

$$\cdot \underline{\left( \ddot{\vec{r}} + \frac{\mu \vec{r}}{r^3} \right) \times \vec{h} = 0}$$

expand :  $\ddot{\vec{r}} \times \vec{h} + \frac{\mu}{r^3} \vec{r} \times \vec{h} = 0$

recognize :  $\frac{d}{dt} (\dot{\vec{r}} \times \vec{h}) = \ddot{\vec{r}} \times \vec{h} + \dot{\vec{r}} \times \dot{\vec{h}} \leftarrow \vec{h} \text{ - conserved}$   
 $= \ddot{\vec{r}} \times \vec{h}$

recognize :  $\frac{\vec{r} \times \vec{h}}{r^3} = \frac{\vec{r} \times (\dot{\vec{r}} \times \vec{v})}{r^3}$   
 $\hookrightarrow \text{use BAC-CAB}$   
 $= \frac{\vec{r} (\dot{\vec{r}} \cdot \vec{v})}{r^3} - \frac{\vec{v} (\dot{\vec{r}} \cdot \vec{r})}{r^3}$   
 $= \frac{\vec{r} (\dot{\vec{r}} \cdot \vec{v})}{r^3} - \frac{\vec{v}}{r}$

recognize :  $\frac{1}{r} \frac{d}{dt} (\vec{r}) = \frac{\vec{v}}{r}$

recognize :  $-\vec{r} \frac{d}{dt} \left( \frac{1}{r} \right) = \frac{\vec{r} (\dot{\vec{r}} \cdot \vec{v})}{r^3}$

substitute :  $-\vec{r} \frac{d}{dt} \left( \frac{1}{r} \right) - \frac{1}{r} \frac{d}{dt} (\vec{r})$   
 $= -\frac{d}{dt} \left( \frac{\vec{r}}{r} \right)$

$$\Rightarrow \ddot{\vec{r}} \times \vec{h} + \frac{\mu}{r^3} \vec{r} \times \vec{h} = 0 \Rightarrow \frac{d}{dt} (\dot{\vec{r}} \times \vec{h}) - \frac{d}{dt} \left( \frac{\mu \vec{r}}{r} \right) = 0$$

$$\Rightarrow \frac{d}{dt} \left( \dot{\vec{r}} \times \vec{h} - \frac{\mu \vec{r}}{r} \right) = 0$$

$\dot{\vec{r}} \times \vec{h} - \frac{\mu \vec{r}}{r} \equiv \mu \vec{e} \quad \text{conserved quantity}$