On the Differential Geometry of the RIC Frame

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This brief note is meant to lay out the differential geometry associated with the RIC frame. The unit vectors that comprise the frame are defined at each step along the object's trajectory in terms of its position $\vec{r}$ and velocity $\vec{v}$ as

$$\hat{R} = \frac{\vec{r}}{|\vec{r}|}\,,$$

$$\hat{C} = \frac{\vec{r}\times\vec{v}}{|\vec{r}\times\vec{v}|}\,,$$

and

$$\hat{I} = \hat{C}\times\hat{R}\,.$$

Since the objects position and velocity change as the object moves, the RIC unit vectors change their orientation as a function of time. In order to fully understand how these vectors change, it is neccessary to analyze their differential geometry. That said, the general structure can be deduced simply based on physical implications as follows.
Generally define

$$\hat{e}_1 \equiv \hat{R}\,,$$

$$\hat{e}_2 \equiv \hat{I}\,,$$

and

$$\hat{e}_3 \equiv \hat{C}\,.$$

With this convenient notation, the time derivatives of the dot-products between these vectors yields

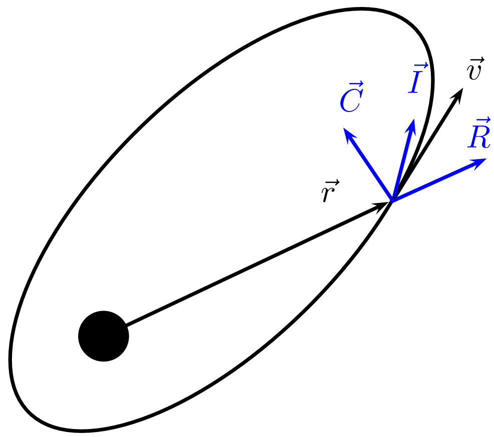
$$\frac{d}{dt}\left(\hat{e}_i\cdot\hat{e}_j\right) = \frac{d}{dt}\delta_{ij} = 0\,,$$

which breaks into two relations

$$\hat{e}_i\cdot\frac{d}{dt}\hat{e}_i = 0$$

and

$$\hat{e}_i\cdot\frac{d}{dt}\hat{e}_j = -\hat{e}_j\cdot\frac{d}{dt}\hat{e}_i\,.$$



Applying these two relations across the RIC frame yields the coupled equations

$$\frac{d}{dt}\begin{bmatrix}\hat{R}\\ \hat{I}\\ \hat{C}\end{bmatrix} = \begin{bmatrix}0 & \alpha & 0\\ -\alpha & 0 & \beta\\ 0 & -\beta & 0\end{bmatrix}\begin{bmatrix}\hat{R}\\ \hat{I}\\ \hat{C}\end{bmatrix},$$

where

$$\alpha = \hat{I}\cdot\frac{d}{dt}\hat{R}$$

and

$$\beta = \hat{C}\cdot\frac{d}{dt}\hat{I}\,.$$

$$\frac{d}{dt}\frac{\vec{r}}{|\vec{r}|} = \frac{\vec{v}}{|\vec{r}|} - \frac{\vec{r}\vec{v}\cdot\vec{r}}{|\vec{r}|^3}$$

$$\hat{I}\cdot\hat{R} = (\hat{C}\times$$