• We wish to prove $S\vec{v}\phi \cdot \vec{F} dV = S\phi \vec{F} \cdot \hat{n} dS - S\phi \vec{v} \cdot \vec{F} dV$ Let $\vec{Q} = \phi \vec{F}$. Then $\vec{v} \cdot \vec{Q} = \phi \vec{v} \cdot \vec{F} + \vec{v}\phi \cdot \vec{F}$. But from the divergence theorem:

St. $\vec{a} dv = S\vec{a} \cdot \vec{n} ds$ Plugging in the expansion of $\vec{v} \cdot \vec{a}$ we get $S(\vec{p} \vec{v} \cdot \vec{r} + \nabla \vec{p} \cdot \vec{r}) dv = S \vec{p} \vec{r} \cdot \hat{n} ds$

or solving for the Dp term

Proof of IUS

Sup. Fdv = SpF. ads - Spr. Fdv