

L2: Basic Cryptography



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Acknowledgement

- ❑ Many slides are from or are revised from the slides of the author of the textbook
 - Matt Bishop, Introduction to Computer Security, Addison-Wesley Professional, October, 2004, ISBN-13: 978-0-321-24774-5. [Introduction to Computer Security @ VSU's Safari Book Online subscription](#)
 - <http://nob.cs.ucdavis.edu/book/book-intro/slides/>

Overview

- ❑ Cryptography as mechanism to enforce security policies
- ❑ Concepts
 - Cryptography, cryptanalysis
- ❑ Basic Cryptography
 - Classical Cryptography
 - Public Key Cryptography
 - Cryptographic Checksums

Overview

❑ Classical Cryptography

- Caesar cipher
- Vigenere cipher
- DES
- AES

❑ Public Key Cryptography

- Diffie-Hellman
- RSA

❑ Cryptographic Checksums

- HMAC

Security Policy and Mechanism

❑ Security policy

- A statement of what is allowed and what is not allowed
- Example
 - ❑ A student may not copy another student's homework
- Can be informal or highly mathematical

❑ Security mechanism

- A method, tool, or procedure for enforcing security policy
- Technical and non-technical
 - ❑ A homework electronic submission system (e.g., Blackboard) enforces who may read a homework submission

Security Mechanisms

- ❑ Cryptographic mechanisms
- ❑ Non-cryptographic mechanisms (system-dependent mechanisms)

Cryptography

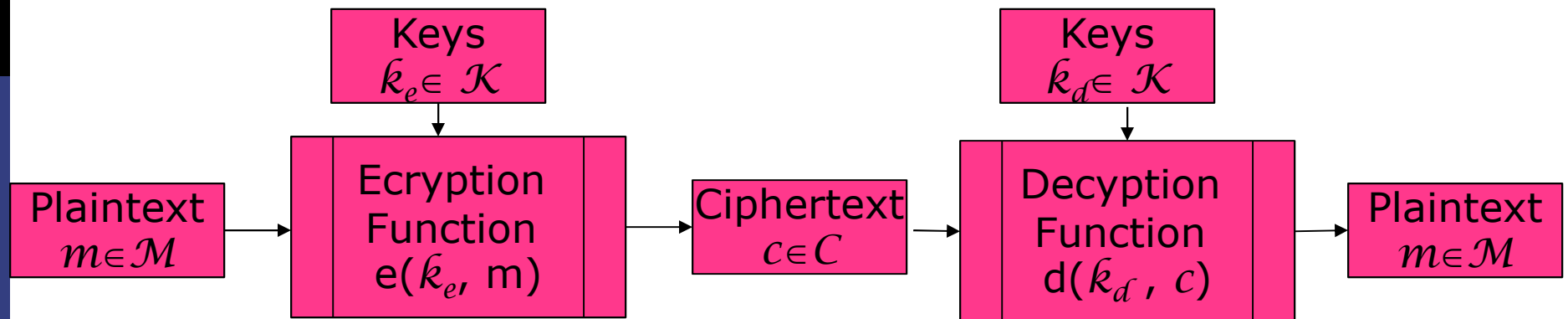
- ❑ Word Origin
 - Greek words
 - “secrete writing”
- ❑ Art & science of concealing meaning

Cryptanalysis

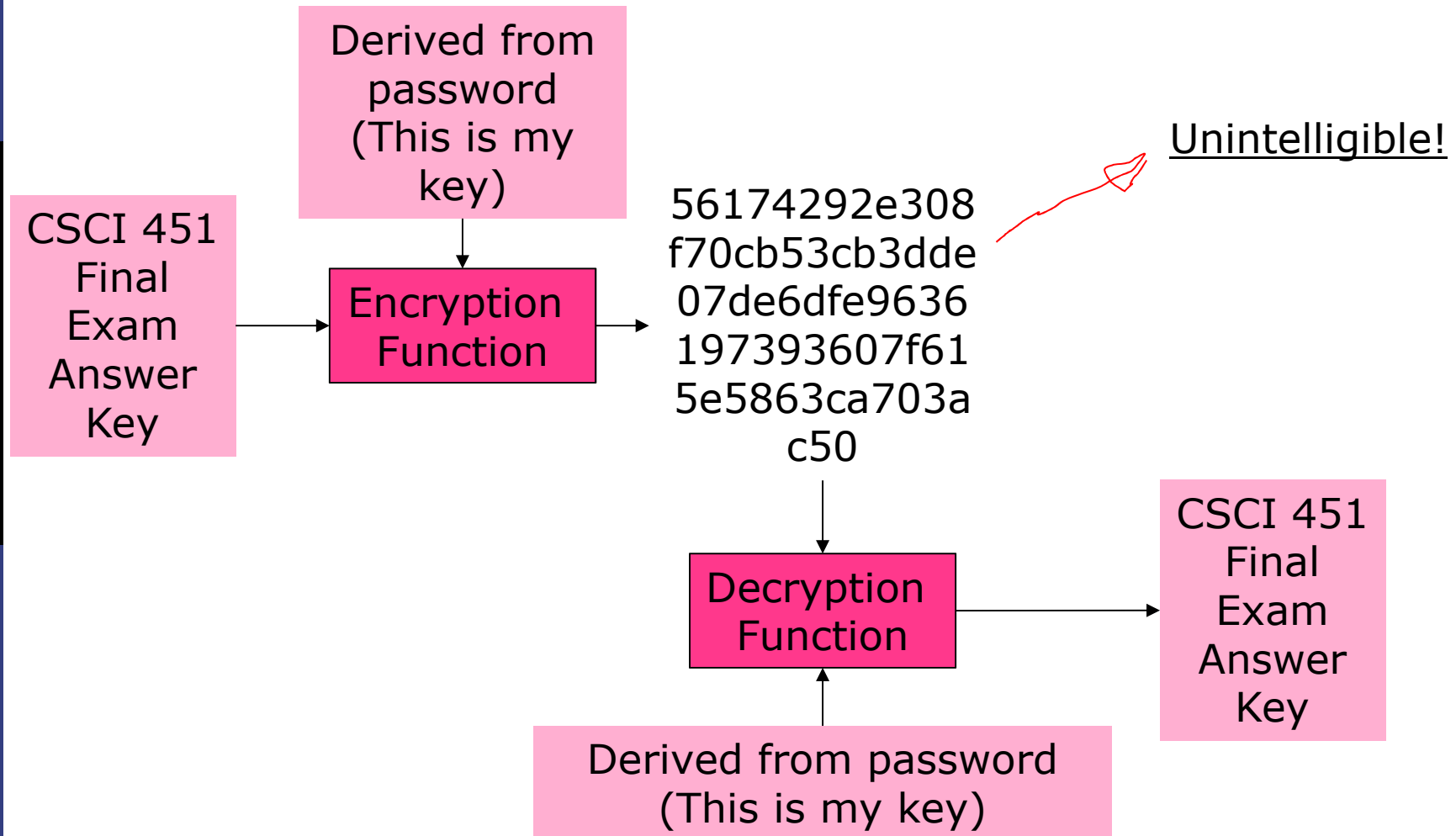
- ❑ Breaking of codes
- ❑ Application
 - World War II
- ❑ Further Reading
 - W. Diffie and M. Hellman. 2006. New directions in cryptography. *IEEE Trans. Inf. Theor.* 22, 6 (September 2006), 644-654. DOI=10.1109/TIT.1976.1055638
<http://dx.doi.org/10.1109/TIT.1976.1055638>

Cryptosystem

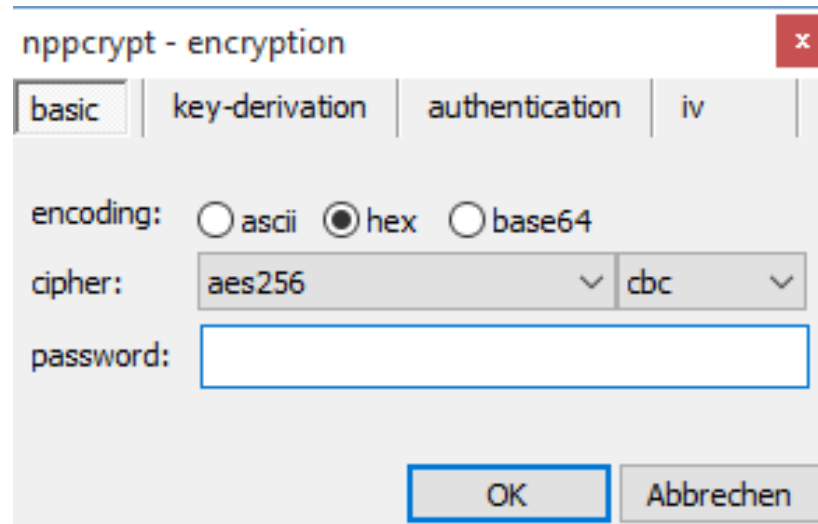
- Quintuple or 5-tuple $(\mathcal{E}, \mathcal{D}, \mathcal{M}, \mathcal{K}, \mathcal{C})$
 - \mathcal{M} set of plaintexts
 - \mathcal{K} set of keys
 - \mathcal{C} set of ciphertexts
 - \mathcal{E} set of encryption functions $e: \mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$
 - \mathcal{D} set of decryption functions $d: \mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$



Example



Example: NotePad++ NPPCrypt Plugin



```
<nppcrypt version="101">
<encryption cipher="aes256" mode="cbc" encoding="base16" />
<random iv="atuPGKigDnTy46fHBPM1vA==" salt="wOiEp1afVtXebE4kMSliFg==" />
<key algorithm="pbkdf2" hash="md5" iterations="1000" />
</nppcrypt>
56174292e308f70cb53cb3dde07de6dfe9636197393607f615e5863ca703ac50
```

Classical Cryptography

- ❑ Sender, receiver share common key
 - Keys may be the same, or trivial to derive from one another
 - Sometimes called *symmetric cryptography*
- ❑ Two basic types
 - Transposition ciphers
 - ❑ Example: Rail Fence Cipher
 - Substitution ciphers
 - ❑ Example: Caesar Cipher
 - Combinations are called *product ciphers*

Transposition Cipher

- ❑ Rearrange letters in plaintext to produce ciphertext
- ❑ Example
 - Rail-Fence Cipher
 - Example
 - ❑ HELLO WORLD becomes HLOOL ELWRD

Rail-Fence Cipher

□ Encryption

- Writing the plaintext in two rows, proceeding down, then across
- Reading the ciphertext across, then down.

Rail-Fence Cipher

❑ Plaintext is HELLO WORLD

■ Rearrange as

HLOOL

ELWRD

■ Cipher-text is HLOOL ELWRD

❑ Mathematically, the key to a transposition cipher is a permutation function.

Attacking Transposition Cipher

- ❑ Mathematically, the key to a transposition cipher is a permutation function.
- ❑ Observation: the permutation does not alter the frequency of plaintext characters
- ❑ Detecting the cipher by comparing character frequencies with a model of the language
 - Anagramming

Anagramming Attack

- ❑ Language Model: tables of n -gram frequencies Input: Cipher-text
- ❑ Method:
 - If 1-gram frequencies match English frequencies, but other n -gram frequencies do not, probably transposition
 - Let $n := 1$
 - Do
 - ❑ $n := n + 1$
 - ❑ Rearrange letters to form n -grams with highest frequencies
 - Until the transposition pattern is found

Example

- ❑ Konheim's diagram table
- ❑ Cipher-text: HLOOLELWRD
- ❑ Frequencies of 2-grams beginning with H
 - HE 0.0305
 - HO 0.0043
 - HL, HW, HR, HD < 0.0010
- ❑ Frequencies of 2-grams ending in H
 - WH 0.0026
 - EH, LH, OH, RH, DH ≤ 0.0002
- ❑ Implies E follows H

Example

- Since “E” follows “H”, we arrange the letters so that each letter in the first block of five letters is adjacent to the corresponding letters in the 2nd block of five letters

- HLOOL ELWRD

- HE

- LL

- OW

- OR

- LD

Substitution Ciphers

- ❑ Change characters in plaintext to produce ciphertext
- ❑ Example
 - Caesar cipher
 - ❑ Plaintext is HELLO WORLD
 - ❑ Change each letter to the third letter following it (X goes to A, Y to B, Z to C)
 - Key is 3, usually written as letter 'D'
 - ❑ Ciphertext is KHOOR ZRUOG
- ❑ More details follow

Caesar Cipher

- Gaius Julius Caesar
(July 100 BC - 15
March 44 BC)
- *“If he had anything
confidential to say, he
wrote it in cipher...”*



Did he invent this also?



Caesar Cipher

- $\mathcal{M} = \{ \text{sequences of letters} \}$
 - The alphabet has N letters
- $\mathcal{K} = \{ i \mid i \text{ is an integer and } 0 \leq i \leq N - 1 \}$
- $\mathcal{E} = \{ E_k \mid k \in \mathcal{K} \text{ and for all letters } m, E_k(m) = (m + k) \bmod N \}$
- $\mathcal{D} = \{ D_k \mid k \in \mathcal{K} \text{ and for all letters } c, D_k(c) = (N + c - k) \bmod N \}$
- $\mathcal{C} = \mathcal{M}$

A Caesar Cipher

- $\mathcal{M} = \{0, 1, 2, \dots, 25\}$
 - Assume English alphabet. The alphabet has $N = 26$ letters, representing each letter by its position in the alphabet
- Choose $k = 3$
- $E_3(m) = (m + k) \bmod 26$
- $D_3(c) = (26 + c - k) \bmod 26$
- $C = \mathcal{M}$

Example: Encryption

□ Plaintext = “HELLO”, i.e.,

■ 7 4 11 11 14

□ $k = 3$

□ Compute ciphertext

■ $7 + 3 \bmod 26 = 10$

■ $4 + 3 \bmod 26 = 7$

■ $11 + 3 \bmod 26 = 14$

■ $11 + 3 \bmod 26 = 14$

■ $14 + 3 \bmod 26 = 17$

■ 10 7 14 14 17

Example

- ❑ Convert the integers back to letters
 - 10 7 14 14 17
- ❑ Ciphertext = “KHOOR”

Example: Decryption

□ Ciphertext = “KHOOR”, i.e.,

■ 10 7 14 14 17

□ $k = 3$

□ Compute plaintext

■ $26 + 10 - 3 \bmod 26 = 7$

■ $26 + 7 - 3 \bmod 26 = 4$

■ $26 + 14 - 3 \bmod 26 = 11$

■ $26 + 14 - 3 \bmod 26 = 11$

■ $26 + 17 - 3 \bmod 26 = 14$

■ 7 4 11 11 14

Example

- ❑ Convert the integers back to letters
 - 7 4 11 11 14
- ❑ Ciphertext = “HELLO”

Attacking the Cipher

❑ Exhaustive search

- If the key space is small enough, try all possible keys until you find the right one
- Caesar cipher has only 26 possible keys (assuming English alphabet)
 - ❑ Exhaustive search is feasible

❑ Statistical analysis

- Compare to 1-gram model of English

Exercise L2-1

- Use Caesar Cipher with $k = 9$, and compute ciphertext for the message below,
 - TROJAN

Exercise L2-2

- ❑ Assume Caesar Cipher, use exhaustive search to find the key for the ciphertext below
 - XUW
- ❑ To determine if your key is correct, read the plaintext using the key guessed to see if it is intelligible.

Exercise L2-3

- ❑ Write a program that computes ciphertext letter from a plaintext letter using Caesar cipher with a given key k
- ❑ Write a program that computes plaintext letter from a given ciphertext letter using Caesar cipher with a given key k .
- ❑ Submit in Blackboard by 10AM, Monday, August 22.

Statistical Attack

- ❑ Compute frequency of each letter in ciphertext:

G	0.1	H	0.1	K	0.1	O	0.3
R	0.2	U	0.1	Z	0.1		

- ❑ Apply 1-gram model of English

- Frequency of characters (1-grams) in English is on next slide

English Letter Frequencies

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

Statistical Analysis

- $f(c)$: frequency of character c in ciphertext
- $d(\tilde{k}_d, c)$: decryption function on ciphertext character c with key \tilde{k}_d
- $\varphi(\tilde{k}_d) = \sum_{0 \leq c \leq 25} f(c)p(d(\tilde{k}_d, c))$: correlation of frequency of letters in ciphertext with corresponding letters in English
 - key is \tilde{k}_d
 - $p(x)$ is frequency of character x in the language
- This correlation should be a maximum when the key k translates to the ciphertext into English, i.e.,
 - $\operatorname{argmax}_{\tilde{k}_d} \varphi(\tilde{k}_d)$

Statistical Analysis on Caesar Cipher

- $f(c)$: frequency of character c in ciphertext
- Considering the Caesar Cipher and English, decryption function is
 - $d_i(c) = 26 + c - i \bmod 26$
- Correlation of frequency of letters in ciphertext with corresponding letters in English becomes
 - $\phi(i) = \sum_{0 \leq c \leq 25} f(c)p(26 + c - i \bmod 26)$
 - $p(x)$ is frequency of character x in English
- Find key i such that $\phi(i)$ is a maximum for all i

Statistical Analysis

- Consider the ciphertext KHOOR ZRUOG
- $f(c)$: frequency of character c in ciphertext

c	$f(c)$	c	$f(c)$	c	$f(c)$	c	c
0	0	7	0.1	13	0	19	0
1	0	8	0	14	0.3	20	0.1
2	0	9	0	15	0	21	0
3	0	10	0.1	16	0	22	0
4	0	11	0	17	0.2	23	0
5	0	12	0	18	0	24	0
6	0.1					25	0

Statistical Analysis

- ❑ Consider the ciphertext KHOOR ZRUOG
- ❑ $f(c)$: frequency of character c in ciphertext
- ❑ $\varphi(i) = \sum_{0 \leq c \leq 25} f(c)p(26 + c - i \bmod 26)$: the correlation
 - ❑ For the cipher text
$$\varphi(i) = 0.1p(26 + 6 - i \bmod 26) + 0.1p(26 + 7 - i \bmod 26) + 0.1p(26 + 10 - i \bmod 26) + 0.3p(26 + 14 - i \bmod 26) + 0.2p(26 + 17 - i \bmod 26) + 0.1p(26 + 20 - i \bmod 26) + 0.1p(26 + 25 - i \bmod 26)$$
 - $p(x)$ is frequency of character x in English
- ❑ Compute $\varphi(i)$ for all i , $0 \leq i \leq 25$
- ❑ Find key i such that $\varphi(i)$ is *large* and decrypted text is *intelligible*

Correlation: $\varphi(i)$ for $0 \leq i \leq 25$

i	$\varphi(i)$	i	$\varphi(i)$	i	$\varphi(i)$	i	$\varphi(i)$
0	0.0482	7	0.0442	13	0.0520	19	0.0315
1	0.0364	8	0.0202	14	0.0535	20	0.0302
2	0.0410	9	0.0267	15	0.0226	21	0.0517
3	0.0575	10	0.0635	16	0.0322	22	0.0380
4	0.0252	11	0.0262	17	0.0392	23	0.0370
5	0.0190	12	0.0325	18	0.0299	24	0.0316
6	0.0660					25	0.0430

Result of Statistical Analysis

□ Most probable keys, based on φ :

- $i = 6, \varphi(i) = 0.0660$
 - plaintext EBIIL TLOLA
- $i = 10, \varphi(i) = 0.0635$
 - plaintext AXEEH PHKEW
- $i = 3, \varphi(i) = 0.0575$
 - plaintext HELLO WORLD
- $i = 14, \varphi(i) = 0.0535$
 - plaintext WTAAD LDGAS

□ Only English phrase is for $i = 3$

- That's the key (3 or 'D')

Problem with Caesar Cipher

- ❑ Key is too short
 - Can be found by exhaustive search
 - Statistical frequencies not concealed well
 - ❑ They look too much like regular English letters
- ❑ So make it longer: long key may obscure the statistics
 - Multiple letters in key
 - Idea is to smooth the statistical frequencies to make cryptanalysis harder

Vigenère Cipher

- ❑ Giovan Battista Bellaso, 1553
- ❑ Use phrase as the key
- ❑ Similar to Caesar cipher, but use each letter from the key to encipher
- ❑ Example

- Message: THE BOY HAS THE BALL

- Key: VIG

- Encipher using Caesar cipher for each letter:

key	VIGVIGVIGVIGVIGV
-----	------------------

plain	THEBOYHASTHEBALL
-------	------------------

cipher	OPKWWECIYOPKWIRG
--------	------------------

Table-Lookup Approach

- ❑ Trade memory for efficiency
- ❑ Store pre-calculated ciphertext for each letter using each possible key letter
 - 26 letters
 - 26 possible keys
 - Table of 26×26

Vigenère Tableau

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	
A	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
B	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A
C	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B
D	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C
E	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D
F	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E
G	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F
H	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G
I	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H
J	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I
K	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J
L	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K
M	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L
N	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M
O	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N
P	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
Q	Q	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
R	R	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q
S	S	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
T	T	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S
U	U	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T
V	V	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
W	W	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V
X	X	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W
Y	Y	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X
Z	Z	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y

Generate the table

In C++

```
for (int key = 0; key < KEY_SPACE_SIZE; key ++)  
{  
    cout << char(key + 'A') << ' ';  
    for (int letter = 0; letter < ALPHABET_SIZE; letter ++)  
    {  
        int ciphertext = (letter + key) % ALPHABET_SIZE;  
        cout << char(ciphertext + 'A') << ' ';  
    }  
    cout << endl;  
}
```

Relevant Parts of Tableau

	<i>G</i>	<i>I</i>	<i>V</i>
<i>A</i>	G	I	V
<i>B</i>	H	J	W
<i>E</i>	L	M	Z
<i>H</i>	N	P	C
<i>L</i>	R	T	G
<i>O</i>	U	W	J
<i>S</i>	Y	A	N
<i>T</i>	Z	B	O
<i>Y</i>	E	H	T

▣ Tableau shown has relevant rows, columns only

▣ Example encipherments:

- key V, letter T: follow V column down to T row (giving "O")
- Key I, letter H: follow I column down to H row (giving "P")

Useful Terms

- ❑ *period*: length of key
 - In earlier example, period is 3
- ❑ *tableau*: table used to encipher and decipher
 - Vigenère cipher has key letters on top, plaintext letters on the left
- ❑ *polyalphabetic*: the key has several different letters
 - Caesar cipher is monoalphabetic

Attacking Vigenère Cipher

□ Approach

- Establish period; call it n
- Break message into n parts, each part being enciphered using the same key letter
- Solve each part
 - You can leverage one part from another

□ We will show each step

Target Ciphertext

- We want to break the Vigenère cipher using the ciphertext:

```
ADQYS MIUSB OXKKT MIBHK IZOOO
EQOOG IFBAG KAUMF VVTAA CIDTW
MOCIO EQOOG BMBFV ZGGWP CIEKQ
HSNEW VECNE DLAAY RWKXS VNSVP
HCEUT QOIOF MEGJS WTPCH AJMOC
HIUIX
```


Establish Period

- ❑ The key is to establish the period
- ❑ Method
 - Using Kasiski method establish initial guesses
 - Using index of coincidence to confirm the guesses

Establish Period: Kasiski

- ❑ Friedrich W. Kasiski: a Prussian cavalry officer
 - *repetitions in the ciphertext occur when characters of the key appear over the same characters in the plaintext (Kasiski, 1863)*

❑ Example:

key	VIGVIGVIGVIGVIGV
plain	THEBOYHASTHEBALL
cipher	<u>OPKW</u> WECIY <u>OPKW</u> IRG

Counting distance 0123456789

Note the key and plaintext line up over the repetitions (underlined). As distance between repetitions is 9, the period is a factor of 9 (that is, 1, 3, or 9)

Repetitions in Example

<i>Letters</i>	<i>Start</i>	<i>End</i>	<i>Distance</i>	<i>Factors</i>
MI	5	15	10	2, 5
OO	22	27	5	5
OEQOOG	24	54	30	2, 3, 5
FV	39	63	24	2, 2, 2, 3
AA	43	87	44	2, 2, 11
MOC	50	122	72	2, 2, 2, 3, 3
QO	56	105	49	7, 7
PC	69	117	48	2, 2, 2, 2, 3
NE	77	83	6	2, 3
SV	94	97	3	3
CH	118	124	6	2, 3

-
- Note that the program counts from 1 and we count from 0 in previous example

Looking For Repetition using Provided Program

- ❑ Note that the program counts from 1; however, we count from 0 in previous example

```
octave>  
findcommonsubstrings('ADQYSMIUSBOXKKTMI BHKIZOOOEQOOGIFBAGKAUMFVVTAACIDTWMOCIOEQOOGBMBFVZGGWPCIEKQHSNEWVEC  
NEDLA AVRWKXSVNSVP HCEUTQOIOFMEGJSWTPCHAJMOCHIUIX', 'v');
```

Start	End	Len	Gap	Letters
6	16	2	10	MI
7	127	2	120	IU
23	28	2	5	OO
23	58	2	35	OO
24	28	2	4	OO
24	58	2	34	OO
27	106	2	79	QO
25	55	6	30	OEQOOG
40	64	2	24	FV
44	88	2	44	AA
46	53	2	7	CI
46	71	2	25	CI
51	123	3	72	MOC
53	71	2	18	CI
54	108	2	54	IO
57	106	2	49	QO
70	118	2	48	PC
78	84	2	6	NE
95	98	2	3	SV
119	125	2	6	CH

```
octave>  
8/17/2016
```

Estimate of Period

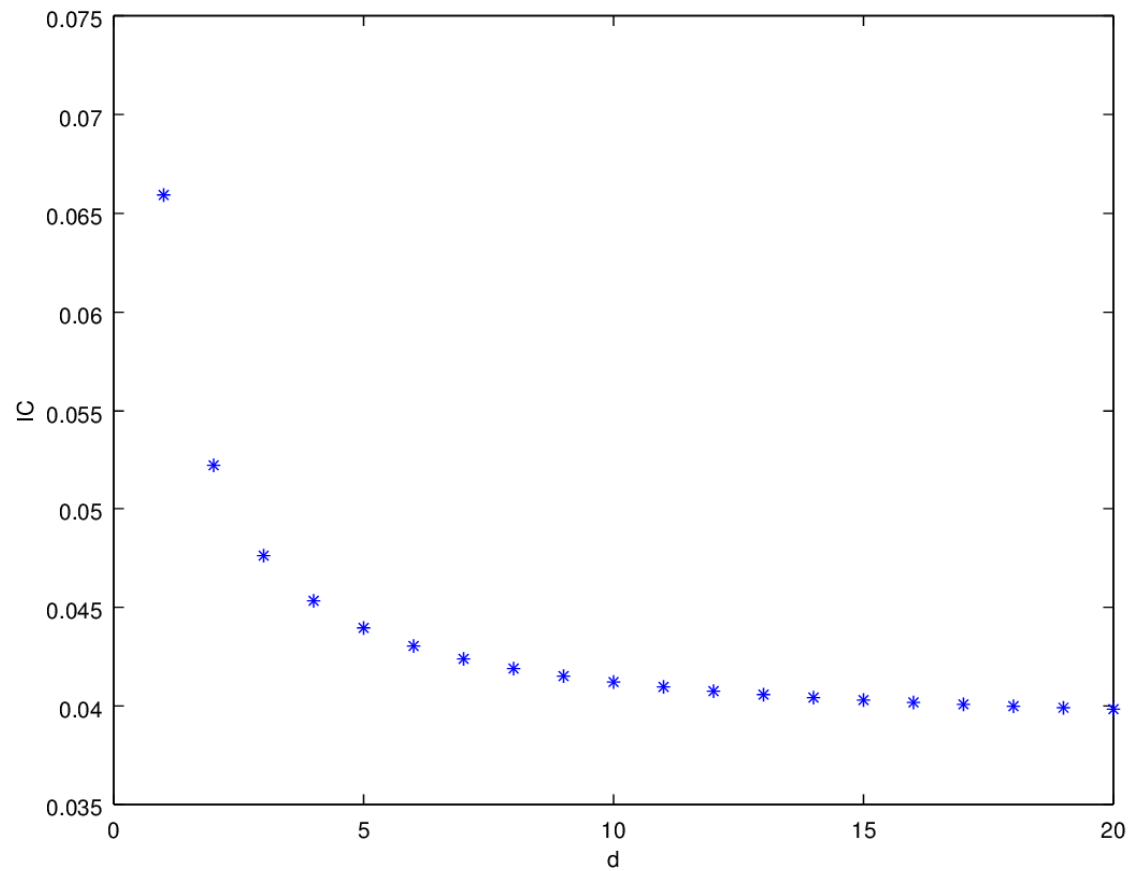
- ❑ OEQOOG is probably not a coincidence
 - It is too long for that
 - Period may be 1, 2, 3, 5, 6, 10, 15, or 30
- ❑ Most others (7/10) have 2 in their factors
- ❑ Almost as many (6/10) have 3 in their factors
- ❑ Begin with period of $2 \times 3 = 6$

Checking on Period

- ❑ Index of coincidence is probability that two randomly chosen letters from ciphertext match
- ❑ Tabulated for different periods for English ciphertexts at different periods (d):
$$IC = 0.065933 / d + 0.038462 (d - 1) / d$$

Period	IC	Period	IC	Period	IC
1	0.066	3	0.047	5	0.044
2	0.052	4	0.045	10	0.041
Large	0.038				

Index of Coincidence for English Ciphertext



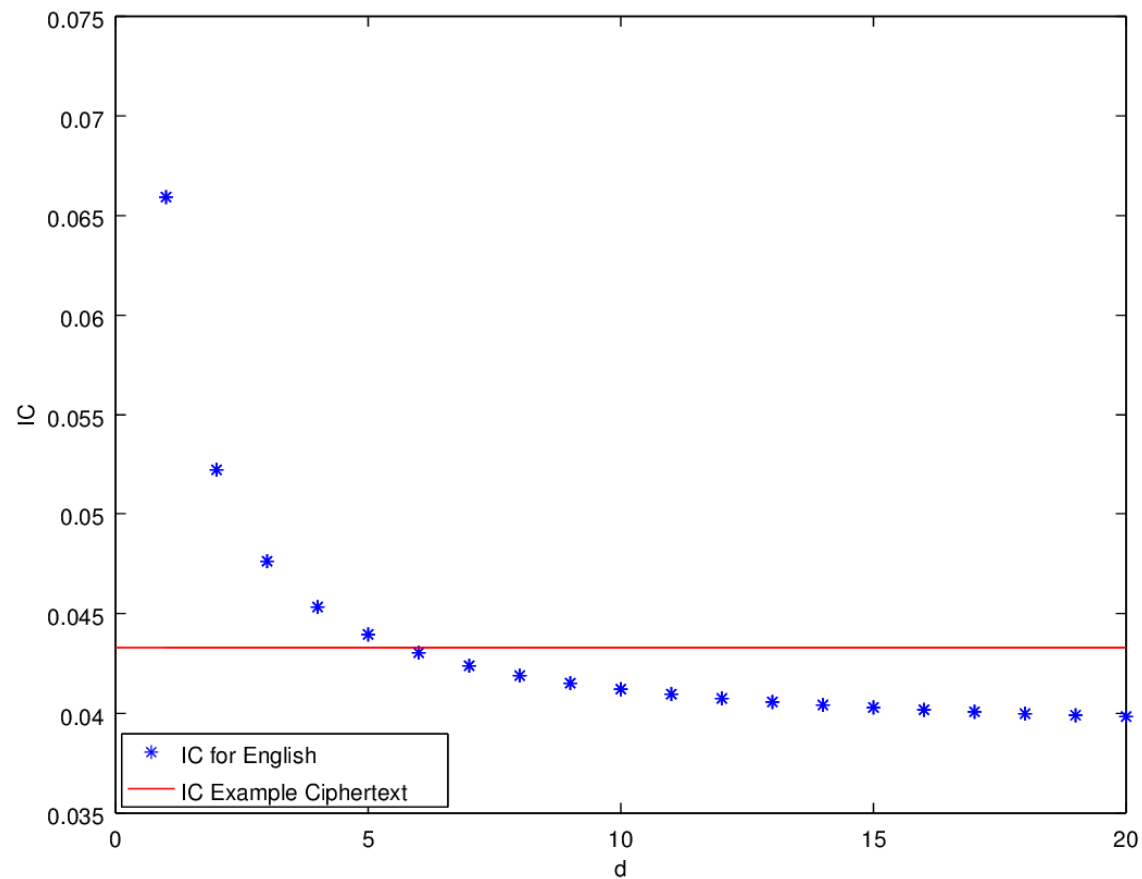
Computing IC

- $IC = [n (n - 1)]^{-1} \sum_{0 \leq i \leq 25} [F_i (F_i - 1)]$
 - where n is length of ciphertext and F_i the number of times character i occurs in ciphertext
- Here, $IC = 0.043$
 - Indicates a key of slightly more than 5
 - A statistical measure, so it can be in error, but it agrees with the previous estimate (which was 6)

Computing IC using Provided Program

```
octave> ciphertext =  
'ADQYSMIUSBOXKKTMIBHKIZOOOEQOOGIFBAGK  
AUMFVVTAACIDTWMOCIOEQOOGBMBFVZGGWPCIE  
KQHSNEWVECNEDLAAVRWKXSVNSVPHCEUTQOIOF  
MEGJSWTPCHAJMOCHIUIX';  
octave> computeic(ciphertext)  
ans = 0.043292  
octave>
```

Confirming Key Length



Splitting Into Alphabets using Estimated Period (Period = 6)

Ciphertext

ADQYS MIUSB OXKKT MIBHK IZOOO EQOOG IFBAG
KAUMF VVTAA CIDTW MOCIO EQOOG BMBFV ZGGWP
CIEKQ HSNEW VECNE DLAAY RWKXS VNSVP HCEUT
QOIOF MEGJS WTPCH AJMOC HIUIX

alphabet 1: AIKHOIATTOBGEEERNEOSAI

alphabet 2: DUKKEFUAWEMGKWDWSUFWJU

alphabet 3: QSTIQBMAMQBWQVLKVTMTMI

alphabet 4: YBMZOAFCOOFPHEAXPQEPOX

alphabet 5: SOIOOGVICOVCSVASHOGCC

alphabet 6: MXBOGKVDIGZINNVVCIJHH

Checking on IC

alphabet 1: AIKHOIATTOBGEEERNEOSAI

alphabet 2: DUKKEFUAWEMGKWDWSUFWJU

alphabet 3: QSTIQBMAMQBWQVLKVTMTMI

alphabet 4: YBMZOAFCOOFPHEAXPQEPOX

alphabet 5: SOIOOGVICOVCSVASHOGCC

alphabet 6: MXBOGKVDIGZINNVVCIJHH

□ ICs

- #1, 0.069; #2, 0.078; #3, 0.078; #4, 0.056; #5, 0.124; #6, 0.043
- Indicate all alphabets have period 1, except #4 and #6; assume statistics off

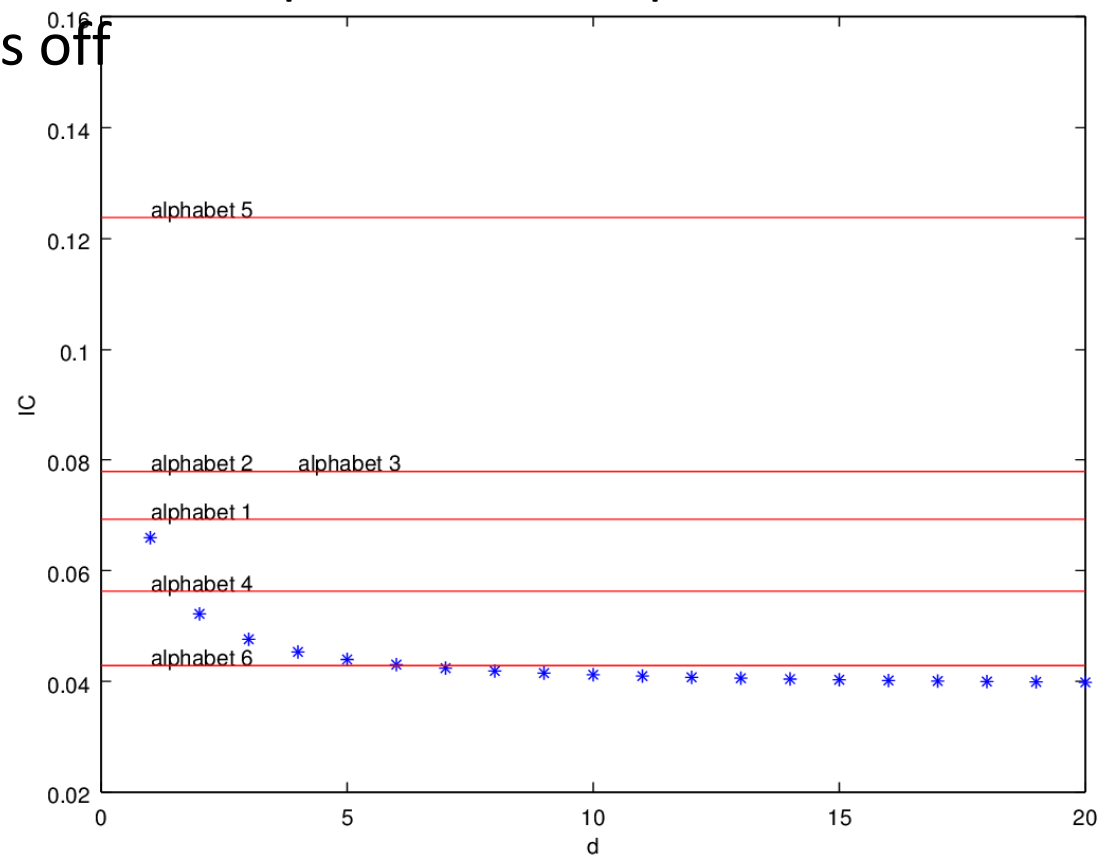
Computing IC using Provided Octave/Matlab Program

```
octave> alphabet1 = ciphertext(1:6:length(ciphertext))
alphabet1 = AIKHOIATTOBGEEERNEOSAI
octave> computeic(alphabet1)
ans = 0.069264
octave> alphabet2 = ciphertext(2:6:length(ciphertext))
alphabet2 = DUKKEFUAWEMGKWDWSUFWJU
octave> computeic(alphabet2)
ans = 0.077922
octave> alphabet3 = ciphertext(3:6:length(ciphertext))
alphabet3 = QSTIQBMAMQBWQVLKVTMTMI
octave> computeic(alphabet3)
ans = 0.077922
octave>
```

.....

Checking on IC

- all alphabets have period 1, except #4 and #6; assume statistics off



Frequency Examination

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	3	1	0	0	4	0	1	1	3	0	1	0	0	1	3	0	0	1	1	2	0	0	0	0	0	0
2	1	0	0	2	2	2	1	0	0	1	3	0	1	0	0	0	0	0	1	0	4	0	4	0	0	0
3	1	2	0	0	0	0	0	2	0	1	1	4	0	0	0	4	0	1	3	0	2	1	0	0	0	0
4	2	1	1	0	2	2	0	1	0	0	0	0	1	0	4	3	1	0	0	0	0	0	0	2	1	1
5	1	0	5	0	0	0	2	1	2	0	0	0	0	0	5	0	0	0	3	0	0	2	0	0	0	0
6	0	1	1	1	0	0	2	2	3	1	1	0	1	2	1	0	0	0	0	0	0	3	0	1	0	1

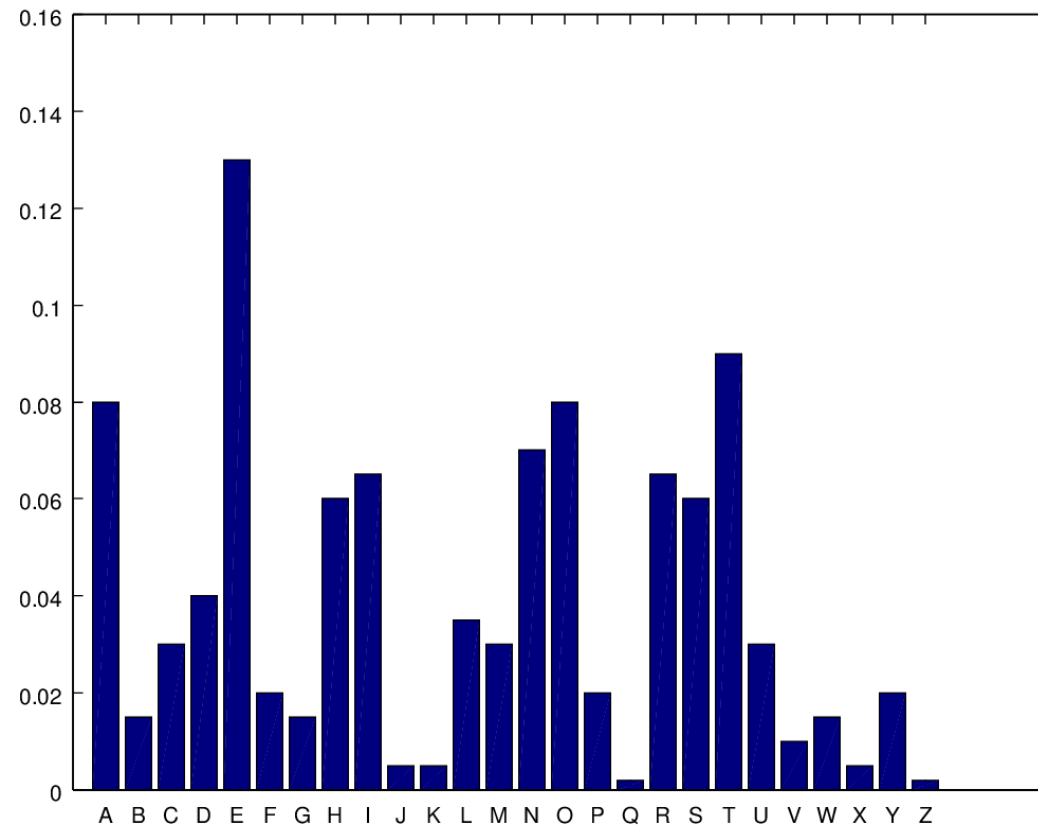
Letter frequencies are (H high, M medium, L low):

HMMMHHMMHHMMMMHHMLHHHMLLLLLL

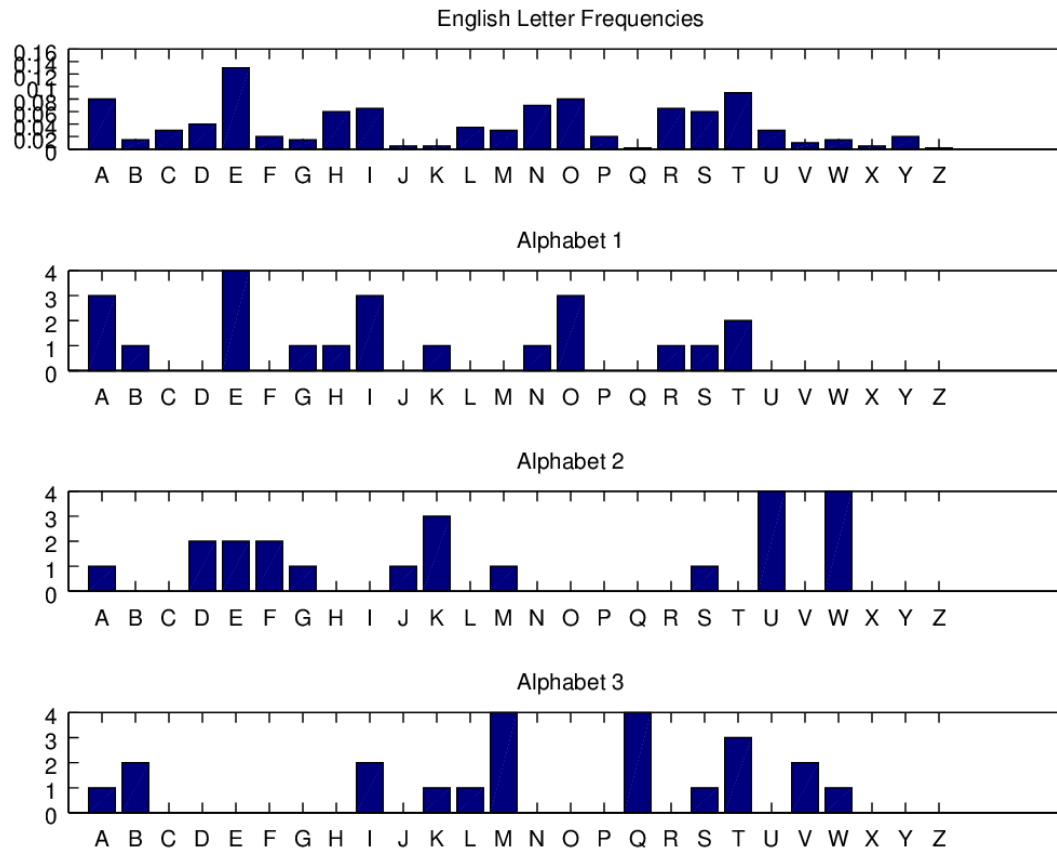
English Letter Frequencies

Letter	Frequency	Letter	Frequency	Letter	Frequency	Letter	Frequency
a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

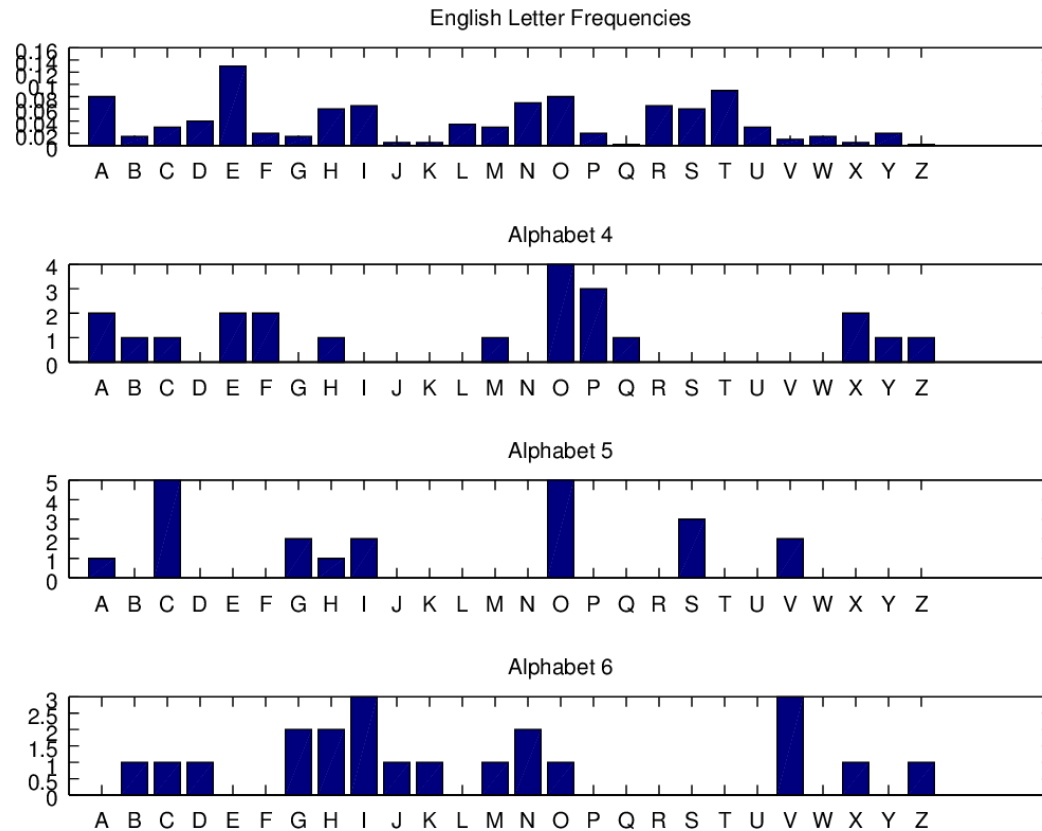
English Letter Frequencies



Guessing Key



Guessing Key



Begin Decryption

- ❑ First matches characteristics of unshifted alphabet
- ❑ Third matches if I shifted to A
- ❑ Sixth matches if V shifted to A
- ❑ Substitute into ciphertext (bold are substitutions)

ADIYS **RIUK**B O**CKKL** MI**GHK** **AZOTO** EI**OO**L
I**F**TAG **PAUE**F V**ATAS** CI**ITW** **EOCNO** EI**OO**L
BMTFV **EGGO**P C**NEKI** HS**SEW** **NECSE** DD**AAA**
RWCXS **ANSNP** H**HEUL** QO**NOF** **EEGOS** WL**PCM**
AJEOC **MIUA**X

Look For Clues

- **AJE** in last line suggests “are”, meaning second alphabet maps A into S:

ALIYS RICKB OCKSL MIGHS AZOTO
MI00L INTAG PACEF VATIS CIITE
EOCNO MI00L BUTFV EGOOP CNESI
HSSEE NECSE LDAAA RECXS ANANP
HHECL QONON EEGOS ELPCM AREOC
MICAX

Next Alphabet

- **MICAX** in last line suggests “mical” (a common ending for an adjective), meaning fourth alphabet maps O into A:

ALIMS	RICKP	OCKSL	AIGHS	ANOTO	MICOL
INTOG	PACET	VATIS	QIITE	ECCNO	MICOL
BUTTV	EGOOD	CNESI	VSSEE	NSCSE	LDOAA
RECLS	ANAND	HHECL	EONON	ESGOS	ELDCM
ARECC	MICAL				

Got It!

- QI means that U maps into I, as Q is always followed by U:

ALIME	RICKP	ACKSL	AUGHS	ANATO	MICAL
INTOS	PACET	HATIS	QUITE	ECONO	MICAL
BUTTH	EGOOD	ONESI	VESEE	NSOSE	LDOMA
RECLE	ANAND	THECL	EANON	ESSOS	ELDOM
ARECO	MICAL				

With Proper Spacing and Punctuation

- ❑ A LIMERICK PACKS LAUGHS ANATOMICAL INTO SPACE THAT IS QUITE ECONOMICAL. BUT THE GOOD ONES I'VE SEEN SO SELDOM ARE CLEAN, AND THE CLEAN ONES SO SELDOM ARE COMICAL.

Lessons Learned

- ❑ Vigenère cipher was once considered unbreakable
- ❑ It is easy to break by hand!
- ❑ Principles of attacks hold for more complex ciphers
 - WordPerfect: encipher a file with a password
 - ❑ Certain fields in the enciphered file contained information internal to WordPerfect
 - ❑ These fields could be predicted
- ❑ Cycles of Attack → Fix → Attack → Fix
- ❑ Stronger ciphers

One-Time Pad

- A variant of Vigenère Cipher
 - The key string is chosen at random
 - The key string is at least as long as the message

Discussion on Attacks

- ❑ Opponent whose goal is to break cryptosystem is the *adversary*
 - Assume adversary knows algorithm used, but not key
- ❑ Three types of attacks:
 - *ciphertext only*: adversary has only ciphertext; goal is to find plaintext, possibly key
 - *known plaintext*: adversary has ciphertext, corresponding plaintext; goal is to find key
 - *chosen plaintext*: adversary may supply plaintexts and obtain corresponding ciphertext; goal is to find key
- ❑ Good cryptosystems protects against all 3 types of attacks

Discussion on Attacks

❑ Mathematical attacks

- Based on analysis of underlying mathematics

❑ Statistical attacks

- Make assumptions about the distribution of letters, pairs of letters (digrams), triplets of letters (trigrams), *etc.*
 - ❑ Called *models of the language*
- Examine ciphertext, correlate properties with the assumptions.

Exercise L2-4

- ❑ Textbook exercise: Question 2 of Chapter 8 in the textbook
- ❑ You may use the provided program `attackcaesar.m`, but must explain your result

Exercise L2-5

❑ Breaking two Vigenère ciphers

- The ciphertext is in *pg.txt* and *tc.txt*
- Use the programs (the example that breaks *pg.txt* follows and you will break *tc.txt* on your own)

❑ Disclaimer

- All programs were tested in Octave, but not in Matlab although they should be mostly fine in Matlab

Attacking Vigenère in Programs (1)

1. Read the ciphertext and find repeating substrings

```
octave> ciphertext = readline('pg.txt');  
octave> computeletterfreq(ciphertext);  
octave> [idx1st, idx2nd, lensubstr, gaps] =  
findcommonsubstrings(ciphertext(1:1000), 'v');  
octave> gaps(lensubstr > 6)  
ans =  
    216    48    78   138    60    12
```

2. Let us now guess the period (the key length): 6

3. Confirm with index of coincidence

```
octave> computeic(ciphertext)  
ans = 0.041854
```


Attacking Vigenère in Programs (2)

4. Now guess the letters in the key

```
octave> guesskey(ciphertext(1:6:end), 'v');  
octave> guesskey(ciphertext(2:6:end), 'v');  
octave> guesskey(ciphertext(1:6:end), 'v');  
octave> guesskey(ciphertext(2:6:end), 'v');  
octave> guesskey(ciphertext(3:6:end), 'v');  
octave> guesskey(ciphertext(4:6:end), 'v');  
octave> guesskey(ciphertext(5:6:end), 'v');  
octave> guesskey(ciphertext(6:6:end), 'v');
```

The key appears to be ASIMOV.

Attacking Vigenère in Programs (3)

5. Decipher the ciphertext

```
octave:34> char(vigenere(ciphertext, 'ASIMOV', 'd'))  
ans =  
THEPROJECTGUTENBERGEBOOKOFMOBYDICKORTHEWHALEBYHERMANM.....
```

What if the result is not intelligible?

Homework L2-1

- ❑ Breaking a Vigenère cipher. The ciphertext is in Exercise 8 of Chapter 8 in the textbook.
- ❑ Show steps, intermediate and final results

Summary

- ❑ Classical Cryptography
 - Caesar cipher
 - Vigenere cipher
- ❑ Attack on Caesar cipher and Vigenere cipher
- ❑ Concepts of cryptanalysis
 - Simple cryptanalysis