Express

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{2005\cdot 2006}$$

in lowest terms.

Solution

Observe that

$$\frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \cdots + \frac{1}{2005 \cdot 2006} = \sum_{k=2}^{2005} \frac{1}{k(k+1)}.$$

Solution

Observe that

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{2005\cdot 2006}=\sum_{k=2}^{2005}\frac{1}{k(k+1)}.$$

Consider that

$$\frac{1}{k(k+1)}=\frac{1+k-k}{k(k+1)}=\frac{k+1}{k(k+1)}-\frac{k}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}.$$

Solution

Observe that

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Consider that

$$\frac{1}{k(k+1)}=\frac{1+k-k}{k(k+1)}=\frac{k+1}{k(k+1)}-\frac{k}{k(k+1)}=\frac{1}{k}-\frac{1}{k+1}.$$

It follows that

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{2005\cdot 2006}=\sum_{k=2}^{2005}\left(\frac{1}{k}-\frac{1}{k+1}\right).$$

Solution (cont.)

Therefore, the series may also be written as

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\left(\frac{1}{4}-\frac{1}{5}\right)+\cdots+\left(\frac{1}{2005}-\frac{1}{2006}\right)$$

which telescopes to $\frac{1}{2}-\frac{1}{2006}$.

Solution (cont.)

Therefore, the series may also be written as

$$\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{3} - \frac{1}{4}\right) + \left(\frac{1}{4} - \frac{1}{5}\right) + \cdots + \left(\frac{1}{2005} - \frac{1}{2006}\right)$$

which telescopes to $\frac{1}{2} - \frac{1}{2006}$. Consequently,

$$\frac{1}{2\cdot 3} + \frac{1}{3\cdot 4} + \cdots + \frac{1}{2005\cdot 2006} = \frac{501}{1003}.$$