- First look the classic FT

$$\mathcal{F}(af+bg)(s) = \int_{-\infty}^{\infty} (af+bg)(t)\, e^{-2\pi i s t}\, dt$$

$$= \int_{-\infty}^{\infty} (af(t)+bg(t))\, e^{-2\pi i s t}\, dt$$

$$= a\int_{-\infty}^{\infty} f(t)\, e^{-2\pi i s t}\, dt + b\int_{-\infty}^{\infty} g(t)\, e^{-2\pi i s t}\, dt$$

$$= a\mathcal{F}f(s) + b\mathcal{F}g(s)$$

or

$$\mathcal{F}(af+bg) = a\mathcal{F}f + b\mathcal{F}g$$

- Now looking at distributions

$$\mathcal{F}(aT+bS)$$

$$\langle \mathcal{F}(aT+bS), \varphi \rangle = \langle aT+bS, \mathcal{F}\varphi \rangle$$

$$= a\langle T, \mathcal{F}\varphi \rangle + b\langle S, \mathcal{F}\varphi \rangle$$

$$= a\langle \mathcal{F}T, \varphi \rangle + b\langle \mathcal{F}S, \varphi \rangle$$

$$= \langle a\mathcal{F}T + b\mathcal{F}S, \varphi \rangle$$

$$\Rightarrow \mathcal{F}(aT+bS) = a\mathcal{F}T + b\mathcal{F}S$$