

• We wish to prove the identity $\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) = 0$

$$\begin{aligned}\vec{\nabla} \cdot (\vec{\nabla} \times \vec{F}) &= \partial_i \epsilon_{ijk} \partial_j F_k \\&= \epsilon_{ijk} \partial_i \partial_j F_k \\&= \frac{1}{2} (\epsilon_{ijk} \partial_i \partial_j F_k + \epsilon_{ikj} \partial_i \partial_j F_k) \\&= \frac{1}{2} (\epsilon_{ijk} \partial_i \partial_j F_k + \epsilon_{jik} \partial_j \partial_i F_k) \\&= \frac{1}{2} (\epsilon_{ijk} \partial_i \partial_j F_k + \epsilon_{jik} \partial_i \partial_j F_k) \\&= \frac{1}{2} (\epsilon_{ijk} - \epsilon_{ijk}) \partial_i \partial_j F_k \\&= 0\end{aligned}$$