

Reynold's Transport

$$\frac{1}{\mathcal{V}\sigma\ell} \frac{D\mathcal{V}\sigma\ell}{Dt} = \vec{\nabla} \cdot \vec{v} \quad \text{Rv}$$

$$\frac{1}{J} \frac{DJ}{Dt} = \vec{\nabla} \cdot \vec{v} \quad \text{RJ}$$

$$\frac{d}{dt} \iiint_{\mathcal{V}(t)} F d\mathcal{V} = \iiint_{\mathcal{V}(t)} \left[\frac{DF}{Dt} + F \vec{\nabla} \cdot \vec{v} \right] d\mathcal{V} = \iiint_{\mathcal{V}(t)} \left(\frac{\partial F}{\partial t} + \vec{\nabla} \cdot (F\vec{v}) \right) d\mathcal{V} \quad \text{Rf}$$

Cauchy Stress Tensor

(total = pressure + sticky)

$$\vec{\tau} = -P\vec{\mathbb{I}} + \vec{\sigma} \quad \text{Cs}$$

Continuity

$$\frac{D\rho}{Dt} + \rho \vec{\nabla} \cdot \vec{v} = 0 \quad \text{lc}$$

$$\frac{d}{dt} \iiint_{\mathcal{V}} \rho d\mathcal{V} = 0 \quad \text{Lc}$$

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v}) = 0 \quad \text{ec}$$

$$\iiint_{\mathcal{V}} \frac{\partial \rho}{\partial t} d\mathcal{V} + \iint_{\mathcal{S}} \rho \vec{v} \cdot \hat{n} d\mathcal{S} = 0 \quad \text{Ec}$$

Momentum

$$\rho \frac{D\vec{v}}{Dt} = \vec{\nabla} \cdot \vec{\tau} + \rho \vec{a}_{body} \quad \text{lm}$$

$$\frac{d}{dt} \iiint_{\mathcal{V}} \rho \vec{v} d\mathcal{V} = \iiint_{\mathcal{V}} \rho \vec{a}_{body} d\mathcal{V} + \iint_{\mathcal{S}} \vec{\tau} \cdot \hat{n} d\mathcal{S} \quad \text{Lm}$$

$$\frac{\partial \rho \vec{v}}{\partial t} + \vec{\nabla} \cdot (\rho \vec{v} \vec{v}) = \vec{\nabla} \cdot \vec{\tau} + \rho \vec{a}_{body} \quad \text{em}$$

$$\iiint_{\mathcal{V}} \frac{\partial (\rho \vec{v})}{\partial t} d\mathcal{V} + \iint_{\mathcal{S}} (\rho \vec{v}) \vec{v} \cdot \hat{n} d\mathcal{S} = \iiint_{\mathcal{V}} \rho \vec{a}_{body} d\mathcal{V} + \iint_{\mathcal{S}} \vec{\tau} \cdot \hat{n} d\mathcal{S} \quad \text{Em}$$

Total Energy

$$\rho \frac{D}{Dt} \left(e + \frac{V^2}{2} \right) = \rho (\vec{a}_{body} \cdot \vec{v} + \dot{Q}) + \vec{\nabla} \cdot (\vec{\tau} \cdot \vec{v} + \vec{q}) \quad \text{le}$$

$$\frac{d}{dt} \iiint_{\mathcal{V}} \rho \left(e + \frac{V^2}{2} \right) d\mathcal{V} = \iiint_{\mathcal{V}} \rho (\vec{a}_{body} \cdot \vec{v} + \dot{Q}) d\mathcal{V} + \iint_{\mathcal{S}} (\vec{\tau} \cdot \vec{v} + \vec{q}) \cdot \hat{n} d\mathcal{S} \quad \text{Le}$$

$$\frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] + \vec{\nabla} \cdot \left[\rho \left(e + \frac{V^2}{2} \right) \vec{v} \right] = \rho (\vec{a}_{body} \cdot \vec{v} + \dot{Q}) + \vec{\nabla} \cdot (\vec{\tau} \cdot \vec{v} + \vec{q}) \quad \text{ee}$$

$$\iiint_{\mathcal{V}} \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] d\mathcal{V} + \iint_{\mathcal{S}} \rho \left(e + \frac{V^2}{2} \right) \vec{v} \cdot \hat{n} d\mathcal{S} = \iiint_{\mathcal{V}} \rho (\vec{a}_{body} \cdot \vec{v} + \dot{Q}) d\mathcal{V} + \iint_{\mathcal{S}} (\vec{\tau} \cdot \vec{v} + \vec{q}) \cdot \hat{n} d\mathcal{S} \quad \text{Ee}$$

Internal Energy

$$\rho \frac{De}{Dt} = \rho \dot{Q} + \vec{\tau} : \nabla \vec{v} \quad \text{lie}$$

- \vec{a}_{body} - body accelerations (e.g. gravity or E&M)
- e - internal energy
- λ - second coefficient of viscosity
- μ - dynamic viscosity
- \hat{n} - outward surface normal
- P - pressure
- \dot{Q} - volumetric heat per mass
- \vec{q} - heat flux
- ρ - fluid density
- $\vec{\sigma}$ - Cauchy stress tensor (aka \vec{T})
- \mathcal{S} - surface
- $\vec{\tau}$ - deviatoric stress tensor
- T - temperature
- \mathcal{V} - volume
- \vec{v} - flow velocity

Movement from Lagrangian (particle) and Eulerian (flux) forms is done with the mnemonic:

Flux = U con + Particle

Equation Label Key for Fluid equations

- lower case first letter - differential form
- upper case first letter - integral form
- first letter 'l' or 'L' - Lagrangian or non-conservation form
- first letter 'e' or 'E' - Eulerian or conservation form
- second letter 'c', 'm', 'e', 'ie' for continuity, momentum, total energy, and internal energy respectively

Newtonian Fluids

$$\vec{\sigma} = \lambda (\vec{\nabla} \cdot \vec{v}) \vec{\mathbb{I}} + \mu (\vec{\nabla} \vec{v} + \vec{\nabla} \vec{v}^T) \quad \text{Nf}$$

Stokes's Hypothesis

$$\lambda = -\frac{2}{3}\mu \quad \text{Sh}$$

Incompressibility

$$\vec{\nabla} \cdot \vec{v} = 0 \quad \text{in}$$

Fourier's Law

$$\vec{q} = -k \nabla T \quad \text{Fl}$$

Thermodynamic Pressure

$$P_{thermo} = -\frac{1}{3} \text{Tr}(\vec{\tau}) = P - \left(\lambda + \frac{2}{3}\mu \right) (\vec{\nabla} \cdot \vec{v}) \quad \text{tp}$$