Proof of E01 | CS | 1/5/08 | 1

We wish to prove

$$\varepsilon_{ijk} = J\,[ijk]$$

- Since $\varepsilon_{ijk} = (\underline{e}_i \times \underline{e}_j)\cdot\underline{e}_k$ it follows that

$\varepsilon_{ijk} = 0$ if $i=j$, $j=k$, or $k=i$ since it would always be possible to get a term like $\underline{e}_i \times \underline{e}_i$, which is identically 0. Thus

$$\varepsilon_{ijk} \sim [ijk]$$

- Now if $i \neq j \neq k$ then $\varepsilon_{123} = (\underline{e}_1 \times \underline{e}_2)\cdot\underline{e}_3$ can be written as

$$\begin{vmatrix} \underline{e}_x & \underline{e}_y & \underline{e}_z \\ [\underline{e}_1]_x & [\underline{e}_1]_y & [\underline{e}_1]_z \\ [\underline{e}_2]_x & [\underline{e}_2]_y & [\underline{e}_2]_z \end{vmatrix} \cdot \underline{e}_3 = \begin{vmatrix} \underline{e}_3\cdot\underline{e}_x & \underline{e}_3\cdot\underline{e}_y & \underline{e}_3\cdot\underline{e}_z \\ [\underline{e}_1]_x & [\underline{e}_1]_y & [\underline{e}_1]_z \\ [\underline{e}_2]_x & [\underline{e}_2]_y & [\underline{e}_2]_z \end{vmatrix}$$

$$= \begin{vmatrix} [\underline{e}_3]_x & [\underline{e}_3]_y & [\underline{e}_3]_z \\ [\underline{e}_1]_x & [\underline{e}_1]_y & [\underline{e}_1]_z \\ [\underline{e}_2]_x & [\underline{e}_2]_y & [\underline{e}_2]_z \end{vmatrix}$$

now switch rows 1 & 2 and then 2 & 3 and then transpose (which doesn't affect the determinant)