Syntax and Semantics

Hui Chen

Computer Science
Virginia State University
Petersburg, Virginia

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Outline

- Backus-Naur Form
  - derivations, parse trees, ambiguity, descriptions of operator precedence and associativity, and extended Backus-Naur Form.
- Attribute grammars
- Operational axiomatic and denotational semantics

Chomsky Hierarchy

- Also called Chomsky-Schützenberger Hierarchy (Noam Chomsky, 1956)

Class	Grammar	Language	Automaton
Type-0	Unrestricted	Recursively enumerable	Turing machine (TM)
Type-1	Context-sensitive	Context-sensitive	Linear-bounded automaton (LBA)
Type-2	Context-free	Context-free	Pushdown automaton (PDA)
Type-3	Regular	Regular	Deterministic finite automaton (DFA)

- A strictly nested sets of classes of formal grammars, i.e.,

$$\text{Type-0} \supset \text{Type-1} \supset \text{Type-2} \supset \text{Type-3}$$

- Context-free and regular grammars are of our primary concern

Context-Free Grammar (CFG)

- A CFG is a quadruple, $G = (V, T, P, S)$ where
  - V: the set of variables or non-terminals
  - T: the set of terminals
  - P: the set of productions of the form $A \rightarrow \gamma$ where A is a single variable, i.e., $A \in V$ and γ is string of terminals and variables, i.e., $\gamma \in (V \cup T)^*$
  - S: the start symbol and $S \in V$
- To describe the grammar of a programming language,
  - Terminals are lexemes or tokens

Example: A Simple Programming Language[1]

- Operators: $+$ and $*$ represent addition and multiplication, respectively
- Arguments are identifiers consisting *only* of letters a, b, and digits 0, 1
- An example statement in the language,

$$(a+b)*(a+b+1)$$

[1] This is an example given in [Hopcroft et al., 2006]

CFG of the Simple Language

- The language can be specified using a CFG as,

$$G = (\{E, I\}, T, P, E)$$

where

  - E and I are the two variables, and E is the start symbol
  - T, the terminals are the set of symbols $\{+, *, (,), a, b, 0, 1\}$
  - P is the productions, i.e.,

1. $E \rightarrow I$
2. $E \rightarrow E + E$
3. $E \rightarrow E * E$
4. $E \rightarrow (E)$
5. $I \rightarrow a$
6. $I \rightarrow a$
7. $I \rightarrow Ia$
8. $I \rightarrow Ib$
9. $I \rightarrow I0$
10. $I \rightarrow I1$

Backus-Naur Form (BNF)

- John Backus (1959) and Peter Naur (1960) developed to describe syntax of ALGOL 58 and 60
- BNF is equivalent to context-free grammars
- Widely used today for describing syntax of programming languages

Production Rules in BNF

- Nonterminals (or variables in CFG, called *abstractions*) are often enclosed in angle brackets
- A start symbol is a special element of the nonterminals of a grammar
- Grammar: a finite non-empty set of rules
- Examples of BNF rules:

$$<\text{ident\_list} > \rightarrow \text{identifier}$$

$$<\text{ident\_list} > \rightarrow \text{identifier}, <\text{ident\_list} >$$

$$<\text{if\_stmt} > \rightarrow \text{if} <\text{logic\_expr}> \ \text{then} <\text{stmt} >$$

More than one RHS

- An abstraction (or a nonterminal symbol) can have more than one right-hand sides
- Example: applying this rule, we can rewrite,

$$<\text{ident\_list} > \to \text{identifier}$$
$$<\text{ident\_list} > \to \text{identifier}, <\text{ident\_list} >$$

as

$$<\text{ident\_list} > \to \text{identifier} \mid \text{identifier}, <\text{ident\_list} >$$

- Another example:

$$<\text{stmt} > \to <\text{single\_stmt} > \ \mid \text{begin} <\text{stmt\_list} > \ \text{end}$$

Lists

- Syntactic lists are described using recursion

$$<\text{ident\_list}> \rightarrow \text{ident} \mid \text{ident}, <\text{ident\_list}>$$

Derivation

- A repeated application of rules, starting with the start symbol and ending with a sentence (all terminal symbols)
  - Every string of symbols in a derivation is a sentential form
  - A sentence is a sentential form that has only terminal symbols
  - A leftmost derivation is one in which the leftmost nonterminal in each sentential form is the one that is expanded
  - A derivation may be neither leftmost nor rightmost

An Example of Derivation

- Given a grammar,

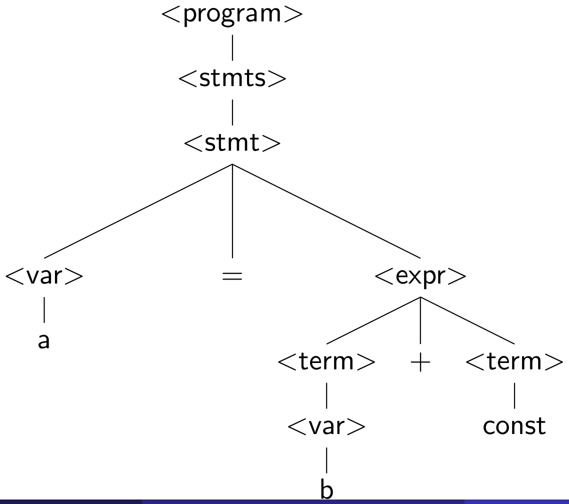
$$
\begin{aligned}
&<\text{program}> \;\to <\text{stmts}> \\
&<\text{stmts}> \;\to <\text{stmt}> \;\mid\; <\text{stmt}> \;;\; <\text{stmts}> \\
&<\text{stmt}> \;\to <\text{var}> \;=\; <\text{expr}> \\
&<\text{var}> \;\to \text{a} \mid \text{b} \mid \text{c} \mid \text{d} \\
&<\text{expr}> \;\to <\text{term}> \;+\; <\text{term}> \;\mid\; <\text{term}> \;\text{-}\; <\text{term}> \\
&<\text{term}> \;\to <\text{var}> \;\mid\; \text{const}
\end{aligned}
$$

- we can have the following derivation,

$$
\begin{aligned}
<program> &\Rightarrow <\text{stmts}> \;\Rightarrow <\text{stmt}> \;\Rightarrow <\text{var}> \;=\; <\text{expr}> \\
&\Rightarrow a = <\text{expr}> \;\Rightarrow a = <\text{term}> \;+ <\text{term}> \\
&\Rightarrow a = <\text{var}> \;+ <\text{term}> \\
&\Rightarrow a = b + <\text{term}>
\end{aligned}
$$

Parse Tree

- A parse tree is a hierarchical representation of a derivation
- Example:



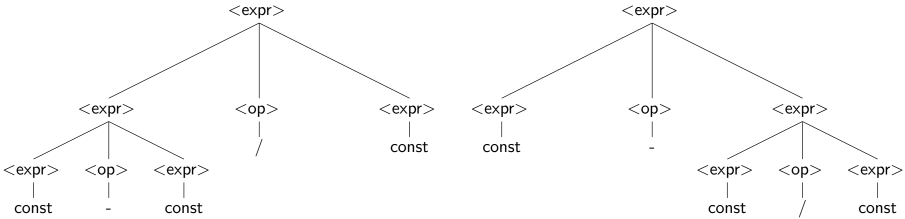
Ambiguity in Grammars

- A grammar is *ambiguous* if and only if it generates a sentential form that has two or more distinct parse trees

Example of Ambiguous Grammar and Parse Trees

$$<\text{expr}> \;\rightarrow <\text{expr}> <\text{op}> <\text{expr}> \mid \text{const}$$

$$<\text{op}> \;\rightarrow / \;\mid\; -$$

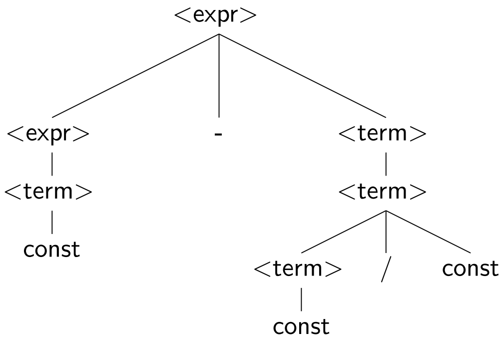


Unambiguous Grammar

- If we use the parse tree to indicate precedence levels of the operators, we cannot have ambiguity
- Example:

$$<\text{expr}> \;\rightarrow <\text{expr}> \; - <\text{term}> \mid <\text{term}>$$

$$<\text{term}> \;\rightarrow <\text{term}> \; / \; \text{const} \mid \text{const}$$



Associativity of Operators

- Operator associativity can also be indicated by a grammar
- Example: compare the following two grammars
  1. Ambiguous grammar

     $$<\text{expr}> \rightarrow <\text{expr}> + <\text{expr}> \mid \text{const}$$

  2. Unambiguous grammar

     $$<\text{expr}> \rightarrow <\text{expr}> + \text{const} \mid \text{const}$$

Extended BNF (EBNF)

- The extensions *do not* enhance the descriptive power of BNF; they only increase its *readability* and *writability*
- Optional parts are placed in brackets [ ], e.g.,

$$\text{<proc\_call>} \rightarrow \text{ident } [(\text{<expr\_list>})]$$

- Alternative parts of RHSs are placed inside ( ) and separated via |, e.g.,

$$\text{<term>} \rightarrow \text{<term>} \ (+|-) \text{ const}$$

- Repetitions (0 or more times) are placed inside {},

$$\text{<ident>} \rightarrow \text{letter } \{ \text{ letter} \mid \text{digit } \}$$

- Can you rewrite the above examples without using extensions?

Recent Variations in EBNF

- Alternative RHSs are put on separate lines
- Use of a : instead of $\rightarrow$
- Use of $_{\text{opt}}$ for optional parts
- Use of $_{\text{oneof}}$ for choices

Discussion on semantics to follow ...

To be continued.

Summary

- BNF and context-free grammars are equivalent meta-languages
  - Well-suited for describing the syntax of programming languages
- An attribute grammar is a descriptive formalism that can describe both the syntax and the semantics of a language
- Three primary methods of semantics description
  - Operation, axiomatic, denotational

References I

Hopcroft, J. E., Motwani, R., and Ullman, J. D. (2006).
Introduction to Automata Theory, Languages, and Computation (3rd Edition).
Addison-Wesley Longman Publishing Co., Inc., Boston, MA, USA.