We wish to prove

$$\varepsilon^{ijk}\varepsilon_{pqr} = \begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix}$$

- First set $(i,j,k) = (p,q,r) = (1,2,3)$

  $\varepsilon^{123}\varepsilon_{123} = 1$ by definition

$$\begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1$$

so the identity holds in this case

- Now note that permutations of the lists (i,j,k) & (p,q,r) is handled by the determinant.

- Now suppose $i = j \neq k$ and $p \neq q \neq r$. Without loss of generality, let $i = j = p$, then

$$\begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} \text{ or } \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$= 0$ since there is a zero column in both cases

this holds if (i,j,k) and (p,q,r) are interchanged.

- Now suppose $i = j \neq k$ $p = q \neq r$. Without loss of generality, let $i = j = p = q$, then

$$\begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_p & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix} = \overset{k=r}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}} \text{ or } \overset{k\neq r}{\begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}} = 0$$

- Now suppose $i = j = k = p$ $\quad p \neq q \neq r$

$$\begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{vmatrix} = 0$$

- Now suppose $i = j = k = p$ $\quad p = q \neq r$

$$\begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix} = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} = 0$$

- Now suppose $i = j = k = p$ $\quad p = q = r$

$$\begin{vmatrix} \delta^i_p & \delta^i_q & \delta^i_r \\ \delta^j_p & \delta^j_q & \delta^j_r \\ \delta^k_p & \delta^k_q & \delta^k_r \end{vmatrix} = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 0$$

- QED