Referee's report on

Block bootstrap consistency under weak assumptions

by Gray Calhoun

This paper gives a proof that a normalized partial sum sequence, with increments drawn by resampling a dependent time series using the moving block or stationary bootstrap, satisfies a functional central limit theorem. This is claimed as an extension of the results due to Gonçalves and White (2002) and Gonçalves and de Jong (2003), following on from the original work on the bootstrap for dependent processes (Kunsch, Politis and Romano, Liu and Singh etc.).

The problem with this claim is that what is proved is not really comparable to the cited work. The above-cited papers, in common with most of this literature, prove uniform convergence in probability of the difference between the conditional probability measure of the statistic computed from the resampled data and the corresponding distribution in the original data; for example, Gonçalves and White (2002) Theorem 2.2 and Gonçalves and de Jong (2003) Theorem 2. This is not done here.

For the result that is given, the weakening of the permitted dependence for the observed series is not an especially striking feature, relying as it does on the fact that the partial sum process consists of independent blocks whose numbers grow with sample size. This is not very difficult or unexpected, given the results of de Jong (1997). A useful contribution would be to further weaken the conditions of (say) Theorems 1 and 2 of Gonçalves and de Jong (2003). Failing that, the paper needs to position itself more clearly as a contribution to the literature on the bootstrap.

Other comments:

1. Page 3, condition 2 of Theorem 1, $\bar{\mu}_n$ is not yet defined. (The definition is in Corollary 2, move it here.)
2. Page 3, Condition 3 of Theorem 2: "uniformly finite"? What does this mean?
3. Pages 3/4. The discussion of the random norming case is not very clear or useful. Either expand this paragraph to give the relevant extensions, or delete it.
4. Page 4, Equation 2. What is ξ? Should this read γ? Otherwise it is not clear what it refers to. I do not find it anywhere in the sequel.
5. Page 5, line before equation (5). How weak is very weak? More explicit conditions required here.

6. Page 6, Lemma 4 Condition 1. The set of block lengths is given the symbol $\mathcal{M}_n$, but this does not appear subsequently. I suspect that it is supposed to appear as the conditioning variables in the expression following equation (6).

7. The procedure described in Lemma 4 appears to be that the assembled blocks are random in length but fixed in number, so that the length of the resampled series is either greater or less than sample size *n*. This might be explained more clearly. At the moment the description leaves the reader wondering what's different.

8. The statement of Lemma 4 should specify convergence in distribution.

9. Page 12, line 3. I don't think that the symbol = surmounted by a 'd' has been defined at this point. Nor for that matter is the role of the identity symbol $\equiv$ very clear.

10. A general remark. The notation is messy, and particular I have in mind that different fonts, such as calligraphic and blackboard bold, are used too whimsically. It is really good practice to let the font convey information to the reader about the type of object it represents. Italic for algebraic quantities including functions, and also sets (caps), Roman for operators, boldface italic for matrices and vectors, and save calligraphic for exotica like sigma-fields, and blackboard bold for spaces. The author knows what (s)he means, but this type of rule makes life a lot easier for the reader who is unfamiliar with the material.