• We wish to prove $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$

$$\vec{\nabla} \times (\vec{\nabla}\phi) = \varepsilon_{ijk}\, \partial_j \partial_k \phi$$

$$= \tfrac{1}{2}(\varepsilon_{ijk}\, \partial_j \partial_k + \varepsilon_{ijk}\, \partial_j \partial_k)\, \phi$$

$$= \tfrac{1}{2}(\varepsilon_{ijk}\, \partial_j \partial_k + \varepsilon_{ikj}\, \partial_k \partial_j)\, \phi \quad \text{relabel}$$

$$= \tfrac{1}{2}(\varepsilon_{ijk}\, \partial_j \partial_k - \varepsilon_{ijk}\, \partial_j \partial_k)\, \phi \quad \text{permute}$$

$$= 0$$

- We wish to prove $\vec{\nabla} \times (\vec{\nabla}\phi) = 0$ using Nφ4 & Nφ5

$$\vec{\nabla} \times (\vec{\nabla}\phi) = \frac{1}{\Omega}\sum_{ijk} \vec{e}_i\,[ijk]\,h_i \frac{\partial}{\partial q^j}\left(h_k \frac{1}{h_k} \frac{\partial}{\partial q^i}\phi\right)$$

$$= \frac{1}{\Omega}\sum_{ijk} \vec{e}_k\,[ijk]\,h_i \frac{\partial^2}{\partial q^i \partial q^j}\phi$$

since $[ijk]$ is anti-symmetric in i,j and $\frac{\partial^2 \phi}{\partial q^i \partial q^j}$ is clearly symmetric in i,j then the contraction is zero