- We wish to prove $\int \nabla\phi \, d\nu = \int \phi \, \hat{n} \, dS$

Start with a vector $\vec{F} = \phi \vec{A}$ where $\vec{A}$ is constant vector.

$$\int \vec{\nabla} \cdot \vec{F} \, d\nu = \int \vec{F} \cdot \hat{n} \, dS$$

then substituting in for $\vec{F}$

$$\int \vec{\nabla} \cdot (\phi \vec{A}) \, d\nu = \int \phi \vec{A} \cdot \hat{n} \, dS$$

Expanding and rearranging yields

$$\vec{A} \cdot \left[\int \vec{\nabla}\phi \, d\nu - \int \phi \hat{n} \, dS \right] = 0$$

Since $\vec{A}$ is arbitrary

$$\int \vec{\nabla}\phi \, d\nu = \int \phi \hat{n} \, dS$$