- We wish to show that in Cartesian coordinates there is $\vec{\nabla} = \partial_x \hat{\imath} + \partial_y \hat{\jmath} + \partial_z \hat{k}$ as an operator such that $div(\vec{F}) = \nabla \cdot \vec{F}$, $grad\,\phi \doteq \nabla\phi$, and $curl\{\vec{F}\} = \vec{\nabla} \times \vec{F}$

Start with $\nabla\phi$; in Cartesian coordinates $h_i = 1$

$$\nabla\phi \doteq (\partial_x \hat{\imath} + \partial_y \hat{\jmath} + \partial_z \hat{k})\,\phi$$ so this checks

Next check $div(\vec{F})$

$$div(\vec{F}) = \sum_i \partial_{q_i} F_i = \nabla \cdot \vec{F}$$

Finally $$curl(\vec{F}) = \sum_{ijk} \hat{e}_i \, \varepsilon_{ijk} \frac{\partial}{\partial q_j} F_k$$

$$= \vec{\nabla} \times \vec{F}$$