- What is the Fourier transform of the derivative of a function f?
  - Start with the identity

$$f(t) = \mathcal{F}^{-1}[\mathcal{F}f(s)]$$

$$= \int_{-\infty}^{\infty} \mathcal{F}f(s)\, e^{2\pi i s t}\, ds$$

- Now take the derivative

$$\frac{d}{dt} f(t) = \frac{d}{dt} \int_{-\infty}^{\infty} \mathcal{F}f(s)\, e^{2\pi i s t}\, ds$$

$$= \int_{-\infty}^{\infty} \mathcal{F}f(s) \frac{d}{dt} e^{2\pi i s t}\, ds$$

$$= \int_{-\infty}^{\infty} \mathcal{F}f(s)\, 2\pi i s\, e^{2\pi i s t}\, ds$$

$$= \int_{-\infty}^{\infty} (2\pi i s\, \mathcal{F}f(s))\, e^{2\pi i s t}\, ds$$

$$= \mathcal{F}^{-1}(2\pi i s\, \mathcal{F}f(s))$$

$$\mathcal{F}f'(s) = 2\pi i s\, \mathcal{F}f(s)$$