This installment was intended to be a relaunching of the recent series of articles on continuum mechanics as a set of columns specialized to fluid mechanics. However, there is a nice presentation of the symmetries of the stress tensor that I developed that is worthy of some daylight. So, this month’s offering is a short but elegant rederivation of a coordinate-free of the stress tensor.

The starting point for the derivation is Cauchy’s tetrahedron,

<fig>

where three sides of the figure coincide with coordinate planes (i.e. $$x-y$$, $$y-z$$, and $$z-x$$ planes) and the fourth side closes the shape. Since we imagine the tetrahedron to be extremely small, the total force exerted on each face is well approximated as the stress at center of each face times the area of the face upon which it acts.

The condition of mechanical equilibrium is that the forces on each face balance:

\[ {\vec t}\_1 \delta S\_1 + {\vec t}\_2 \delta S\_2 + {\vec t}\_3 \delta S\_3 + {\vec t} \delta S = 0 \; . \]

By their orientation, the faces that coincide with a coordinate plane have unit normals that relate to the basis vectors as

\[ {\hat n}\_1 = - {\hat e}\_1 \; , \]vertigo

\[ {\hat n}\_2 = - {\hat e}\_2 \; , \]

and

\[ {\hat n}\_3 = - {\hat e}\_3 \; . \]

From basic geometry, the area of each of these planes are related to the area of the