- Start with the canonical form of a linear 2^{nd} order ODE

$$y''(x) + P(x)\,y'(x) + Q(x)\,y(x) = 0$$

- Denote the two trial solutions as $y_1(x)$ & $y_2(x)$ and construct the Wronskian

$$W[y_1(x), y_2(x)] = y_1(x)\,y_2'(x) - y_2(x)\,y_1'(x)$$

- Next take the derivative of W

$$\frac{dW}{dx} = y_1'(x)\,y_2'(x) + y_1(x)\,y_2''(x) - y_2'(x)\,y_1'(x) - y_2(x)\,y_1''(x)$$

$$= y_1(x)\,y_2''(x) - y_2(x)\,y_1''(x)$$

- Now solve the ODE for the second derivative and substitute in

$$\frac{dW}{dx} = y_1(x)\left[-P(x)\,y_2'(x) - Q(x)\,y_2(x)\right] - y_2(x)\left[-P(x)\,y_1'(x) - Q(x)\,y_1(x)\right]$$

$$= -P(x)\left[y_1(x)\,y_2'(x) - y_2(x)\,y_1'(x)\right] - Q(x)\left[y_1(x)\,y_2(x) - y_2(x)\,y_1(x)\right]$$

$$= -P(x)\,W(x)$$

$$\boxed{\frac{dW}{dx} + P(x)\,W(x) = 0}$$

- Now solve the first order equation for $W(x)$

$$\boxed{W(x) = W(x_0)\exp\left\{-\int_{x_0}^{x} P(\xi)\,d\xi\right\}}$$

- Since the exponential factor can never be zero, the quality of the Wronskian is governed by $W(x_0)$.

- If $W(x_0) = 0$ then $y_1(x_0)\,y_2'(x_0) = y_2(x_0)\,y_1'(x_0)$

or $y_1(x_0)/y_1'(x_0) = y_2(x_0)/y_2'(x_0)$

which implies that $y_1(x_0) = k\,y_2(x_0)$ & $y_1'(x_0) = k\,y_2'(x_0)$ which in turn means that $y_1(x)$ & $y_2(x)$ are linearly dependent.