- We wish to prove $\int \vec{\nabla} \times \vec{F}\, d\tau = \int (\hat{n} \times \vec{F})\, dS$

Start with a vector $\vec{G} = \vec{A} \times \vec{F}$ where $\vec{A}$ is a constant vector.
Now apply the divergence theorem to $\vec{G}$

$$\int \vec{\nabla} \cdot \vec{G}\, d\tau = \int \vec{G} \cdot \hat{n}\, dS$$

Now substitute in for $\vec{G}$ and expand the LHS

$$\int \vec{\nabla} \cdot (\vec{A} \times \vec{F})\, d\tau = -\int \vec{A} \cdot (\vec{\nabla} \times \vec{F})\, d\tau \quad \text{(using D◊8)}$$

similarly the expansion of the RHS yields

$$\int \vec{G} \cdot \hat{n}\, dS = \int (\vec{A} \times \vec{F}) \cdot \hat{n}\, dS = \int (\vec{F} \times \hat{n}) \cdot \vec{A}\, dS$$

Equating both sides

$$-\int \vec{A} \cdot (\vec{\nabla} \times \vec{F})\, d\tau = \int \vec{A} \cdot (\vec{F} \times \hat{n})\, dS$$

or

$$\vec{A} \cdot \int (\vec{\nabla} \times \vec{F})\, d\tau = \vec{A} \cdot \int (\hat{n} \times \vec{F})\, dS$$

Re-arranging yields

$$\vec{A} \cdot \left[\int (\vec{\nabla} \times \vec{F})\, d\tau - \int (\hat{n} \times \vec{F})\, dS \right] = 0$$

Since $\vec{A}$ is arbitrary

$$\int \vec{\nabla} \times \vec{F}\, d\tau = \int (\hat{n} \times \vec{F})\, dS$$