• We wish to prove Green's 2nd theorem

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dv = \int_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n}\, dS$$

Start with Green's 1st theorem and subtract the same with $\phi \leftrightarrow \psi$

$$\int_V (\phi \nabla^2 \psi + \nabla \phi \cdot \nabla \psi)\, dv - \int_V (\psi \nabla^2 \phi + \nabla \psi \cdot \nabla \phi)\, dv$$

$$= \int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dv$$

but the LHS is also written as

$$\int_S \phi \nabla \psi \cdot \hat{n}\, dS - \int_S \psi \nabla \phi \cdot \hat{n}\, dS = \int_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n}\, dS$$

Thus

$$\int_V (\phi \nabla^2 \psi - \psi \nabla^2 \phi)\, dv = \int_S (\phi \nabla \psi - \psi \nabla \phi) \cdot \hat{n}\, dS$$