L2: Basic Cryptography

Hui Chen, Ph.D.
Dept. of Engineering & Computer Science
Virginia State University
Petersburg, VA 23806

Acknowledgement

- Many slides are from or are revised from the slides of the author of the textbook
  - Matt Bishop, Introduction to Computer Security, Addison-Wesley Professional, October, 2004, ISBN-13: 978-0-321-24774-5. Introduction to Computer Security @ VSU's Safari Book Online subscription
  - http://nob.cs.ucdavis.edu/book/book-intro/slides/

Overview

- Cryptography as mechanism to enforce security policies
- Concepts
  - Cryptography, cryptanalysis
- Basic Cryptography
  - Classical Cryptography
  - Public Key Cryptography
  - Cryptographic Checksums

Overview

- Classical Cryptography
  - Caesar cipher
  - Vigènere cipher
  - DES
  - AES
- Public Key Cryptography
  - Diffie-Hellman
  - RSA
- Cryptographic Checksums
  - HMAC

Security Policy and Mechanism

- Security policy
  - A statement of what is allowed and what is not allowed
  - Example
    - A student may not copy another student’s homework
  - Can be informal or highly mathematical
- Security mechanism
  - A method, tool, or procedure for enforcing security policy
  - Technical and non-technical
    - A homework electronic submission system (e.g., Blackboard) enforces who may read a homework submission

Security Mechanisms

- Cryptographic mechanisms
- Non-cryptographic mechanisms (system-dependent mechanisms)

Cryptography

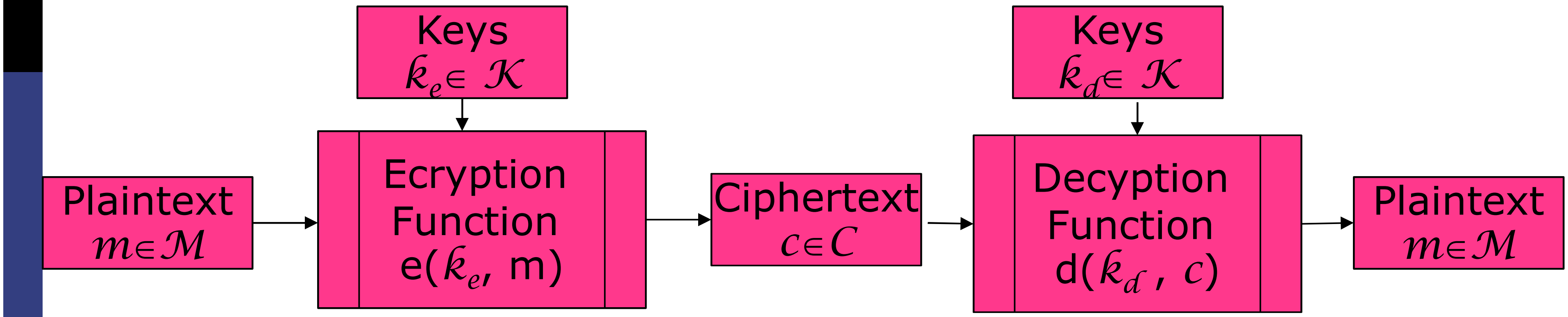
- Word Origin
  - Greek words
  - “secrete writing”
- Art & science of concealing meaning

Cryptanalysis

- Breaking of codes
- Application
  - World War II
- Further Reading
  - W. Diffie and M. Hellman. 2006. New directions in cryptography. *IEEE Trans. Inf. Theor.* 22, 6 (September 2006), 644-654. DOI=10.1109/TIT.1976.1055638 http://dx.doi.org/10.1109/TIT.1976.1055638

Cryptosystem

- Quintuple or 5-tuple ($\mathcal{E}$, $\mathcal{D}$, $\mathcal{M}$, $\mathcal{K}$, $\mathcal{C}$)
  - $\mathcal{M}$ set of plaintexts
  - $\mathcal{K}$ set of keys
  - $\mathcal{C}$ set of ciphertexts
  - $\mathcal{E}$ set of encryption functions *e*: $\mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$
  - $\mathcal{D}$ set of decryption functions *d*: $\mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$



Keys $k_e \in \mathcal{K}$

Keys $k_d \in \mathcal{K}$

Plaintext $m \in \mathcal{M}$

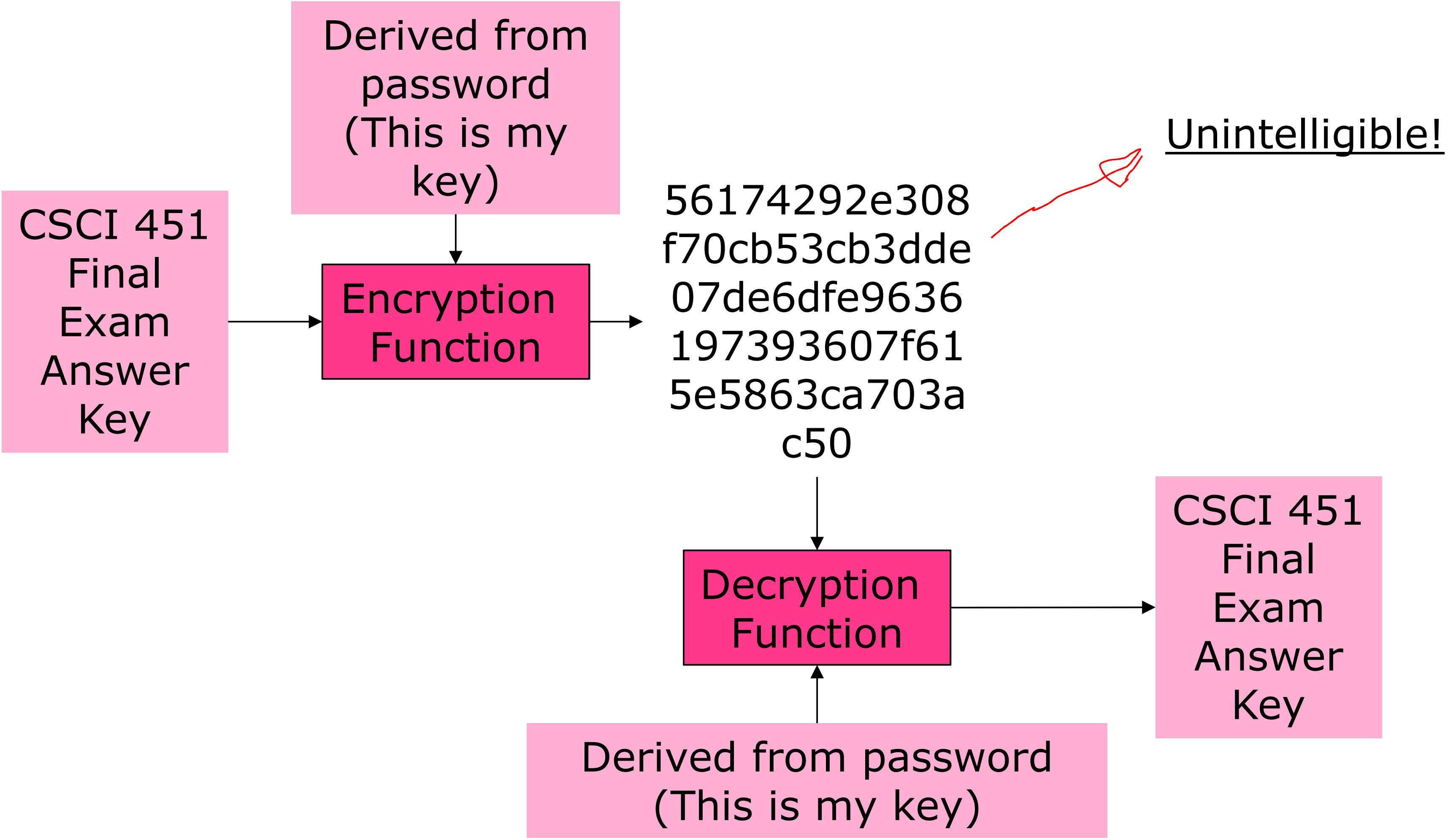
Ecryption Function e(k_e, m)

Ciphertext $c \in \mathcal{C}$

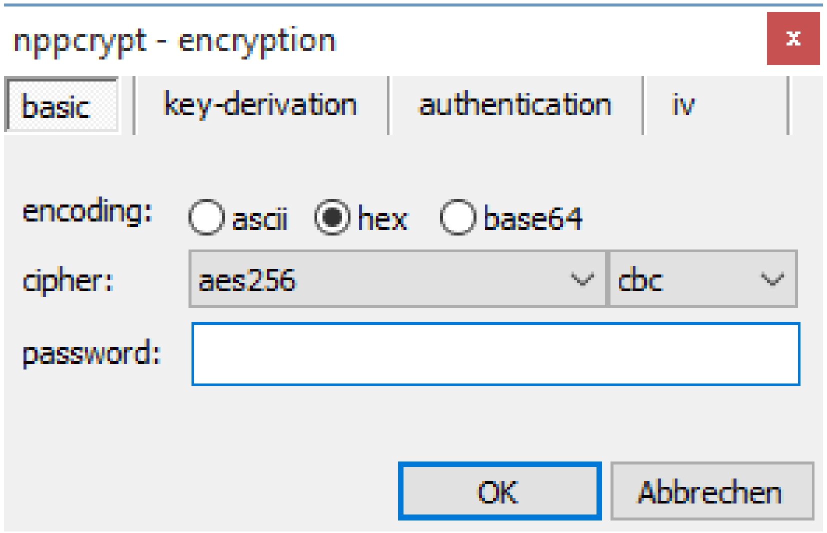
Decyption Function d(k_d , c)

Plaintext $m \in \mathcal{M}$

Example


Derived from password (This is my key)
CSCI 451 Final Exam Answer Key
Encryption Function
56174292e308 f70cb53cb3dde 07de6dfe9636 197393607f61 5e5863ca703a c50
Unintelligible!
Decryption Function
CSCI 451 Final Exam Answer Key
Derived from password (This is my key)

Example: NotePad++ NPPCrypt Plugin



encoding: ○ ascii ● hex ○ base64

cipher: aes256 cbc

password:

```
<nppcrypt version="101">
<encryption cipher="aes256" mode="cbc" encoding="base16" />
<random iv="atuPGKigDnTy46fHBPM1vA==" salt="wOiEp1afVtXebE4kMSliFg==" />
<key algorithm="pbkdf2" hash="md5" iterations="1000" />
</nppcrypt>
56174292e308f70cb53cb3dde07de6dfe9636197393607f615e5863ca703ac50
```

Classical Cryptography

- Sender, receiver share common key
  - Keys may be the same, or trivial to derive from one another
  - Sometimes called *symmetric cryptography*
- Two basic types
  - Transposition ciphers
    - Example: Rail Fence Cipher
  - Substitution ciphers
    - Example: Caesar Cipher
  - Combinations are called *product ciphers*

Transposition Cipher

- Rearrange letters in plaintext to produce ciphertext
- Example
  - Rail-Fence Cipher
  - Example
    - `HELLO WORLD` becomes `HLOOL ELWRD`

Rail-Fence Cipher

- Encryption
  - Writing the plaintext in two rows, proceeding down, then across
  - Reading the ciphertext across, then down.

Rail-Fence Cipher

- Plaintext is `HELLO WORLD`
  - Rearrange as

```
HLOOL
ELWRD
```

  - Cipher-text is `HLOOL ELWRD`
- Mathematically, the key to a transposition cipher is a permutation function.

Attacking Transposition Cipher

- Mathematically, the key to a transposition cipher is a permutation function.
- Observation: the permutation does not alter the frequency of plaintext characters
- Detecting the cipher by comparing character frequencies with a model of the language
  - Anagramming

Anagramming Attack

- Language Model: tables of n-gram frequencies Input: Cipher-text
- Method:
  - If 1-gram frequencies match English frequencies, but other *n*-gram frequencies do not, probably transposition
  - Let n := 1
  - Do
    - n := n + 1
    - Rearrange letters to form *n*-grams with highest frequencies
  - Until the transposition pattern is found

Example

- Konheim's diagram table
- Cipher-text: `HLOOLELWRD`
- Frequencies of 2-grams beginning with H
  - HE 0.0305
  - HO 0.0043
  - HL, HW, HR, HD < 0.0010
- Frequencies of 2-grams ending in H
  - WH 0.0026
  - EH, LH, OH, RH, DH ≤ 0.0002
- Implies E follows H

Example

- Since “E” follows “H”, we arrange the letters so that each letter in the first block of five letters is adjacent to the corresponding letters in the 2nd block of five letters
  - `HLOOL ELWRD`
  - `HE`
  - `LL`
  - `OW`
  - `OR`
  - `LD`

Substitution Ciphers

- Change characters in plaintext to produce ciphertext
- Example
  - Caesar cipher
    - Plaintext is `HELLO WORLD`
    - Change each letter to the third letter following it (X goes to A, Y to B, Z to C)
      - Key is 3, usually written as letter 'D'
    - Ciphertext is `KHOOR ZRUOG`
- More details follow

Caesar Cipher

- Gaius Julius Caesar (July 100 BC - 15 March 44 BC)
- *“If he had anything confidential to say, he wrote it in cipher...”*

Did he invent this also?

Caesar Cipher

- $\mathcal{M}$ = { sequences of letters }
  - The alphabet has N letters
- $\mathcal{K}$ = { i | i is an integer and $0 \leq i \leq N - 1$ }
- $\mathcal{E}$ = { E_k | $k \in \mathcal{K}$ and for all letters m, $E_k(m) = (m + k) \bmod N$ }
- $\mathcal{D}$ = { D_k | $k \in \mathcal{K}$ and for all letters c, $D_k(c) = (N + c - k) \bmod N$ }
- $\mathcal{C} = \mathcal{M}$

A Caesar Cipher

- $\mathcal{M} = \{0, 1, 2, \ldots, 25\}$
  - Assume English alphabet. The alphabet has N = 26 letters, representing each letter by its position in the alphabet
- Choose k = 3
- $E_3(m) = (m + k) \bmod 26$
- $D_3(c) = (26 + c - k) \bmod 26$
- $\mathcal{C} = \mathcal{M}$

Example: Encryption

- Plaintext = "HELLO", i.e.,
  - 7 4 11 11 14
- k = 3
- Compute ciphertext
  - 7 + 3 mod 26 = 10
  - 4 + 3 mod 26 = 7
  - 11 + 3 mod 26 = 14
  - 11 + 3 mod 26 = 14
  - 14 + 3 mod 26 = 17
  - 10 7 14 14 17

Example

- Convert the integers back to letters
  - 10 7 14 14 17
- Ciphertext = “KHOOR”

Example: Decryption

- Ciphertext = “KHOOR”, i.e.,
  - 10 7 14 14 17
- k = 3
- Compute plaintext
  - 26 + 10 – 3 mod 26 = 7
  - 26 + 7 - 3 mod 26 = 4
  - 26 + 14 - 3 mod 26 = 11
  - 26 + 14 - 3 mod 26 = 11
  - 26 + 17 - 3 mod 26 = 14
  - 7 4 11 11 14

Example

- Convert the integers back to letters
  - 7 4 11 11 14
- Ciphertext = “HELLO”

Attacking the Cipher

- Exhaustive search
  - If the key space is small enough, try all possible keys until you find the right one
  - Caesar cipher has only 26 possible keys (assuming English alphabet)
    - Exhaustive search is feasible
- Statistical analysis
  - Compare to 1-gram model of English

Exercise L2-1

- Use Caesar Cipher with k = 9, and compute ciphertext for the message below,
  - TROJAN

Exercise L2-2

- Assume Caesar Cipher, use exhaustive search to find the key for the ciphertext below
  - XUW
- To determine if your key is correct, read the plaintext using the key guessed to see if it is intelligible.

Exercise L2-3

- Write a program that computes ciphertext letter from a plaintext letter using Caesar cipher with a given key k
- Write a program that computes plaintext letter from a giver ciphertext letter using Caesar cipher with a given key k.
- Submit in Blackboard by 10AM, Monday, August 22.

Statistical Attack

- Example:
  - Breaking the ciphertext “KHOOR ZRUOG”
- Procedure:
  1. Compute *f(c)*, frequency of each letter in ciphertext
  2. Obtain unigram model of English
  3. Compute $\varphi(i)$, $0 \leq i \leq 25$, the correlation of the frequency of each letter in the ciphertext with the character frequencies in English, $\varphi(i)$, $0 \leq i \leq 25$
  4. Sort $\varphi(i)$, compute plaintext for i when $\varphi(i)$ from the greatest to the smallest, until the correct key is found

Step 1: Compute *f(c)*

- Breaking the ciphertext `KHOOR ZRUOG`
- *Step 1: compute f*(*c*) frequency of character *c* in the ciphertext

c	*f(c)*	*c*	*f(c)*	*c*	*f(c)*	*c*	*f(c)*
0	0	7	0.1	13	0	19	0
1	0	8	0	14	0.3	20	0.1
2	0	9	0	15	0	21	0
3	0	10	0.1	16	0	22	0
4	0	11	0	17	0.2	23	0
5	0	12	0	18	0	24	0
6	0.1					25	0.1

Step 2: Obtain Unigram Model of English (Frequencies of Letters)

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

Step 3: Compute Correlation $\varphi(i)$ for Caesar Cipher

- $f(c)$: frequency of character c in ciphertext
- $p(26 + c - i \bmod 26)$: frequency of character 26 + c – i mod 26 in the unigram model given i
  - Since the decryption function of Caesar Cipher is
  - $d_i(c) = 26 + c - i \bmod 26$
- Correlation of frequency of letters in ciphertext with corresponding letters in English becomes
  - $\varphi(i) = \Sigma_{0 \le c \le 25} f(c)p(26 + c - i \bmod 26)$
  - $p(x)$ is frequency of character x in English
- See next slide for example

Example: Correlation: $\varphi(4)$

c	*f(c)*	*c*	*f(c)*	*c*	*f(c)*	*c*	*f(c)*
0	0	7	0.1	13	0	19	0
1	0	8	0	14	0.3	20	0.1
2	0	9	0	15	0	21	0
3	0	10	0.1	16	0	22	0
4	0	11	0	17	0.2	23	0
5	0	12	0	18	0	24	0
6	0.1					25	0.1

- $\varphi(4) = \Sigma_{0 \le c \le 25}\, f(c)p(26 + \mathrm{c} - 4 \bmod 26)$

  $= f(6)p(26 + 6 - 4 \bmod 26) + f(7)p(26 + 7 - 4 \bmod 26) + f(10)p(26 + 10 - 4 \bmod 26)$

  $+ f(14)p(26 + 14 - 4 \bmod 26) + f(17)p(26 + 17 - 4 \bmod 26)$

  $+ f(20)p(26 + 20 - 4 \bmod 26) + f(25)p(26 + 25 - 4 \bmod 26)$

  $= 0.1\ p(26 + 6 - 4 \bmod 26) + 0.1\ p(26 + 7 - 4 \bmod 26) + \ldots$

Example: Correlation: φ(*4*) (continued)

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
a	0.080	h	0.060	n or 13	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c or 2	0.030	j	0.005	p	0.020	v	0.010
d or 3	0.040	k or 10	0.005	q or 16	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g or 6	0.015					z	0.002

- $\varphi(4) = \Sigma_{0 \le c \le 25}\, f(c)p(26 + c - 4 \bmod 26)$

$= f(6)p(2) + f(7)p(3) + f(10)p(6) + f(14)p(10) + f(17)p(13) + f(20)p(16) + f(25)*p(21)$

$= 0.1*0.030+0.1*0.040+0.1*0.015+0.3*0.005+0.2*0.070+0.1*0.002+0.1*0.010$

$= 0.0252$

Correlation: $\varphi(i)$ for $0 \leq i \leq 25$

i	$\varphi(i)$	i	$\varphi(i)$	i	$\varphi(i)$	i	$\varphi(i)$
0	0.0482	7	0.0442	13	0.0520	19	0.0315
1	0.0364	8	0.0202	14	0.0535	20	0.0302
2	0.0410	9	0.0267	15	0.0226	21	0.0517
3	0.0575	10	0.0635	16	0.0322	22	0.0380
4	0.0252	11	0.0262	17	0.0392	23	0.0370
5	0.0190	12	0.0325	18	0.0299	24	0.0316
6	0.0660					25	0.0430

Step 4: Test Results

- Most probable keys, based on φ:
  - *i* = 6, φ(*i*) = 0.0660
    - plaintext `EBIIL TLOLA`
  - *i* = 10, φ(*i*) = 0.0635
    - plaintext `AXEEH PHKEW`
  - *i* = 3, φ(*i*) = 0.0575
    - plaintext `HELLO WORLD`
  - *i* = 14, φ(*i*) = 0.0535
    - plaintext `WTAAD LDGAS`
- Only English phrase is for *i* = 3
  - That’s the key (3 or ‘D’)

Statistical Attack on Caesar Cipher

- $f(c)$: frequency of character c in ciphertext
- d(k_d, c): decryption function on ciphertext character c with key k_d
- $\varphi(k_d) = \Sigma_{0 \le c \le 25} f(c)p(\mathrm{d}(k_d, c))$: correlation of frequency of letters in ciphertext with corresponding letters in English
  - key is k_d
  - $p(x)$ is frequency of character x in the language
- This correlation should be a maximum when the key *k* translates to the ciphertext into English, i.e.,
  - argmax $_{k_d}$ $\varphi(k_d)$

Problem with Caesar Cipher

- Key is too short
  - Can be found by exhaustive search
  - Statistical frequencies not concealed well
    - They look too much like regular English letters
- So make it longer: long key may obscure the statistics
  - Multiple letters in key
  - Idea is to smooth the statistical frequencies to make cryptanalysis harder

Vigenère Cipher

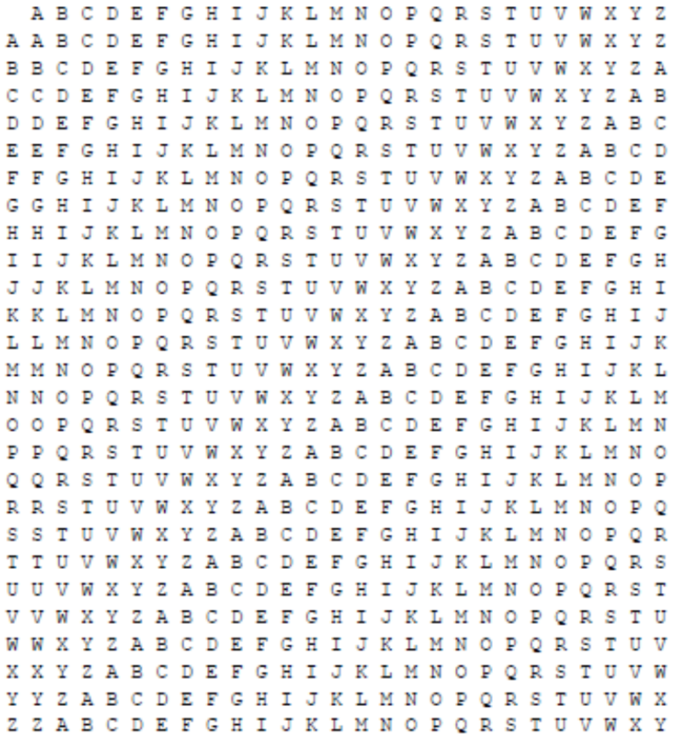
- Giovan Battista Bellaso, 1553
- Use phrase as the key
- Similar to Caesar cipher, but use each letter from the key to encipher
- Example
  - Message: `THE BOY HAS THE BALL`
  - Key: `VIG`
  - Encipher using Caesar cipher for each letter:

```
key     VIGVIGVIGVIGVIGV
plain   THEBOYHASTHEBALL
cipher  OPKWWECIYOPKWIRG
```

Table-Lookup Approach

- Trade memory for efficiency
- Store pre-calculated ciphertext for each letter using each possible key letter
  - 26 letters
  - 26 possible keys
  - Table of 26 × 26

Vigenère Tableau



	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Generate the table

```
In C++

for (int key = 0; key < KEY_SPACE_SIZE; key ++)
{
  cout << char(key + 'A') << ' ';
  for (int letter = 0; letter < ALPHABET_SIZE; letter ++)
  {
    int ciphertext = (letter + key) % ALPHABET_SIZE;
    cout << char(ciphertext + 'A') << ' ';
  }
  cout << endl;
}
```

Relevant Parts of Tableau

	G	*I*	*V*
A	G	I	V
B	H	J	W
E	L	M	Z
H	N	P	C
L	R	T	G
O	U	W	J
S	Y	A	N
T	Z	B	O
Y	E	H	T

- Tableau shown has relevant rows, columns only
- Example encipherments:
  - key V, letter T: follow V column down to T row (giving “O”)
  - Key I, letter H: follow I column down to H row (giving “P”)

Useful Terms

- *period*: length of key
  - In earlier example, period is 3
- *tableau*: table used to encipher and decipher
  - Vigenère cipher has key letters on top, plaintext letters on the left
- *polyalphabetic*: the key has several different letters
  - Caesar cipher is monoalphabetic

Attacking Vigenère Cipher

- Approach
  - Establish period; call it *n*
  - Break message into *n* parts, each part being enciphered using the same key letter
  - Solve each part
    - You can leverage one part from another
- We will show each step

Target Ciphertext

- We want to break the Vigenère cipher using the ciphertext:

```
ADQYS MIUSB OXKKT MIBHK IZOOO
EQOOG IFBAG KAUMF VVTAA CIDTW
MOCIO EQOOG BMBFV ZGGWP CIEKQ
HSNEW VECNE DLAAV RWKXS VNSVP
HCEUT QOIOF MEGJS WTPCH AJMOC
HIUIX
```

Establish Period

- The key is to establish the *period*
- Method
  - Using Kasiski method establish initial guesses
  - Using index of coincidence to confirm the guesses

Establish Period: Kasiski

- Friedrich W. Kasiski: a Prussian cavalry officer
  - *repetitions in the ciphertext occur when characters of the key appear over the same characters in the plaintext (Kasiski, 1863)*
- Example:

```
           key    VIGVIGVIGVIGVIGV
           plain  THEBOYHASTHEBALL
           cipher OPKWWECIYOPKWIRG
Counting distance 0123456789
```

Note the key and plaintext line up over the repetitions (underlined). As distance between repetitions is 9, the period is a factor of 9 (that is, 1, 3, or 9)

Repetitions in Example

Letters	***Start***	***End***	***Distance***	***Factors***
MI	5	15	10	2, 5
OO	22	27	5	5
OEQOOG	24	54	30	2, 3, 5
FV	39	63	24	2, 2, 2, 3
AA	43	87	44	2, 2, 11
MOC	50	122	72	2, 2, 2, 3, 3
QO	56	105	49	7, 7
PC	69	117	48	2, 2, 2, 2, 3
NE	77	83	6	2, 3
SV	94	97	3	3
CH	118	124	6	2, 3

Estimate of Period

- OEQOOG is probably not a coincidence
  - It is too long for that
  - Period may be 1, 2, 3, 5, 6, 10, 15, or 30
- Still too many to try, how may we try less?
  - Examining the table in the previous slide, how many repetitions have 2 in their factors? And how many have 3 in their factors?
    - 8/11 have 2 in their factors
    - 7/11 have 3 in their factors
- Begin with period of 2 × 3 = 6

Checking on Period using IC

- Index of coincidence (IC) is probability that two randomly chosen letters from ciphertext match
- Tabulated for different periods for English ciphertexts at different periods (d):

IC = 0.065933 / d + 0.038462 (d – 1) /d

Period	IC	Period	IC	Period	IC
1	0.066	3	0.047	5	0.044
2	0.052	4	0.045	10	0.041
Large	0.038				

Computing IC

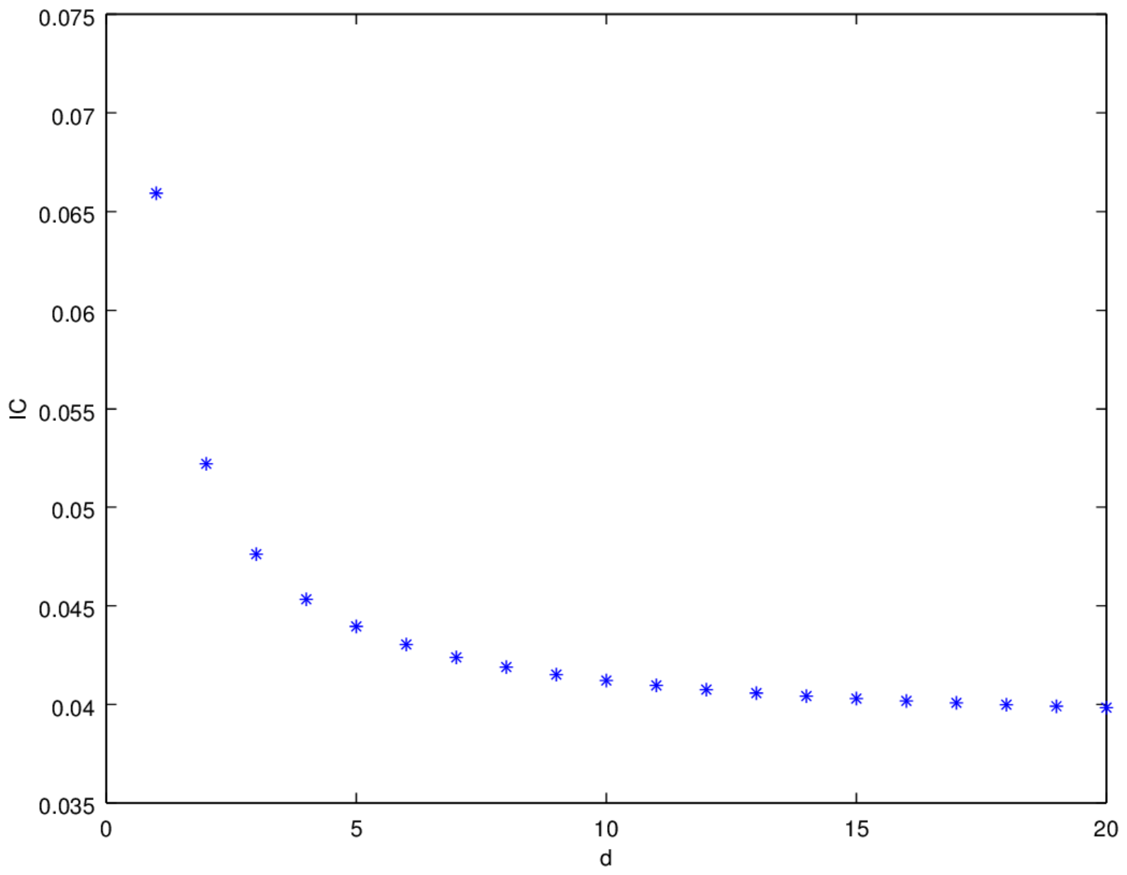
- $IC = \frac{1}{N(N-1)} \sum_{i=0}^{25} F_i(F_i - 1)$

  where *N* is length of ciphertext and *F*$_i$ the frequency of character *i* (i.e., the number of times character *i*) occurs in ciphertext

- Example: “ABCAD”
  - N = 5
  - Frequencies: A: 2; B: 1; C: 1; D:1; the rest: 0

$$IC = \frac{1}{5(5-1)}[2(2-1) + 1(1-1) + 1(1-1) + 1(1-1)] = \frac{1}{20}2 = \frac{1}{10} = 0.1$$

Index of Coincidence for English Ciphertext



Checking on Period using IC

- IC = 0.043 for the ciphertext
- Compare it with IC of English
  - Indicates a key of slightly more than 5
  - A statistical measure, so it can be in error, but it agrees with the previous estimate (which was 6)

Splitting Into Alphabets using Estimated Period (Period = 6)

Ciphertext

```
ADQYS MIUSB OXKKT MIBHK IZOOO EQOOG IFBAG
KAUMF VVTAA CIDTW MOCIO EQOOG BMBFV ZGGWP
CIEKQ HSNEW VECNE DLAAV RWKXS VNSVP HCEUT
QOIOF MEGJS WTPCH AJMOC HIUIX
```

alphabet 1: AIKHOIATTOBGEEERNEOSAI

alphabet 2: DUKKEFUAWEMGKWDWSUFWJU

alphabet 3: QSTIQBMAMQBWQVLKVTMTMI

alphabet 4: YBMZOAFCOOFPHEAXPQEPOX

alphabet 5: SOIOOGVICOVCSVASHOGCC

alphabet 6: MXBOGKVDIGZINNVVCIJHH

Checking on Period using IC (Again)

alphabet 1: `AIKHOIATTOBGEEERNEOSAI`

alphabet 2: `DUKKEFUAWEMGKWDWSUFWJU`

alphabet 3: `QSTIQBMAMQBWQVLKVTMTMI`

alphabet 4: `YBMZOAFCOOFPHEAXPQEPOX`

alphabet 5: `SOIOOGVICOVCSVASHOGCC`

alphabet 6: `MXBOGKVDIGZINNVVCIJHH`

- If the guessed period is correct, each alphabet should exhibit the "same" IC as that of English at period 1
  - #1, 0.069; #2, 0.078; #3, 0.078; #4, 0.056; #5, 0.124; #6, 0.043
  - Indicate all alphabets have period 1, *except* #4 and #6; assume statistics off

Begin Decryption

- First matches characteristics of unshifted alphabet
- Third matches if I shifted to A
- Sixth matches if V shifted to A
- Substitute into ciphertext (bold are substitutions)

AD**I**YS **RI**U**K**B O**CK**K**L** MI**GH**K **A**ZO**TO** E**I**OO**L**

IF**T**AG **PA**U**E**F V**AT**A**S** CI**IT**W **E**OC**NO** E**I**OO**L**

BM**T**FV **EG**G**O**P C**NE**K**I** HS**SE**W **N**EC**SE** D**D**AA**A**

RW**C**XS **AN**S**N**P H**HE**U**L** QO**NO**F **E**EG**OS** W**L**PC**M**

AJ**E**OC **MI**U**A**X

Look For Clues

- **A**J**E** in last line suggests “are”, meaning second alphabet maps A into S:

  ALIYS **RICK**B O**CKSL** MI**GHS** **A**ZO**TO**

  MIOO**L** **INT**AG **PACE**F V**ATIS** CI**ITE**

  EOC**NO** **MI**OO**L** **BUT**FV **EGOO**P C**NESI**

  HS**SEE** **N**EC**SE** **LD**AA**A** **REC**XS **ANAN**P

  H**HECL** QO**NON** **E**EG**OS** **EL**PC**M** **ARE**OC

  MICAX

Next Alphabet

- **MICA**X in last line suggests “mical” (a common ending for an adjective), meaning fourth alphabet maps O into A:

ALIMS **RICKP** O**CKSL** **A**I**GHS** **AN**O**TO** **MIC**O**L**

INTOG **PACET** V**ATIS** **Q**I**ITE** **EC**C**NO** **MIC**O**L**

BUTTV **EGOOD** C**NESI** **V**S**SEE** **NS**C**SE** **LDO**A**A**

RECLS **ANAND** H**HECL** **E**O**NON** **ES**G**OS** **ELD**C**M**

ARECC **MICAL**

Got It!

- QI means that U maps into I, as Q is always followed by U:

```
ALIME RICKP ACKSL AUGHS ANATO MICAL
INTOS PACET HATIS QUITE ECONO MICAL
BUTTH EGOOD ONESI VESEE NSOSE LDOMA
RECLE ANAND THECL EANON ESSOS ELDOM
ARECO MICAL
```

With Proper Spacing and Punctuation

- A LIMERICK PACKS LAUGHS ANATOMICAL INTO SPACE THAT IS QUITE ECONOMICAL. BUT THE GOOD ONES I'VE SEEN SO SELDOM ARE CLEAN, AND THE CLEAN ONES SO SELDOM ARE COMICAL.

Lessons Learned

- Vigenère cipher was once considered unbreakable
- It is easy to break by hand!
- Principles of attacks hold for more complex ciphers
  - WordPerfect: encipher a file with a password
    - Certain fields in the enciphered file contained information internal to WordPerfect
    - These fields could be predicted
- Cycles of Attack → Fix → Attack → Fix
- Stronger ciphers

Sample Programs to Ease the Process

- Find them at the class website.
- They were tested in Octave. With minor revision, they Should run in Matlab.

Looking For Repetition using Provided Program

- Note that the program counts from 1; however, we count from 0 in previous example

```
octave>
findcommonsubstrings('ADQYSMIUSBOXKKTMIBHKIZOOOEQOOGIFBAGKAUMFVVTAACIDTWMOCIOEQOOGBMBFVZGGWPCIEKQHSNEWVEC
NEDLAAVRWKXSVNSVPHCEUTQOIOFMEGJSWTPCHAJMOCHIUIX', 'v');
Start   End     Len     Gap     Letters
  6      16       2      10     MI
  7     127       2     120     IU
 23      28       2       5     OO
 23      58       2      35     OO
 24      28       2       4     OO
 24      58       2      34     OO
 27     106       2      79     QO
 25      55       6      30     OEQOOG
 40      64       2      24     FV
 44      88       2      44     AA
 46      53       2       7     CI
 46      71       2      25     CI
 51     123       3      72     MOC
 53      71       2      18     CI
 54     108       2      54     IO
 57     106       2      49     QO
 70     118       2      48     PC
 78      84       2       6     NE
 95      98       2       3     SV
119     125       2       6     CH
octave>
```

Computing IC using Provided Program

```
octave> ciphertext =
'ADQYSMIUSBOXKKTMIBHKIZOOOEQOOGIFBAGK
AUMFVVTAACIDTWMOCIOEQOOGBMBFVZGGWPCIE
KQHSNEWVECNEDLAAVRWKXSVNSVPHCEUTQOIOF
MEGJSWTPCHAJMOCHIUIX';
octave> computeic(ciphertext)
ans =  0.043292
octave>
```

Computing IC using Provided Octave/Matlab Program

```
octave> alphabet1 = ciphertext(1:6:length(ciphertext))
alphabet1 = AIKHOIATTOBGEEERNEOSAI
octave> computeic(alphabet1)
ans =  0.069264
octave> alphabet2 = ciphertext(2:6:length(ciphertext))
alphabet2 = DUKKEFUAWEMGKWDWSUFWJU
octave> computeic(alphabet2)
ans =  0.077922
octave> alphabet3 = ciphertext(3:6:length(ciphertext))
alphabet3 = QSTIQBMAMQBWQVLKVTMTMI
octave> computeic(alphabet3)
ans =  0.077922
octave>
```

……

English Letter Frequencies

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

One-Time Pad

- A variant of Vigenère Cipher
  - The key string is chosen at random
  - The key string is at least as long as the message

Discussion on Attacks

- Opponent whose goal is to break cryptosystem is the *adversary*
  - Assume adversary knows algorithm used, but not key
- Three types of attacks:
  - *ciphertext only*: adversary has only ciphertext; goal is to find plaintext, possibly key
  - *known plaintext*: adversary has ciphertext, corresponding plaintext; goal is to find key
  - *chosen plaintext*: adversary may supply plaintexts and obtain corresponding ciphertext; goal is to find key
- Good cryptosystems protects against all 3 types of attacks

Discussion on Attacks

- Mathematical attacks
  - Based on analysis of underlying mathematics
- Statistical attacks
  - Make assumptions about the distribution of letters, pairs of letters (digrams), triplets of letters (trigrams), *etc.*
    - Called *models of the language*
  - Examine ciphertext, correlate properties with the assumptions.

Exercise L2-4

- □ Textbook exercise: Question 2 of Chapter 8 in the textbook
- □ You may use the provided program attackcaesar.m, but must explain your result

Exercise L2-5

- Breaking two Vigenère ciphers
  - The ciphertext is in *pg.txt* and *tc.txt*
  - Use the programs (the example that breaks *pg.txt* follows and you will break *tc.txt* on your own)
- Disclaimer
  - All programs were tested in Octave, but not in Matlab although they should be mostly fine in Matlab

Attacking Vigenère in Programs (1)

1. Read the ciphertext and find repeating substrings

```
octave> ciphertext = readline('pg.txt');
octave> computeletterfreq(ciphertext);
octave> [idx1st, idx2nd, lensubstr, gaps] =
findcommonsubstrings(ciphertext(1:1000), 'v');
octave> gaps(lensubstr > 6)
ans =
   216    48    78   138    60    12
```

2. Let us now guess the period (the key length): 6

3. Confirm with index of coincidence

```
octave> computeic(ciphertext)
ans =  0.041854
```

Attacking Vigenère in Programs (2)

4. Now guess the letters in the key

```
octave> guesskey(ciphertext(1:6:end), 'v');
octave> guesskey(ciphertext(2:6:end), 'v');
octave> guesskey(ciphertext(1:6:end), 'v');
octave> guesskey(ciphertext(2:6:end), 'v');
octave> guesskey(ciphertext(3:6:end), 'v');
octave> guesskey(ciphertext(4:6:end), 'v');
octave> guesskey(ciphertext(5:6:end), 'v');
octave> guesskey(ciphertext(6:6:end), 'v');
```

The key appears to be ASIMOV.

Attacking Vigenère in Programs (3)

5. Decipher the ciphertext

```
octave:34> char(vigenere(ciphertext, 'ASIMOV', 'd'))
ans =
THEPROJECTGUTENBERGEBOOKOFMOBYDICKORTHEWHALEBYHERMANM……
```

What if the result is not intelligible?

Homework L2-1

- Breaking a Vigenère cipher. The ciphertext is in Exercise 8 of Chapter 8 in the textbook.
- Show steps, intermediate and final results

Summary

- Classical Cryptography
  - Caesar cipher
  - Vigènere cipher
- Attack on Caesar cipher and Vigènere cipher
- Concepts of cryptanalysis
  - Simple cryptanalysis