

Special functional:  $\phi(x)[\phi(y)] = \int dy \phi(y) \delta(x-y)$

$$\frac{\delta \phi(x)[\phi(y)]}{\delta \phi(z)} = \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \left\{ \int dy [\phi(y) + \varepsilon \delta(z-y)] \delta(x-y) - \int dy \phi(y) \delta(x-y) \right\}$$

$$= \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} \left\{ \int dy \phi(y) \delta(x-y) + \varepsilon \int dy \delta(z-y) \delta(x-y) - \int dy \phi(y) \delta(x-y) \right\}$$

$$= \lim_{\varepsilon \rightarrow 0} \int dy \delta(z-y) \delta(x-y)$$

$$= \delta(z-x) = \delta(x-z)$$

dropping the ' $[\phi(y)]$ ' we get

$$\boxed{\frac{\delta \phi(x)}{\delta \phi(z)} = \delta(x-z)}$$