- Want to determine the action of the shift operator on distributional pairing.

$$\langle \tau_{\pm b} f, \varphi \rangle = \int_{-\infty}^{\infty} (\tau_{\pm b} f)(x)\, \varphi(x)\, dx$$

$$= \int_{-\infty}^{\infty} f(x \mp b)\, \varphi(x)\, dx$$

$$q = x \mp b \qquad dq = dx$$

$$= \int_{-\infty}^{\infty} f(q)\, \varphi(q \pm b)\, dq$$

$$= \int_{-\infty}^{\infty} f(q)\, \tau_{\mp b} \varphi(q)\, dq$$

$$= \langle f, \tau_{\mp b} \varphi \rangle$$