Start with the product of two Fourier Transforms

$$\mathcal{F}g(s)\,\mathcal{F}f(s) = \int_{-\infty}^{\infty} g(t) e^{-2\pi i s t}\,dt \int_{-\infty}^{\infty} f(\tau) e^{-2\pi i s \tau}\,d\tau$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t) f(\tau) e^{-2\pi i s (t+\tau)}\,dt\,d\tau$$

$$= \int_{-\infty}^{\infty} d\tau\, f(\tau) \left[\int_{-\infty}^{\infty} g(t) e^{-2\pi i s (t+\tau)}\,dt \right]$$

$q = t + \tau \qquad dq = dt$ inner integral only

$$= \int_{-\infty}^{\infty} d\tau\, f(\tau) \int_{-\infty}^{\infty} g(q-\tau) e^{-2\pi i s q}\,dq$$

$$= \int_{-\infty}^{\infty} dq\, e^{-2\pi i s q} \int_{-\infty}^{\infty} d\tau\, g(q-\tau) f(\tau)$$

$$= \int_{-\infty}^{\infty} dq\, e^{-2\pi i s q} (g*f)(q)$$

rename dummy

$$= \int_{-\infty}^{\infty} (g*f)(t)\, e^{-2\pi i s t}\,dt$$

$$= \mathcal{F}(g*f)(s)$$