- We wish to prove the operator identity

$$\int dl\, \hat{t} \circ q \doteq \int ds\, (\hat{n} \times \vec{\nabla}) \circ q$$

where '$\circ$' is any allowed product (ordinary, dot, or cross) and q is a compatible scalar or vector.

a) From Stokes theorem ( )

$$\int dl\, \hat{t} \cdot \vec{F} = \int ds\, (\hat{n} \times \vec{\nabla}) \cdot \vec{F}$$

but $(\hat{n} \times \vec{\nabla}) \cdot \vec{F} = \varepsilon_{ijk}\, n_i\, \partial_j F_k = \hat{n} \cdot (\vec{\nabla} \times \vec{F})$

$$\int dl\, \hat{t} \cdot \vec{F} = \int ds\, \hat{n} \cdot (\vec{\nabla} \times \vec{F})$$

b) From I06

$$\int dl\, \hat{t}\, \phi = \int ds\, (\hat{n} \times \nabla\phi) = \int ds (\hat{n} \times \vec{\nabla})\phi$$

c) From I07

$$\int dl\, \hat{t} \times \vec{F} = \int ds\, (\hat{n} \times \vec{\nabla}) \times \vec{F}$$

QED