Non-life Insurance Mathematics HW1.

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LEV-function $E[X;x]$ properties:

As 1. and 4. were already proven, here is the proof for 2, 3. and 5. property.

2. $EX = E[X;x] + e(x)(1-F_X(x))$

$$e(x) = \frac{\int_x^\infty (1-F_X(t))\,dt}{1-F_X(x)}$$

$$EX = \int_0^x [1-F_X(y)]\,dy + \frac{\int_x^\infty [1-F_X(y)]\,dy}{\cancel{1-F_X(x)}} \cancel{(1-F_X(x))} =$$

$$= \int_0^x [1-F_X(y)]\,dy + \int_x^\infty [1-F_X(y)]\,dy = \int_0^\infty [1-F_X(y)]\,dy =$$

$$= E[X;\infty] = E(\min(X;\infty)) = EX$$

3. Similarly to the end of 2. property proof:

if $x \to \infty$: $\lim_{x\to\infty} E[X;x] = \lim_{x\to\infty} E(\min(X;x)) =$

$$= E(\min(X;\infty)) = EX$$

5. Using 2. property we know that

$$EX = E[X;x] + e(x)(1-F_X(x))$$

So $E[X;x] = EX - e(x)(1-F_X(x))$ | $e(x) = \frac{\int_x^\infty [1-F_X(y)]\,dy}{1-F_X(x)}$

Therefore

let $z = aX+b$

$$E[aX+b;x] = E(aX+b) - \int_x^\infty [1-F_Z(y)]\,dy =$$

$$= aEX + b - a\int_{\frac{x-b}{a}}^\infty [1-F_X(y)]\,dy =$$

$$= a\int_0^\infty [1-F_X(y)]\,dy - a\int_{\frac{x-b}{a}}^\infty [1-F_X(y)]\,dy + b =$$

$$= a\int_0^{\frac{x-b}{a}} [1-F_X(y)]\,dy + b = aE\left[X;\frac{x-b}{a}\right] + b$$