- How does the Fourier Transform change if the time domain is scaled $t \to at$

$$\mathcal{F}[f(at)](s) = \int_{-\infty}^{\infty} f(at)\, e^{-2\pi i s t}\, dt \qquad a > 0$$

$$t = \frac{q}{a} \qquad dt = \frac{dq}{a}$$

$$= \int_{-\infty}^{\infty} f(q)\, e^{-2\pi i s q/a}\, \frac{dq}{a}$$

$$= \frac{1}{a} \mathcal{F} f\left(\frac{s}{a}\right)$$

and

$$\mathcal{F}[f(at)](s) = \int_{-\infty}^{\infty} f(at)\, e^{-2\pi i s t}\, dt \qquad a < 0$$

$$t = \frac{q}{a} \qquad dt = \frac{dq}{a}$$

$$= \int_{\infty}^{-\infty} f(q)\, e^{-2\pi i s q/a}\, \frac{dq}{a}$$

$$= -\frac{1}{a} \int_{-\infty}^{\infty} f(q)\, e^{-2\pi i s/a\, q}\, dq$$

$$= -\frac{1}{a} \mathcal{F} f(s/a)$$

$$\mathcal{F}[f(at)](s) = \frac{1}{|a|} \mathcal{F} f\left(\frac{s}{a}\right)$$