Algebraic Laws for Regular Expressions

- Associativity
  - $(L + M) + N = L + (M + N)$
  - $(LM)N = L(MN)$
- Commutativity
  - $L + M = M + L$
  - $LM \neq ML$
- Identities and Annilhilators
  - $\phi + L = L + \phi = L$
  - $\varepsilon L = L\varepsilon = L$
  - $\phi L = L\phi = \phi$

Algebraic Laws for Regular Expressions

- Distributive Laws
  - $L(M + N) = LM + LN$
  - $(M + N)L = (M + N)L$
- Idempotent Law
  - $L + L = L$
- Laws Involving Closures
  - $(L^*)^* = L^*$
  - $\phi^* = \varepsilon$
  - $\varepsilon^* = \varepsilon$
  - $L^+ = LL^* = L^*L$
  - $L^* = L^+ + \varepsilon$

Test Regular Expression Law

- Test whether E = F:
  - Convert E and F to concrete regular expressions C and D, respectively, by replacing each variable by a concrete symbol
  - Test whether L(C) = L(D). If so, then E = F is a law, and if not, then it is not a law.
- This only works for the three basic operations. This may not work for additional operations.

Pumping Lemma

Let L be a regular language. Then, there is a constant n such that if z is any string in L, and $|z| \geq n$, there exist three strings u, v, w so that

- $z = uvw$
- $|uv| \leq n$
- $|v| \geq 1$
- for all $i \geq 0$, uv^iw is in L.

Applications of Pumping Lemma

- Pumping lemma is mainly used to prove some language is not a regular language.

1. Select the language L you wish to prove nonregular.
2. Pick up any integer n.
3. Select a string z in L. $|z|$ must be larger than n.
4. Break z into u, v, and w in all possible ways so that $|uv| \leq n$ and $|v| \geq 1$.
5. Prove uv^iw for a given i for example 2 is not in L. From pumping lemma, L is not a regular language.

Example

1. Prove $a^i b^i$ is not a regular language.
2. Assume that $a^i b^i$ is accepted by an FA with n states.
3. Let's consider the string $a^n b^n$.
4. For any u, v, w such that $|uv| <= n$ and $|v| >= 1$. We have $u = a^l$ and $v = a^m$ $(m >= 1)$ and $w = a^k b^n$ and $l + m + k = n$.
5. $uv^2w = a^l a^m a^m a^k b^n = a^{l+2m+k} b^n$ is not in L, because $l+2m+k > n$. According pumping lemma, L is not a regular language.

Closure Properties of Regular Languages

- A class of languages is closed under a particular operation if application of this operation to languages in this class results in a language also in this class.
- Theorem:

  The regular languages are closed under union, concatenation, and Kleene closure.
- Theorem:

  The regular languages are closed under complementation.
- Theorem:

  The regular languages are closed under intersection.