• We wish to expand $\vec{\nabla}(\vec{F}\cdot\vec{G})$ so that the vector operator operates only on one vector. This is the equivalent of the Leibnizt relation.

$$\vec{\nabla}(\vec{F}\cdot\vec{G}) = \vec{\nabla}(F_i G_i) = \partial_j (F_i G_i)$$

$$= G_i \partial_j F_i + F_i \partial_j G_i$$

$$\{G_i \partial_j F_i\}_x = G_x \partial_x F_x + G_y \partial_x F_y + G_z \partial_x F_z$$

$$= G_x \partial_x F_x + G_y \partial_y F_x + G_z \partial_z F_x$$

$$- G_y \partial_y F_x - G_z \partial_z F_x + G_y \partial_x F_y + G_z \partial_x F_z$$

$$= (\vec{G}\cdot\vec{\nabla}) F_x + G_y (\partial_x F_y - \partial_y F_x) + G_z (\partial_x F_z - \partial_z F_x)$$

$$= (\vec{G}\cdot\vec{\nabla}) F_x + G_y \{\vec{\nabla}\times\vec{F}\}_z - G_z \{\vec{\nabla}\times\vec{F}\}_y$$

$$= (\vec{G}\cdot\vec{\nabla}) F_x + \{\vec{G}\times(\vec{\nabla}\times\vec{F})\}_x$$

$$\Rightarrow \boxed{\vec{\nabla}(\vec{F}\cdot\vec{G}) = (\vec{G}\cdot\vec{\nabla})\vec{F} + \vec{G}\times(\vec{\nabla}\times\vec{F}) + (\vec{F}\cdot\vec{\nabla})\vec{G} + \vec{F}\times(\vec{\nabla}\times\vec{G})}$$

• We wish to expand $\vec{\nabla}(\vec{F}\cdot\vec{G})$ so that the vector operator acts only on one vector. This is the equivalent of the Leibniz relation.

$$\vec{\nabla}(\vec{F}\cdot\vec{G}) = G_i\partial_j F_i + F_i\partial_j G_i$$

$$= G_i\partial_j F_i + G_j\partial_j F_i + F_i\partial_j G_i + F_j\partial_j G_i$$

$$- G_j\partial_j F_i - F_j\partial_j G_i$$

$$(\vec{G}\cdot\vec{\nabla})\vec{F} + \vec{G}\times(\vec{\nabla}\times\vec{F}) \doteq G_j\partial_j F_i + \varepsilon_{ijk} G_j \{\vec{\nabla}\times\vec{F}\}_k$$

$$= G_j\partial_j F_i + \varepsilon_{ijk} G_j \varepsilon_{klm}\partial_l F_m$$

$$= G_j\partial_j F_i + \varepsilon_{ijk}\varepsilon_{lmk} G_j\partial_l F_m$$

$$= G_j\partial_j F_i + (\delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}) G_j\partial_l F_m$$

$$= G_j\partial_j F_i + G_j\partial_i F_j - G_j\partial_j F_i$$

$$= G_j\partial_i F_j$$