Proof of E04 | CJ | 12/26/07

We wish to prove

$$\varepsilon^{ijk}\varepsilon_{pqk} = (\delta^i_p\,\delta^j_q - \delta^i_q\,\delta^j_p)$$

- Start from

$$\varepsilon^{ijk}\varepsilon_{pqr} = \begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix}$$

expand the determinant

$$\delta^i_p(\delta^j_q\delta^k_r - \delta^j_r\delta^k_q) + \delta^i_q(\delta^j_r\delta^k_p - \delta^j_p\delta^k_r)$$
$$+\,\delta^i_r(\delta^j_p\delta^k_q - \delta^j_q\delta^k_p)$$

set $i = p$ $\quad(\delta^i_i = 3)$

$$3(\delta^j_q\delta^k_r - \delta^j_r\delta^k_q) + \delta^i_q(\delta^j_r\delta^k_i - \delta^j_i\delta^k_r)$$
$$+\,\delta^p_r(\delta^j_p\delta^k_q - \delta^j_q\delta^k_p)$$

$$= 3(\delta^j_q\delta^k_r - \delta^j_r\delta^k_q) + (\delta^j_r\delta^k_q - \delta^j_q\delta^k_r)$$
$$+\,(\delta^j_r\delta^k_q - \delta^j_q\delta^k_r)$$

$$= 3(\delta^j_q\delta^k_r - \delta^j_r\delta^k_q) - 2(\delta^j_q\delta^k_r - \delta^j_r\delta^k_q)$$

$$= \delta^j_q\delta^k_r - \delta^j_r\delta^k_q$$