

• Fourier transform of a shifted tempered dist.

$$\begin{aligned}\langle \mathcal{F}(\tau_{\pm b} \mathcal{T}), \varphi \rangle &= \langle \tau_{\pm b} \mathcal{T}, \widehat{\mathcal{F}} \varphi \rangle \\ &= \langle \mathcal{T}, \tau_{\mp b} \widehat{\mathcal{F}} \varphi \rangle \\ &= \langle \mathcal{T}, \widehat{\mathcal{F}}(\varphi e^{\mp 2\pi i b t}) \rangle \\ &= \langle \widehat{\mathcal{F}} \mathcal{T}, \varphi e^{\mp 2\pi i b t} \rangle \\ &= \langle e^{\mp 2\pi i b x} \mathcal{T}, \varphi \rangle\end{aligned}$$

$$\Rightarrow \mathcal{F}(\tau_{\pm b} \mathcal{T}) = e^{\mp 2\pi i b x} \mathcal{T}$$