

**To inscribe an equilateral and equiangular  
hexagon in a given circle.**

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ .

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ .

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ .

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ .

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ .

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal. Likewise,  $AO$ ,  $AC$ , and  $AD$  are radii of the same circle, so they are equal.



To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal. Likewise,  $AO$ ,  $AC$ , and  $AD$  are radii of the same circle, so they are equal. It follows that  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ ,  $OF$ ,  $AC$ , and  $AD$  are equal.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal. Likewise,  $AO$ ,  $AC$ , and  $AD$  are radii of the same circle, so they are equal. It follows that  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ ,  $OF$ ,  $AC$ , and  $AD$  are equal. Observe that triangles  $ACO$  and  $ADO$  are equilateral.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal. Likewise,  $AO$ ,  $AC$ , and  $AD$  are radii of the same circle, so they are equal. It follows that  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ ,  $OF$ ,  $AC$ , and  $AD$  are equal. Observe that triangles  $ACO$  and  $ADO$  are equilateral. Thus, the angles of triangle  $ACO$  are equal to one another.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal. Likewise,  $AO$ ,  $AC$ , and  $AD$  are radii of the same circle, so they are equal. It follows that  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ ,  $OF$ ,  $AC$ , and  $AD$  are equal. Observe that triangles  $ACO$  and  $ADO$  are equilateral. Thus, the angles of triangle  $ACO$  are equal to one another. By SSS, triangles  $ACO$  and  $ADO$  are congruent.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction

Given a circle, construct its center  $O$  by III.1, then draw the diameter  $AB$ . Construct the circle centered at  $A$  with radius  $AO$ . Label the intersections of these two circles as  $C$  and  $D$ . Draw the diameters  $CE$  and  $DF$ . Join  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$ . As they are radii of the same circle,  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ , and  $OF$  are equal. Likewise,  $AO$ ,  $AC$ , and  $AD$  are radii of the same circle, so they are equal. It follows that  $OA$ ,  $OB$ ,  $OC$ ,  $OD$ ,  $OE$ ,  $OF$ ,  $AC$ , and  $AD$  are equal. Observe that triangles  $ACO$  and  $ADO$  are equilateral. Thus, the angles of triangle  $ACO$  are equal to one another. By SSS, triangles  $ACO$  and  $ADO$  are congruent. Hence, the angles of triangle  $ADO$  equal the angles of triangle  $ACO$ .

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another. Therefore, by vertical angles, angles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$ , and  $BOF$  are equal.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another. Therefore, by vertical angles, angles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$ , and  $BOF$  are equal. By SAS, it follows that triangles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$  and  $BOF$  are congruent.



To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another. Therefore, by vertical angles, angles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$ , and  $BOF$  are equal. By SAS, it follows that triangles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$  and  $BOF$  are congruent. Thus, sides  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$  equal one another, and angles  $DAC$ ,  $ACF$ ,  $CFB$ ,  $FBE$ ,  $BED$  and  $EDA$  equal one another.

To inscribe an equilateral and equiangular hexagon in a given circle.

## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another. Therefore, by vertical angles, angles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$ , and  $BOF$  are equal. By SAS, it follows that triangles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$  and  $BOF$  are congruent. Thus, sides  $AC$ ,  $CF$ ,  $FB$ ,  $BE$ ,  $ED$  and  $DA$  equal one another, and angles  $DAC$ ,  $ACF$ ,  $CFB$ ,  $FBE$ ,  $BED$  and  $EDA$  equal one another. Consequently, we have inscribed an equilateral and equiangular hexagon in a given circle.