Referee report:

Block bootstrap consistency under weak assumptions
by Gray Calhoun

This paper sets out to prove some useful results on the properties of the block bootstrap. The essential trick is to use the fact that the bootstrap resamples are drawn from independently sampled blocks, a procedure that parallels the “Bernstein sums” method of showing the CLT in dependent processes.

My main comment on the paper relates to the array notation used throughout. Specifically, what is it for?

In the several cited articles on the CLT and FCLT for heterogeneous processes, array notation is used as a flexible way of introducing normalizations. Thus, in a typical application we would have $X_{nt} = X_t / \sqrt{n}$, or perhaps some other power of *n* in the denominator, and with arrays we don’t need to be specific. In these applications, note that the mean of the process is typically zero.

However, in the present case the means of the processes are nonzero and represented with array notation as μ_{nt}. The sample mean is denoted (evidently, since it’s not actually defined anywhere in the paper!) as $\bar{X}_n = n^{-1}\sum_{t=1}^{n} X_{nt}$ with population counterpart $\bar{\mu}_n = n^{-1}\sum_{t=1}^{n} \bar{\mu}_{nt}$. Given the division by sample size *n* the role of the array notation is not to introduce normalization, because we don’t usually want to compute the sample mean of a normalized sequence. What therefore is the purpose of the array notation?

It’s not helpful to claim increased generality of the results unless some specific application, where it is useful, can be presented. I am of course aware that various works cited in the paper use the same convention. However, it is never a good practice to copy what other authors do, as an alternative to justifying it. My feeling is that, in connection with the estimation of the mean of a sequence, array notation needs to be carefully justified. Without such justification, it looks ambiguous and messy.

My suggestion is that the author should ask himself what changes to the results would follow from replacing X_{nt} by X_t and μ_{nt} by μ_t throughout, and the bounding constants and sub-sigma fields similarly. If some essential generality is thereby lost, then it should be possible to cite specific examples where it is needed. Discussing these would be a very useful addition to the paper. Otherwise, the added clarity of a simpler representation would be well worth having.

If the array notation is to be retained, on the other hand, then it would be worth asking whether (for example) the factor 1/*n* appearing in expressions such as (4) and (10) are needed. They don’t appear in the counterpart expressions in the papers by de Jong and Davidson being cited as sources. What’s different here?

Other comments:

1. In the statement of Lemma 2 (p. 13) $\mathcal{G}_n$ is a sigma-field of events, so in what sense are c_{nt} (scalar constant?) and Δ_n (real random variable?) elements of $\mathcal{G}_n$? I think something else is intended here?

2. It's customary in the this type of work to define the probability space in which the action takes place, say $(\Omega, \mathcal{F}, P)$, so that subsequently defined sigma fields can be identified as subsets of $\mathcal{F}$. I think that being explicit about this is a convention worth adhering to.