- We are really defining the Laplacian as $\nabla^2 = \partial_x^2 + \partial_y^2 + \partial_z^2$ and wish to show that this is the same as $\vec{\nabla} \cdot (\vec{\nabla})$

$$\vec{\nabla}\phi = \partial_x \phi \hat{i} + \partial_y \phi \hat{j} + \partial_z \phi \hat{k} = \partial_i \phi$$

$$\vec{\nabla} \cdot (\vec{\nabla}\phi) = \partial_i \partial_i \phi = (\partial_x^2 + \partial_y^2 + \partial_z^2)\phi$$

this uses NΦ8

We wish to use NΦ4 and NΦ6, to show that NΦ7 results, which effectively shows

$$\nabla^2 = \nabla \cdot \nabla$$

in orthogonal coordinates

$$\vec{\nabla} \cdot \vec{\nabla} \phi = \sum_i \frac{1}{\Omega} \frac{\partial}{\partial q^i} \left(\frac{\Omega}{h_i} \frac{1}{h_i} \frac{\partial}{\partial q^i} \phi \right) = \frac{1}{\Omega} \sum_i \frac{\partial}{\partial q^i} \left(\frac{\Omega}{h_i^2} \frac{\partial}{\partial q^i} \phi \right)$$

which is just NΦ7.