- Want the Fourier transform of a shifted signal

$$\mathcal{F}(\tau_{\mp b} f)(s) = \int_{-\infty}^{\infty} \tau_{\mp b} f(t)\, e^{-2\pi i s t}\, dt$$

$$= \int_{-\infty}^{\infty} f(t \mp b)\, e^{-2\pi i s t}\, dt$$

$$q = t \mp b \qquad dq = dt$$

$$= \int_{-\infty}^{\infty} f(q)\, e^{-2\pi i s (q \pm b)}\, dq$$

$$= \int_{-\infty}^{\infty} f(q)\, e^{-2\pi i s q}\, e^{\mp 2\pi i s b}\, dq$$

$$= e^{\mp 2\pi i s b} \int_{-\infty}^{\infty} f(q)\, e^{-2\pi i s q}\, dq$$

$$= e^{\mp 2\pi i s b}\, \mathcal{F} f$$

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