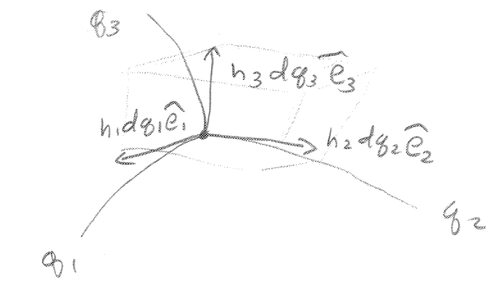
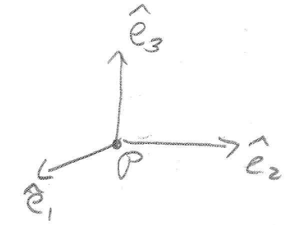
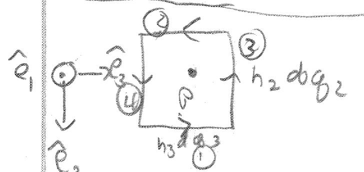
• We wish to show that $\text{curl}(\vec{F}) = \frac{1}{\Omega}\sum_{ijk} \hat{e}_i \, \varepsilon_{ijk} \, h_i \frac{\partial}{\partial q_j}(h_k F_k)$
where $\Omega = h_1 h_2 h_3 \quad h_i = |\hat{e}_i|$





Volume @ $P = \Omega \, dq_1 \, dq_2 \, dq_3$

$$\boxed{\text{curl}(\vec{F}) \cdot \hat{n}_C = \lim_{S \to 0} \frac{1}{S} \int_C \vec{F} \cdot \hat{t} \, dl}$$

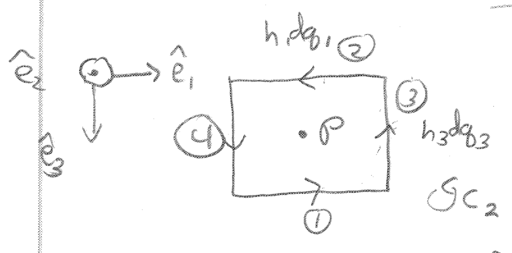


$\hat{e}_1$ component

Side ①: $\hat{t} = \hat{e}_3 \qquad \vec{F}\cdot\hat{t}\,dl = F_3 h_3 (q_1, q_2 + \frac{dq_2}{2}, q_3)\, dq_3$
Side ②: $\hat{t} = -\hat{e}_3 \qquad \vec{F}\cdot\hat{t}\,dl = -F_3 h_3 (q_1, q_2 - \frac{dq_2}{2}, q_3)\, dq_3$
Side ③: $\hat{t} = -\hat{e}_2 \qquad \vec{F}\cdot\hat{t}\,dl = F_2 h_2 (q_1, q_2, q_3 + \frac{dq_3}{2})\, dq_2$
Side ④: $\hat{t} = \hat{e}_2 \qquad \vec{F}\cdot\hat{t}\,dl = F_2 h_2 (q_1, q_2, q_3 - \frac{dq_3}{2})\, dq_2$

$$\int_{C_1} \vec{F}\cdot\hat{t}\,dl = \left(F_3 h_3 (q_2 + 1/2) - F_3 h_3 (q_2 - 1/2)\right) dq_3$$
$$+ \left(-F_2 h_2 (q_3 + 1/2) + F_2 h_2 (q_3 - 1/2)\right) dq_2$$
$$= \left(\frac{\partial}{\partial q_2}(F_3 h_3) - \frac{\partial}{\partial q_3}(F_2 h_2)\right) dq_2 \, dq_3$$

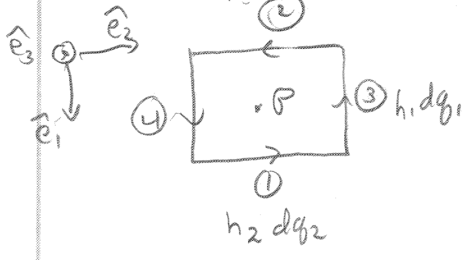
$\hat{e}_2$ component



$$\int_{C_2} \vec{F}\cdot\hat{t}\,dl = \left(\overset{①}{F_1 h_1 (q_3 + 1/2)} - \overset{②}{F_1 h_1 (q_3 - 1/2)}\right) dq_1$$
$$+ \left(\underset{③}{-F_3 h_3 (q_1 + 1/2)} + \underset{④}{F_3 h_3 (q_1 - 1/2)}\right) dq_3$$

$$\int_{C_2} \vec{F}\cdot\hat{t}\,dl = \left(\frac{\partial}{\partial q_3}(F_1 h_1) - \frac{\partial}{\partial q_1}(F_3 h_3)\right) dq_1 \, dq_3$$

$\hat{e}_3$ component



$$\int_{C_3} \vec{F}\cdot\hat{t}\,dl = \left(F_2 h_2 (q_1 + 1/2) - F_2 h_2 (q_1 - 1/2)\right) dq_2$$
$$+ \left(-F_1 h_1 (q_2 + 1/2) + F_1 h_1 (q_2 - 1/2)\right) dq_1$$

$$\int_C \vec{F}\cdot\hat{t}\,dl = \left(\frac{\partial}{\partial q_1}(F_2 h_2) - \frac{\partial}{\partial q_2}(F_1 h_1)\right) dq_1 \, dq_2$$

Now we simply need to 'pack' these formulae into a compact form:

$$\vec{\nabla}\times\vec{F} = \left(\frac{\partial}{\partial q_2}(F_3h_3) - \frac{\partial}{\partial q_3}(F_2h_2)\right)\hat{e}_1/h_2h_3$$

$$+\left(\frac{\partial}{\partial q_3}(F_1h_1) - \frac{\partial}{\partial q_1}(F_3h_3)\right)\hat{e}_2/h_3h_1$$

$$+\underbrace{\left(\frac{\partial}{\partial q_1}(F_2h_2) - \frac{\partial}{\partial q_2}(F_1h_1)\right)}_{\sigma_i}\hat{e}_3/h_1h_2$$

$$= h_1\sigma_1\hat{e}_1/\Omega + h_2\sigma_2\hat{e}_2/\Omega + h_3\sigma_3\hat{e}_3/\Omega$$

$$\sigma_i = \sum_{jk}\varepsilon_{ijk}\,\partial q_j(F_kh_k)$$

$$\sigma_1 = \varepsilon_{123}\,\partial q_2(F_3h_3) + \varepsilon_{132}\,\partial q_3(F_2h_2)$$

$$= \partial q_2(F_3h_3) - \partial q_3(F_2h_2)$$

$$\boxed{\vec{\nabla}\times\vec{F} = \sum_i \frac{h_i\hat{e}_i}{\Omega}\sum_{jk}\varepsilon_{ijk}\,\partial q_j(F_kh_k)}$$