zero time prediction for no prediction equivalence
    if not start_with_prediction:
        Ts_arr[0] = 0
    # Make sure the sensor state list actually is a sequence
    sensor state seq = sensor state or [None] * K
    init immstate = self.init filter state(init immstate)
    immstate_upd = init_immstate
    immstate pred list = []
    immstate_upd_list = []
    estimates = []
    for z_k, Ts_k, ss_k in zip(Z, Ts_arr, sensor_state_seq):
        immstate_pred = self.predict(immstate_upd, Ts_k)
        immstate_upd = self.update(z_k, immstate_pred, sensor_state=ss_k)
        immstate pred list.append(immstate pred)
        immstate upd list.append(immstate upd)
        estimates.append(self.estimate(immstate upd))
    return immstate_pred_list, immstate_upd_list, estimates
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