Special functional: $\phi'(x)[\phi(y)] = \int dy\, \phi'(y)\, \delta(x-y)$

$$\frac{\delta\phi'(x)[\phi(y)]}{\delta\phi(z)} = \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\Big\{ \int dy \frac{d}{dy}\left[\phi(y)+\varepsilon\,\delta(z-y)\right]\delta(x-y) - \int dy \left[\frac{d}{dy}\phi(y)\right]\delta(x-y) \Big\}$$

$$= \lim_{\varepsilon\to 0} \frac{1}{\varepsilon}\Big\{ \int dy \left[\frac{d}{dy}\phi(y)\right]\delta(x-y) + \int dy\, \varepsilon\left[\frac{d}{dy}\delta(z-y)\right]\delta(x-y) - \int dy \left[\frac{d}{dy}\phi(y)\right]\delta(x-y) \Big\}$$

$$= \lim_{\varepsilon\to 0} \int dy \left[\frac{d}{dy}\delta(z-y)\right]\delta(x-y)$$

$$= \frac{d}{dx}\delta(z-x) = \frac{d}{dx}\delta(x-z)$$

dropping the '$[\phi(y)]$' we get

$$\frac{\delta\phi'(x)}{\delta\phi(z)} = \frac{d}{dx}\delta(x-z)$$