Report of the revision of "Block bootstrap consistency under weak assumptions," by Gray Calhoun

This paper has been thoroughly revised. In his letter, the author acknowledges the changes that were made. It is still my opinion that in terms of subject matter and exposition, this paper is in principle suitable for publication in *Econometric Theory.* However, I still had trouble going through the author's arguments. Below is a list of points at which I got stuck; I would expect that the author will be able to clarify many of these issues, but some seem to be genuine mistakes.

1. Setup of Theorem 1: it is not clear to me from the text how exactly the last block that ranges up to n is dealt with. Is $K_{n,J_n} = n$?

2. Page 7, Equations (16), (17) and (18): I do not understand why (16) is implied by Hall and Heyde's CLT by (17) and (18). This may be due to my lack of exposure to the bootstrap literature. It seems that the author applies a conditional version of a CLT that is unconditional; however, the exact argument escapes me. I seem to be able to derive the author's result by working directly with Hall and Heyde's proof, but I failed to see how the shortcut that the author uses works. Some clarification or a good reference may be needed here.

3. Equation below equation (19): I cannot quite see this. There is the issue of the last block, but that must be fixable because it is asymptotically negligible; but I also do not understand how this formula works overall. Obviously by (12),

$$E^*(g(Z^*_{nj}|\mathcal{M}_n) = E^*(g(\frac{1}{\sqrt{n}}\sum_{t=K_{n,j-1}+1}^{K_{nj}}(X^*_{nt} - \bar{X}))|\mathcal{M}_n)$$

and by (14) and (15),

$$\frac{1}{n}\sum_{\tau=0}^{n-1} g(Z_n(\tau, M_{nj})) = \frac{1}{n}\sum_{\tau=0}^{n-1} g(\frac{1}{\sqrt{n}}\sum_{t\in I_n(\tau,M_{nj})}(X_{nt} - \mu))$$

$$\approx \frac{1}{n}\sum_{\tau=0}^{n-1} g(\frac{1}{\sqrt{n}}\sum_{t=\tau+1}^{\tau+M_{nj}}(X_{nt} - \mu))$$

where the $\approx$ indicates that I have ignored the last block. However, I could not see why both expressions are identical. One odd aspect of the last formula is that it will refer to values of X_{nt} for $t > n$ for values of j for which $M_{nj} \geq 1$ (this can be seen by choosing $\tau = n - 1$).

4. p. 8, middle: "by definition": perhaps explain why the expression is $O_{a.s.}(1)$ here.

5. p. 8: reference to Van der Vaart's Lemma 2.11: this lemma involves convergence of a distribution to a continuous distribution, while your setting involves convergence in probability. Therefore, perhaps you should indicate how exactly the extension can be made to your setting.

6. Lemma 3: usually, uniform integrability of Y_n is defined as

$$\limsup_{K\to\infty} \limsup_{n\to\infty} E|X_n| I(|Y_n| > K) = 0.$$

   Are you claiming here that for the $X_n(\tau, m)$ of Lemma 3, we have

$$\limsup_{K\to\infty} \limsup_{\tau\to\infty} \limsup_{m\to\infty} \limsup_{n\to\infty} E|X_n(\tau, m)| I(|X_n(\tau, m)| > K) = 0?$$

   If so, it is not clear to me that the single-index results from Davidson and de Jong have been translated into the triple index result needed here; that is, I have trouble with the last sentence of the proof of Lemma 3.

7. Start of Theorem 2: I do not see why Lemma 5 has the stated implication.

8. p. 9, bottom: this result is an error. Davidson's theorem 15.14 is a standard maximal inequality in the spirit of Kolmogorov's inequality. Therefore, it cannot deliver the stated result, because a maximal inequality such as 15.14 only maximizes over one index only.

9. Lemma 6: this result is obtained from direct reference to existing proofs. Therefore as far as I can see, the proof as stated leaves open the possibility that C, N and ε depend on $\mathcal{M}_n$.