FT18: Proof
$(k*g)*f = k*(g*f)$

6/18/11

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- Want to prove that convolution is associative

$$(k*g)*f = \int_{-\infty}^{\infty} (k*g)(t-\tau)\, f(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} k(a-\alpha)\, g(\alpha)\, d\alpha \right]_{a=t-\tau} f(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} d\tau \left[\int_{-\infty}^{\infty} d\alpha \;\, k(t-\tau-\alpha)\, g(\alpha) \right] f(\tau)$$

$$\alpha = q - \tau \qquad d\alpha = dq$$

$$= \int_{-\infty}^{\infty} d\tau \left[\int_{-\infty}^{\infty} dq \;\, k(t-q)\, g(q-\tau) \right] f(\tau)$$

$$= \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} dq \;\, k(t-q)\, g(q-\tau)\, f(\tau)$$

$$= \int_{-\infty}^{\infty} dq \;\, k(t-q) \int_{-\infty}^{\infty} g(q-\tau)\, f(\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} dq \; k(t-q)\; (g*f)(q)$$

$$= k*(g*f)$$