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## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another.

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## Construction (cont.)

As angle  $COA$  equals angle  $AOD$ , by I.13, angles  $FOC$ ,  $COA$ , and  $AOD$  are equal to one another. Therefore, by vertical angles, angles  $FOC$ ,  $COA$ ,  $AOD$ ,  $DOE$ ,  $EOB$ , and  $BOF$  are equal.

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