

Reynold's Transport

$$\frac{1}{\rho \sigma \ell} \frac{D V \sigma \ell}{D t} = \vec{V} \cdot \vec{V}$$

Rv

$$\frac{1}{J} \frac{D J}{D t} = \vec{V} \cdot \vec{V}$$

RJ

$$\frac{d}{dt} \iiint_{V(t)} F \, dV = \iiint_{V(t)} \left[\frac{DF}{Dt} + F \vec{V} \cdot \vec{V} \right] \, dV = \iiint_{V(t)} \left(\frac{\partial F}{\partial t} + \vec{V} \cdot (F \vec{V}) \right) \, dV$$

Rf

Cauchy Stress Tensor
(total = pressure + sticky)

$$\vec{\tau} = -P \mathbb{I} + \vec{\sigma}$$

Cs

Continuity

$$\frac{D\rho}{Dt} + \rho \vec{V} \cdot \vec{V} = 0$$

lc

$$\rho \frac{D\vec{V}}{Dt} = \vec{V} \cdot \vec{\tau} + \rho \vec{a}_{body}$$

lm

$$\frac{d}{dt} \iiint_V \rho \, dV = 0$$

Lc

$$\frac{d}{dt} \iiint_V \rho \vec{V} \, dV = \iiint_V \rho \vec{a}_{body} \, dV + \iint_S \vec{\tau} \cdot \hat{n} \, dS$$

Lm

$$\frac{\partial \rho}{\partial t} + \vec{V} \cdot (\rho \vec{V}) = 0$$

ec

$$\frac{\partial \rho \vec{V}}{\partial t} + \vec{V} \cdot (\rho \vec{V} \vec{V}) = \vec{V} \cdot \vec{\tau} + \rho \vec{a}_{body}$$

em

$$\iiint_V \frac{\partial \rho}{\partial t} \, dV + \iint_S \rho \vec{V} \cdot \hat{n} \, dS = 0$$

Ec

$$\iiint_V \frac{\partial(\rho \vec{V})}{\partial t} \, dV + \iint_S (\rho \vec{V}) \vec{V} \cdot \hat{n} \, dS = \iiint_V \rho \vec{a}_{body} + \iint_S \vec{\tau} \cdot \hat{n} \, dS$$

Em

Momentum

$$\rho \frac{D}{Dt} \left(e + \frac{V^2}{2} \right) = \rho (\vec{a}_{body} \cdot \vec{V} + \dot{Q}) + \vec{V} \cdot (\vec{\tau} \cdot \vec{V} + \vec{q})$$

le

$$\rho \frac{De}{Dt} = \rho \dot{Q} + \vec{\tau} : \nabla \vec{V}$$

lie

$$\frac{d}{dt} \iiint_V \rho \left(e + \frac{V^2}{2} \right) \, dV = \iiint_V \rho (\vec{a}_{body} \cdot \vec{V} + \dot{Q}) \, dV + \iint_S (\vec{\tau} \cdot \vec{V} + \vec{q}) \cdot \hat{n} \, dS$$

Le

$$\frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] + \vec{V} \cdot \left[\rho \left(e + \frac{V^2}{2} \right) \vec{V} \right] = \rho (\vec{a}_{body} \cdot \vec{V} + \dot{Q}) + \vec{V} \cdot (\vec{\tau} \cdot \vec{V} + \vec{q})$$

ee

$$\iiint_V \frac{\partial}{\partial t} \left[\rho \left(e + \frac{V^2}{2} \right) \right] \, dV + \iint_S \rho \left(e + \frac{V^2}{2} \right) \vec{V} \cdot \hat{n} \, dS = \iiint_V \rho (\vec{a}_{body} \cdot \vec{V} + \dot{Q}) \, dV + \iint_S (\vec{\tau} \cdot \vec{V} + \vec{q}) \cdot \hat{n} \, dS$$

Ee

- \vec{a}_{body} - body accelerations (e.g. gravity or E&M)
- e - internal energy
- λ - second coefficient of viscosity
- μ - dynamic viscosity
- \hat{n} - outward surface normal
- P - pressure
- Q - volumetric heat per mass
- \vec{q} - heat flux
- ρ - fluid density
- $\vec{\sigma}$ - Cauchy stress tensor (aka \vec{T})
- S - surface
- $\vec{\tau}$ - deviatoric stress tensor
- T - temperature
- V - volume
- \vec{V} - flow velocity

Movement from Lagrangian (particle) and Eulerian (flux) forms is done with the mnemonic:

Flux = U con + Particle

Newtonian Fluids

$$\vec{\sigma} = \lambda (\vec{V} \cdot \vec{V}) \mathbb{I} + \mu (\vec{V} \vec{V} + \vec{V} \vec{V}^T)$$

Nf

Stokes's Hypothesis

$$\lambda = -\frac{2}{3} \mu$$

Sh

$$P_{thermo} = -\frac{1}{3} \text{Tr}(\vec{\tau}) = P - \left(\lambda + \frac{2}{3} \mu \right) (\vec{V} \cdot \vec{V})$$

tp

Incompressibility

$$\vec{V} \cdot \vec{V} = 0$$

in

Fourier's Law

$$\vec{q} = -k \nabla T$$

Fl

Thermodynamic Pressure