Report of the second revision of “Block bootstrap consistency under weak assumptions,” by Gray Calhoun

This paper has been thoroughly revised again. In his letter, the author acknowledges the changes that were made.

1. Corollary 1: Assuming there exists an estimator $\hat{\sigma}_n^2(\gamma)$ such that $\sup_\gamma |\sigma_n^2(\gamma)/\hat{\sigma}_n^2(\gamma) - 1| \xrightarrow{p} 0$ may not be innocent. After all, for an i.i.d. sequence of mean zero, finite variance ε_i, we could have

$$\hat{\sigma}_n^2(\gamma) = n^{-1} \sum_{t=1}^{[\gamma n]} \varepsilon_i^2 \quad \text{and} \quad \sigma_n^2(\gamma) = [\gamma n]\sigma^2/n.$$

Then

$$\sup_{0 \le \gamma \le 1} |\sigma_n^2(\gamma)/\hat{\sigma}_n^2(\gamma) - 1|$$

$$= \sup_{0 \le \gamma \le 1} |1/([n\gamma])^{-1} \sum_{t=1}^{[\gamma n]} \varepsilon_i^2 - 1| = \sup_{k:1 \le k \le n} |1/(k^{-1} \sum_{t=1}^{k} \varepsilon_i^2) - 1|$$

and the last expression converges a.s. to a non degenerate random variable. It is not clear to me what exactly the author wants to achieve by dividing by $\hat{\sigma}_n(\gamma)$. Clearly such a division achieves that the resulting process is $N(0,1)$ and not $N(0,\gamma)$ for each γ, which means that the limit is not Wiener measure, and therefore I did not understand how the resulting process can end up at W^*, which is undefined, but is suggested through the use of the letter W to be some type of Wiener measure.

2. p. 11, first equation: there is an expectation of an absolute value there; should the absolute value be there?

3. p. 12, Lemma 1: typo: “$\sum_{i=1}^{J_n-1} < n$” should read “$\sum_{i=1}^{J_n-1} M_{ni} < n$”

4. On page 7, I could not verify that (18) holds; that is, I could not see that none of the X_{nt}^* is not counted twice or excluded in the $\sum_{j=1}^{J_n} Z_{nj}^*$.

5. At this time, I still do not understand the reasoning of the author when it comes to conditioning issues. The author to his credit has added explanations, and very likely the problem is simply my lack of experience with bootstrap issues. However, on page 9

for example, the author says that Lemma 5 implies that there exists a finite, monotone function $B(\cdot)$ that has a limit of 0. As I read Lemma 5, I would think that this $B(\cdot)$ depends on $\mathcal{M}$. Similarly, on page 15, the author applies Lemma 6, Equation 37; yet that result was derived for an unconditional expectation appearing on the spot where the author now uses $E_{\mathcal{M}}$. In short, I am still confused by some of the conditioning issues, but am willing to assume that the problem is with me and not with the author.