Probability & Statistics.

Statistics:- It is game of numbers.
↳ how to analyze, present, manupliate & collect info to reach appropiate conclusion.

Types:- descriptive & stationary

Dictionary words:-

(i) → Population (ii) → Sample

(iii) → Data (iv) → Types of data (v) → Variable

population:-

(i) Each and every indiviual belonging to our target (object we are discussing) (all)

Sample:- (selected)

(ii) some portion or part of that population (target)

Data:-

(iii) info we are collecting from indiviual

(iv) Types of data

(i) Qualitative (based on characters)

(ii) Quantitative (based on #'s)

- Countinuous (measure) (height)
- discreat (count-able)

(v) Variable

indiviual to indiviual (character of object varies (they are quantities)

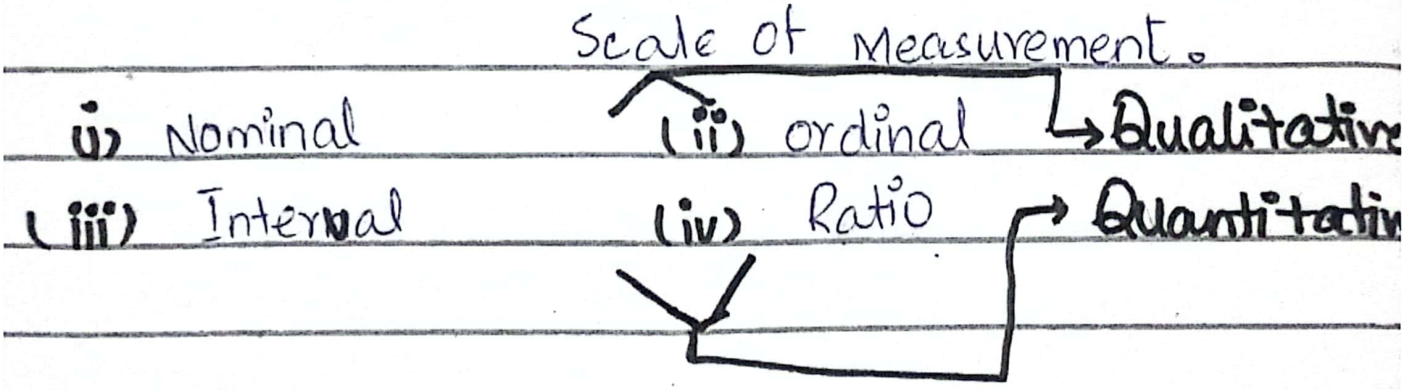
→ not fixed

→ Qualitative
→ Quantitative
> variable Types

→ Numeric value of papulation ⇒ parameter

→ Numeric value of sample ⇒ statistics

Level of Measurements:-



Nominal: defines only names (no ranking no other info)
→ just give one info not much more. (selective info)

Ordinal: upgrade nomial (provide rank)
⇒ Order ho sakay

Interval: if presence of zero object (object exist karta hai)

Ratio:- if **absence** of object with zero value → (object exists hi nahi karta ho.)

→ zero means nothing.

Example of level of Measurement:

~~Nawaz Sharief~~

Nawaz shareef
Imran Khan > Nominal

Imran is better than Nawaz → ordinal

Imran won the election by 4 ~~votes~~ → interval

Imran ~~kes~~ is bigger in height than nawaz → ratio

Cricket:

Babar Shahdab → Nominal

Shahdab is player while babar is captain → G

Shahdab catches 5 catchs in each match than baber → interval

Shahdab lives far away from babar → ratio

Descriptive Statistics:

↳ Collection
↳ Analyzation
↳ Presentation < Table / Graph / diagram

Source of data

Primary Secondary

⇒ **Primary:** data collected by user (First-hand.
(Khud ki study kai liay kaam (on-de
e.g: survey, QNA

⇒ **Secondary:** Ready mate.data (Data alread
collected
e.g: Internet, webserver

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Frequency distribution (Table Form of dataset)

→ At First Find range (calculate)

range is the difference between highest number and the minimum value of dataset.

→ Number of classes (K)

$$K = 1 + 3.32 \log N$$

N is the number of datapoint.

→ always round the number to get accurate answer

→ decide / talk about class interval.

(how it is divided

h = Range / # of classes

h is the ratio first twosteps

Prepare a frequency distribution For this score given 42, 22, 55, 18, 50, 10, 33, 29, 17, 29, 29, 27, 34, 15, 40, 42, 40, 41, 35, 27, 44, 31, 38, 19, 54, 55, 38, 19, 20, 30, 42, 59, 15, 19, 27, 23, 40, 32, 28, 51

① higest = 59 lowest = 10

Range = 49

$K = 1 + 3.32 \log(40) = 6.31 \approx 7$

$$h = \frac{R}{K} = \frac{49}{7} = 7$$

∴ aisa starting point lo jis mai lowest number series ka lazmi aiy.

Classes (U.C.L – L.C.L)	Tallybar	freq	class boundary	class mid point
10 - 16	III	3	9.5 – 16.5	13
17 - 23	𝍸 II	8	16.5 – 23.5	20
24 - 30	𝍸 III	8	23.5 – 30.5	27
31 - 37	𝍸	5	30.5 – 37.5	34
38 - 44	𝍸 𝍸	10	37.5 – 44.5	41
45 - 51	II	2	44.5 – 51.5	48
52 - 58	III	3	51.5 – 58.5	55
59 - 65	I	1	58.5 – 65.5	62
~~66 - 72~~		$\Sigma f = 40$		

class boundary:- $\frac{\text{upper class limit (next)} - \text{Lower class lim (Previous)}}{2}$

$$= \frac{17 - 16}{2} = \frac{1}{2}$$

replace the answer as in subtract if from all

upper class limit and all it in L.C.L

$= 10 - 0.5 = 9.5$

$= 16 + 0.5 = 16.5$

Class Mid Point / Class Mark (X)

pick any one (classes or boundaries).

$$= \frac{\text{upper class Limit} + \text{lower class}}{2}$$

$$= \frac{10+16}{2} = \frac{26}{2} = 13$$

F:- Previous sara add hota hai

comlative freq (F)	decumulative DF	R.F f_0 =	Pre Angle $f_0 + f_1$
		$f_2 = f_0 + f_1 + f_2$ ---	
→ 3	37+3=40	7.5 %	27 Relative = $\frac{f}{\Sigma f} \times 1$
3+8=11	29+8=37	20 %	72
11+8=19	21+8=29	20 %	72 $\theta = \frac{f}{\Sigma f} \times 3$
19+5=24	16+5=21	12.5 %	45
24+10=34	6+10=16	25 %	90
34+2=36	4+2=6	5 %	18
36+3=39	1+3=4	7.5 %	27
39+1=40	1	2.5 %	9

∴ decumaltive reverse order mai hoti.

Graphical Representation:-

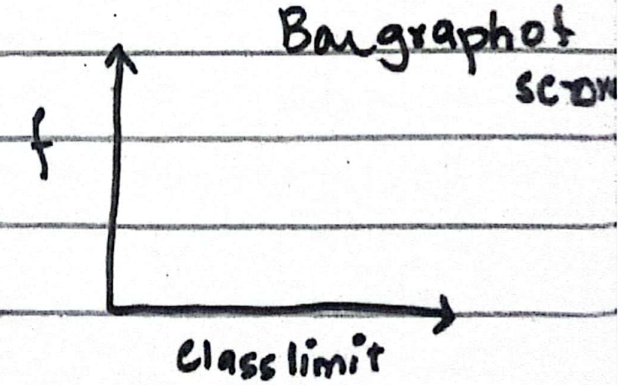
→ Bar graph → ~~poly~~
→ histogram
→ frequency polygam / O-Give
→ pie chart
→ Box & whisker plot

→ aik hi nazar mai trend daso.

1- Bar graph:

→ To check itemsale.
→ enrollement trend

simple graph we polt frequency on y-axis and X- class limit (year, mon against jis kai dataset hai.

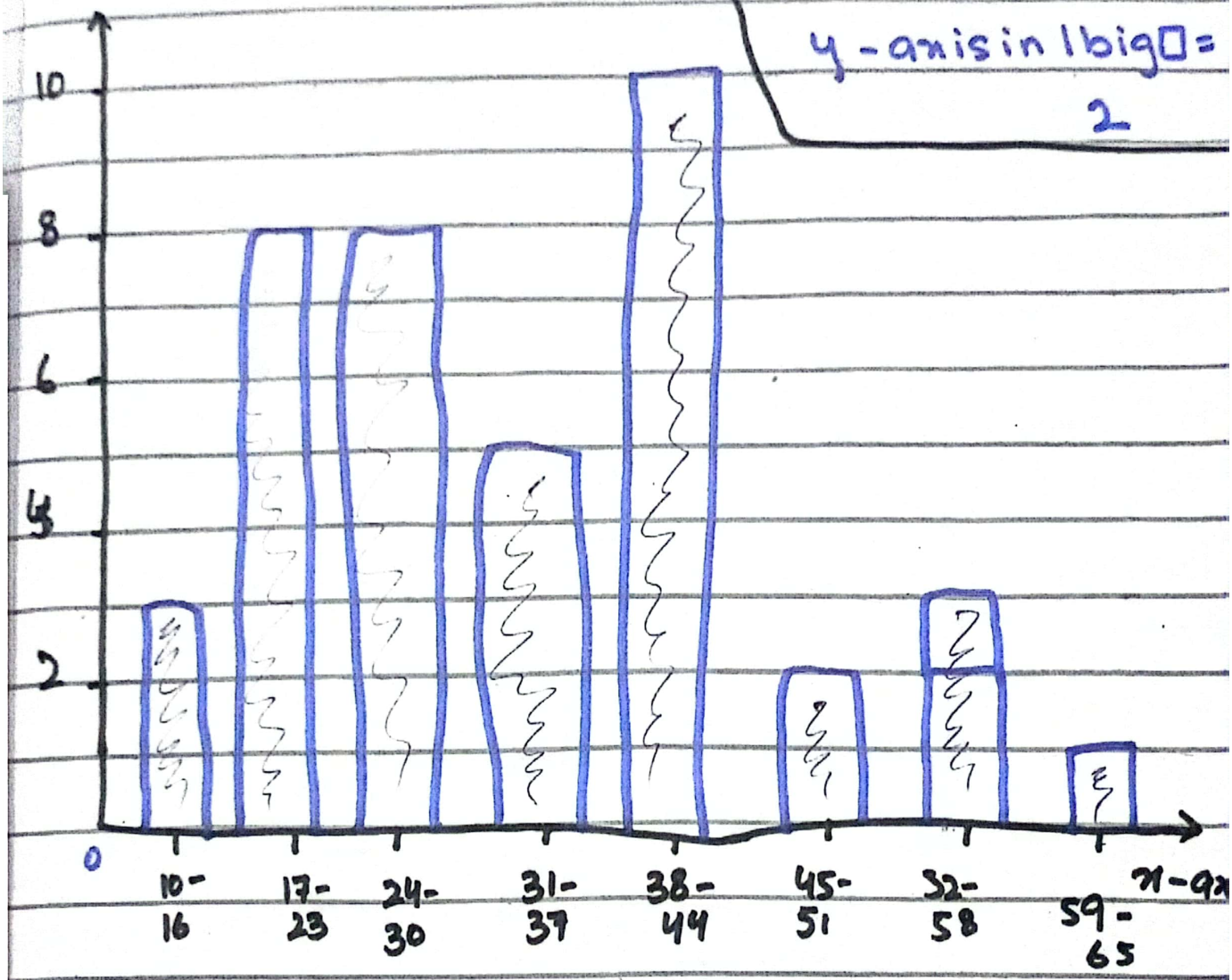


Bar graph.

Scale

x-axis in 1 big □ = 1 class

y-axis in 1 big □ = 2



The max score is range = 38 - 44

min score = 59-65

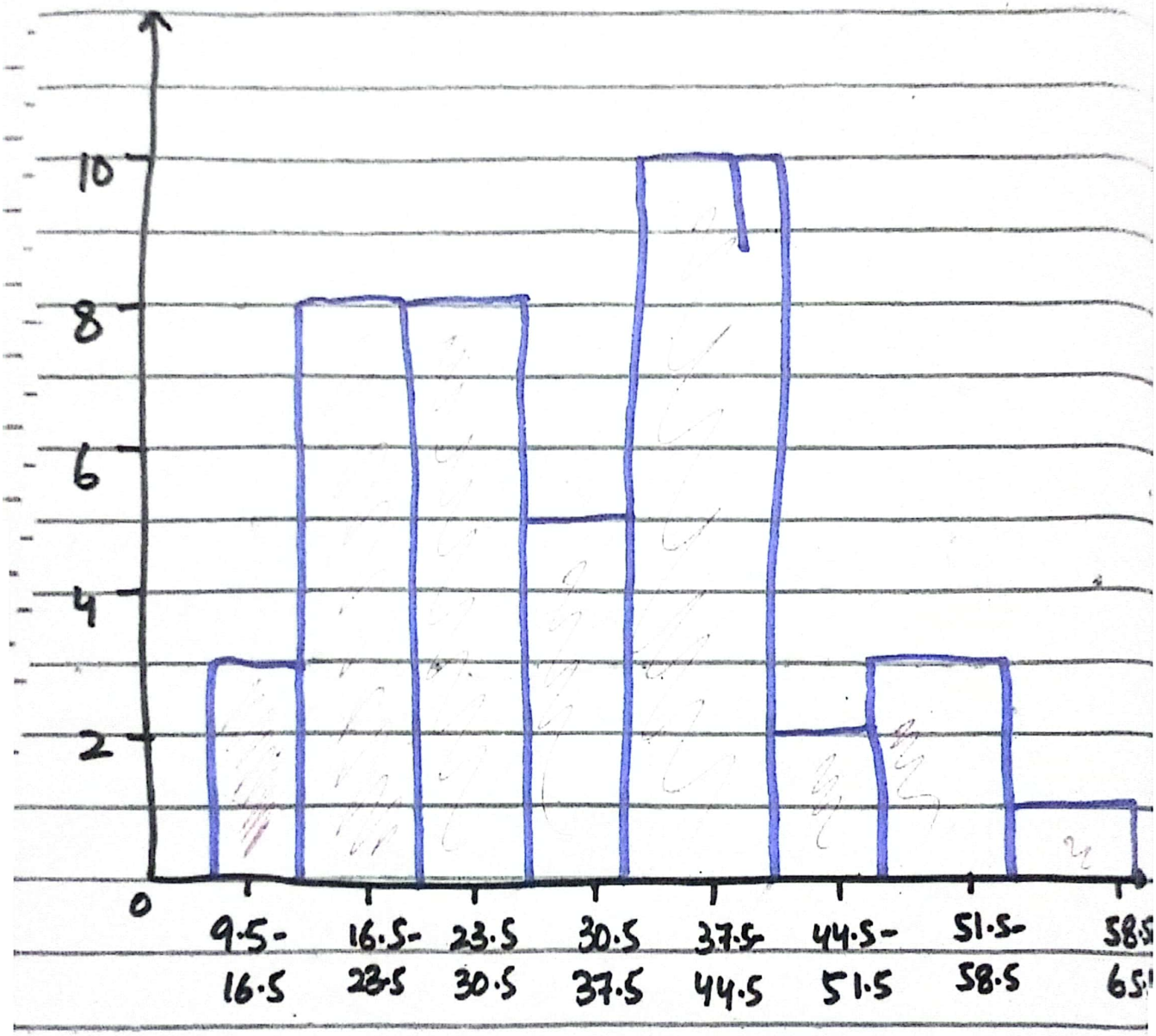
class with same score = 17 - 23, 24 - 30 → 8

10 - 16, 52 - 58 → 3

Histogram:-

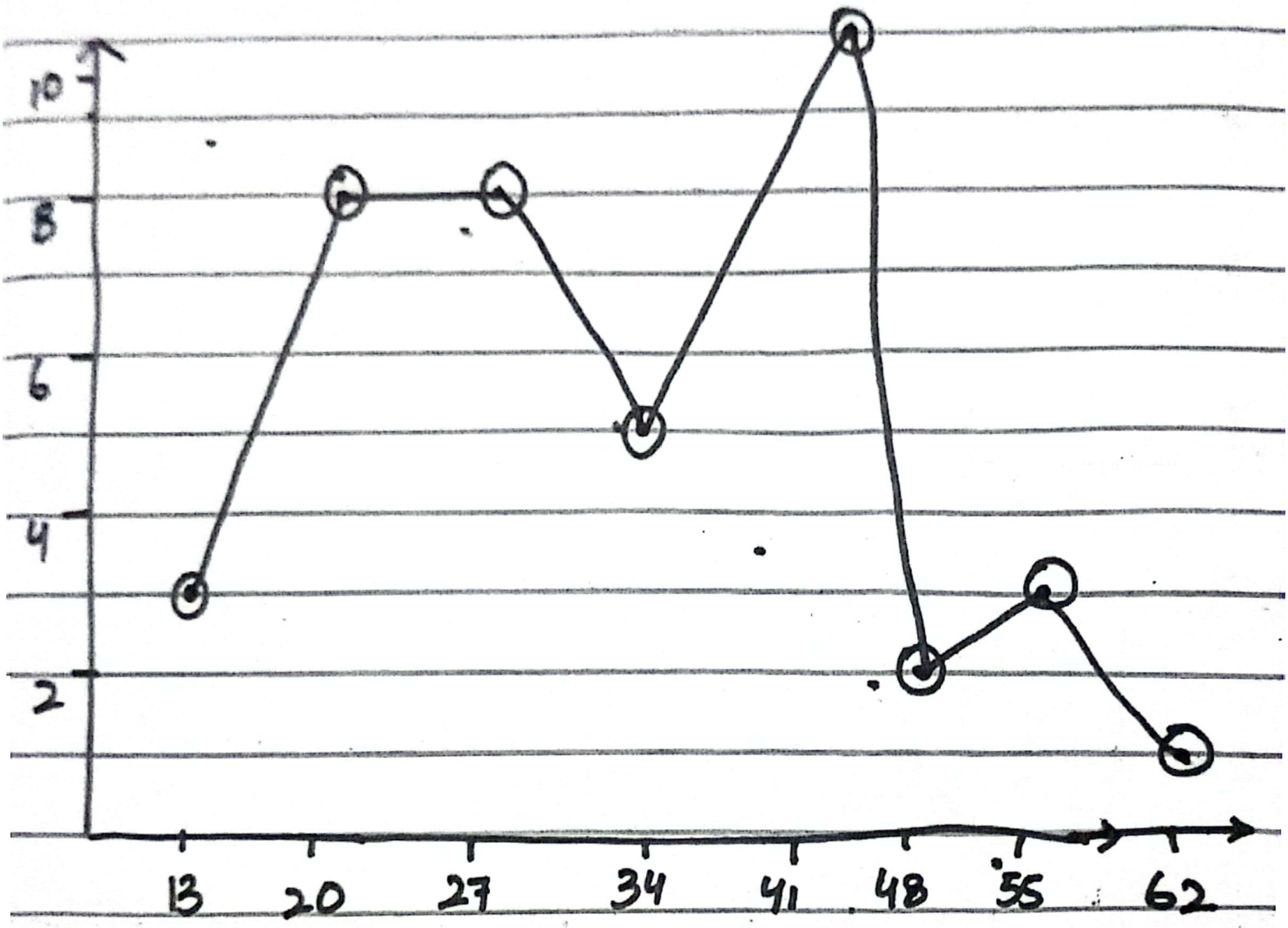
Take class boundaries (joint (jump)) nahi hota.

$Y \rightarrow f$ $X \rightarrow C.B$



Frequency Polygone.

Take x-axis Midpoint / class Mark



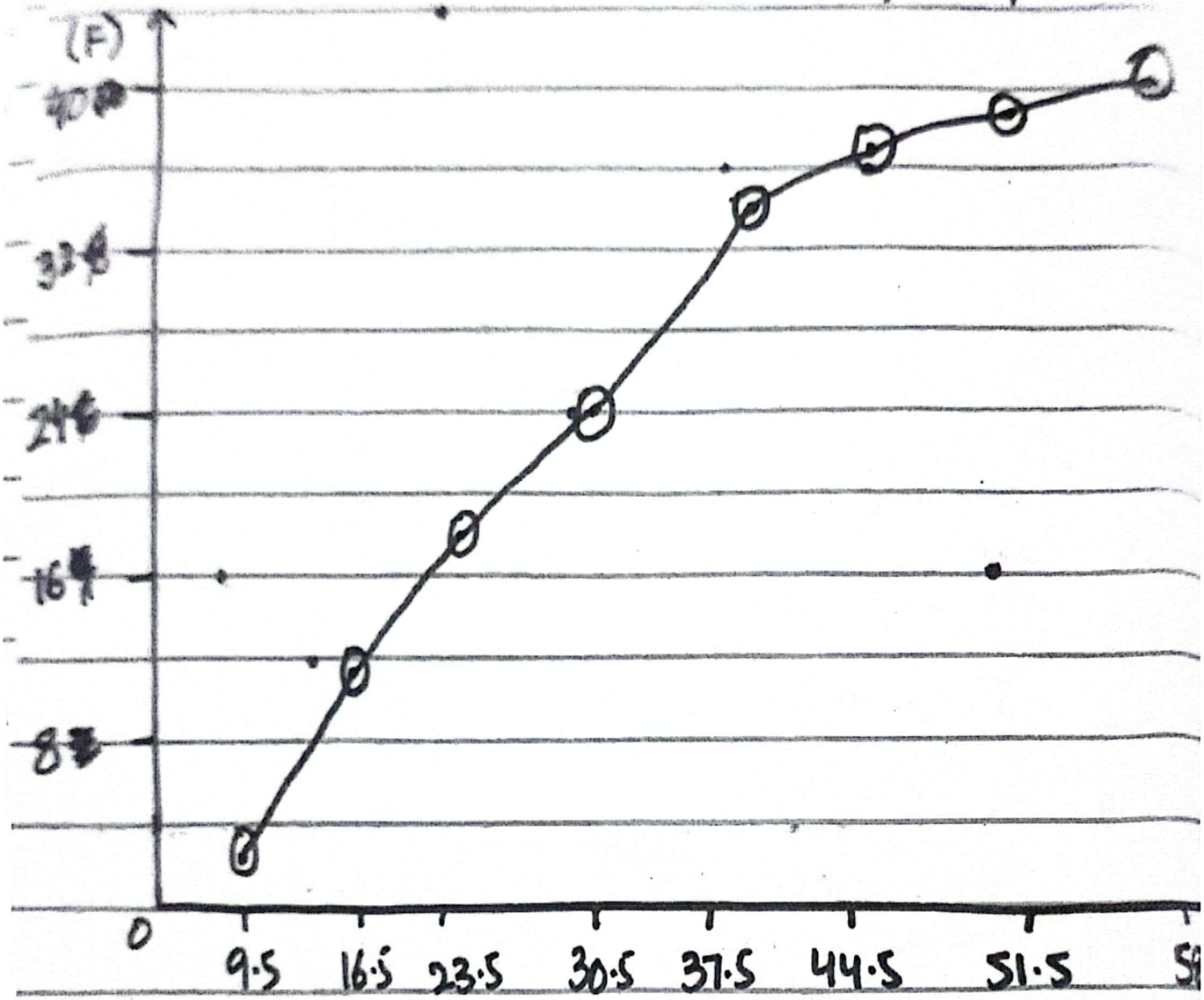
along x-axis = 1 big □ = 1 class

along - y-axis = 2

O-give →

Take class boundary Lower part o

Y- axis ⇒ comulative frequency

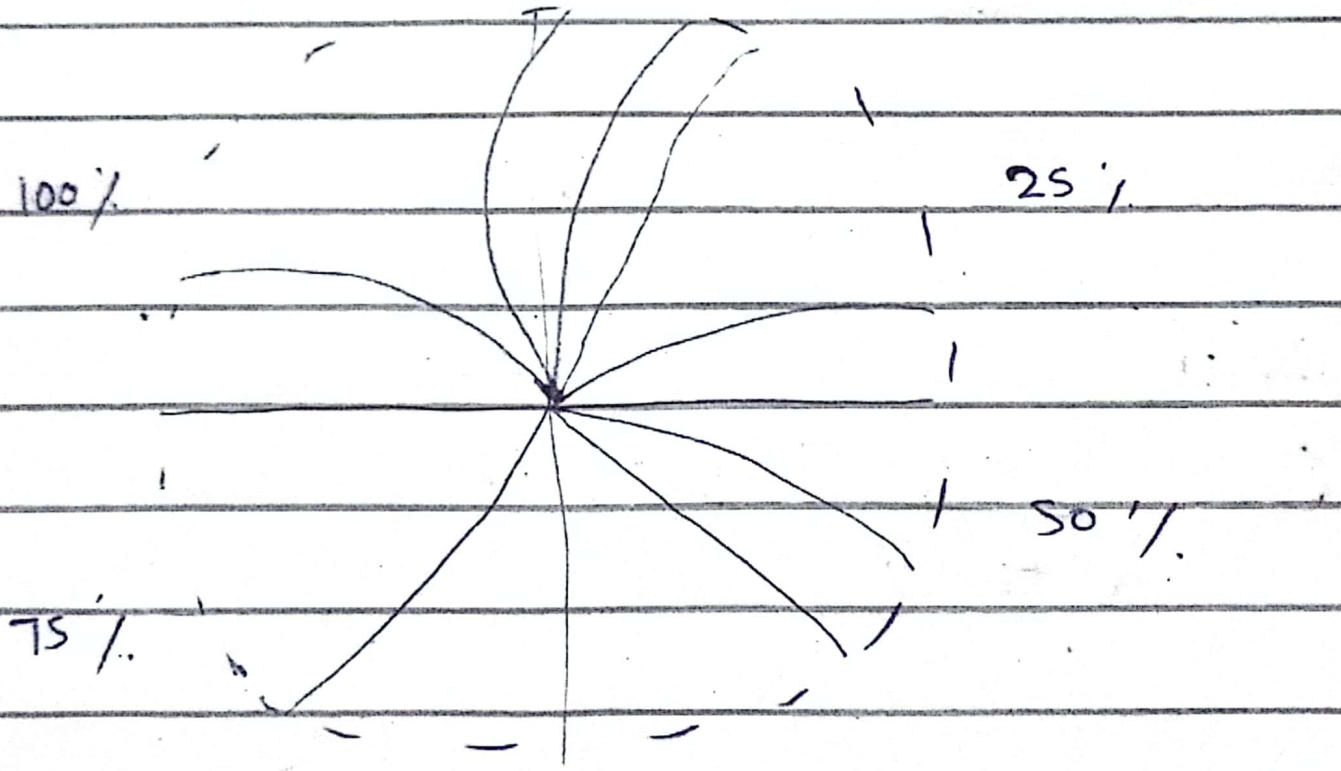


L-C.B

Pie Chart.

We Take relative frequency (data & ratio)
draw circle (To compute first we find
pie angle = $\frac{\text{freq}}{\Sigma f} \times 360^\circ$

draw - with D (use in geometry).



Box & whisker Plot (Random

5- number summary plot

(Min, Max, Q_1, Q_2, Q_3)

quartile

① arrange in ascending order

20, 12, 17, 20, 14, 11, 13, 18, 10, 11

⇒ 10, 11, 11, 12, 13, 14, 17, 18, 20, 20

Min = 10 Max = 20

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ obs. of an array

$= \frac{10+1}{4} = 2.75$ (b/w 2 points hai)

$= \frac{11+11}{2} = \boxed{11}$

$Q_2 = \left(\frac{2n+1}{4}\right) = \frac{2(10)+1}{4} = 5.25$

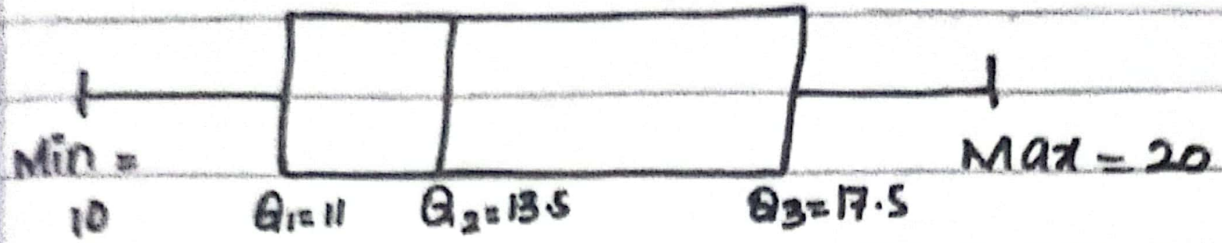
$= \frac{13+14}{2} = \boxed{13.5}$

$Q_3 = \frac{3n+1}{4} = \frac{31}{4} = 7.75$

$= \frac{17+18}{2} = \boxed{17.5}$

Min = 10 Max = 20 $Q_1 = 11$ $Q_2 = 13.5$

$Q_3 = 17.5$



if $Q_3 - Q_2 = Q_2 - Q_1$ Normal (Mid)

if $Q_3 - Q_2 > Q_2 - Q_1$

if $Q_3 - Q_2 < Q_2 - Q_1$

Ascending Order:-

12, 14, 15, 15, 16, 16, 18, 18, 19, 20, 21, 24, 24, 2
25, 25, 26, 26, 27, 27, 28, 30, 33, 34, 35, 36,
36, 37, 38, 39, 41, 42, 43, 45, 46, 46, 46, 48,
48, 52, 53, 57, 58, 59, 59, 60, 60, 63, 63, 6

Min = 12 **Max = 64**

$$Q_1 = \frac{50+1}{4} = \frac{51}{4} = 12.75$$

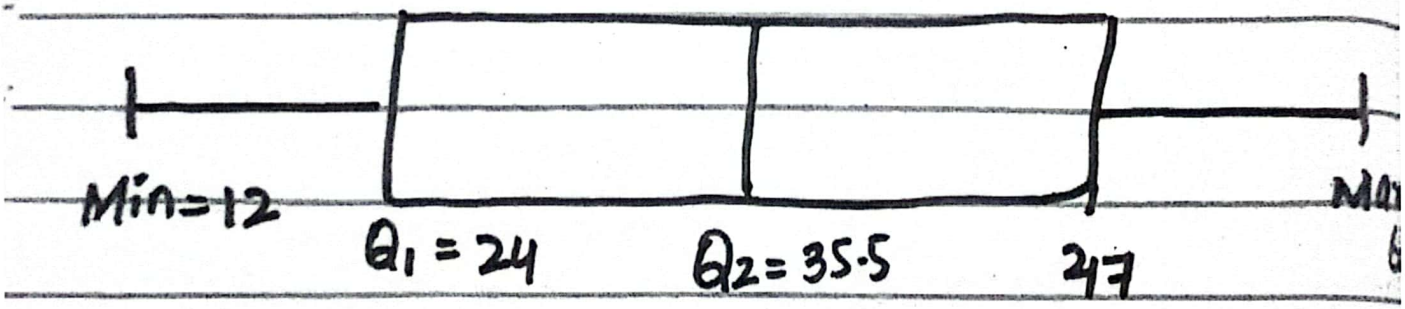
$$\frac{24+24}{2} = 24$$

$$Q_2 = \frac{2(50)+1}{4} = 25.25$$

$$= \frac{35+36}{2} = \frac{71}{2} = 35.5$$

$$Q_3 = \frac{3(50)+1}{4} = 37.75$$

$$= \frac{46+48}{2} = \frac{94}{2} = 47$$



→ Normal (Case III) skewness.

Measures of Central tendancy

It represent the centre of the dataset & most commonly used measures are known as mean, median, modle, quantize, skewnes, Kurtosis

Mean — AM (2) GM (3) H·M (4) W·M

Quatiles: Quartiles (2) deciles (3) Percentile

Mean:- (Average)

Arithematic (Simple Mean)

Geometric (rate, ratio, %)

Harmonic " "

Weighted (% har value kai sath weight atlach)

$$\text{A·M} = \bar{X} = \frac{\Sigma X}{n}$$ (for ungroup data)

X = 20, 12, 17, 20, 14, 11, 13, 18, 10, 11

$\Sigma X = 146$

$$\bar{X} = \frac{146}{10} = 14.6$$

On average score = 14.6

Harmonic mean: reciprocal of values

The reciprocal of Mean is known as H.M

$$H.M = \frac{n}{\Sigma\left(\frac{1}{x}\right)}$$

$$= \left(\frac{1}{20}\right) + \left(\frac{1}{12}\right) + \text{---} + \left(\frac{1}{11}\right)$$

$$= 0.72$$

$$= \frac{10}{0.72} = 13.8$$

Geometric: The n^{th} product of the values with n^{th} root is known as G.M

$$G.M = (X_1 X_2 \ldots X_n)^{1/n}$$

or

$$= \text{Antilog}\left[\frac{\Sigma \log(x)}{n}\right]$$

Find log x =

$$\Rightarrow \log X = 11.5$$

G.M

$$= \text{Antilog}\left[\frac{11.5}{10}\right]$$

= Antilog [1.15]

= 14.1

Mam walay question:

$$A.M = \bar{x} = \frac{\Sigma x}{n} = \frac{1319}{40}$$

$$= 32.975$$

$$X.M = \frac{n}{\Sigma\left(\frac{1}{x}\right)} = \frac{40}{0.29 + 0.28 + 0.19 + 0.17 + 0.19}$$

$$= \frac{40}{1.43} = 27.835$$

$$G.M = (2.2781 \times 10^{59})^{1/40}$$

$$= 30.47$$

Measures of Central Tendancy

For ungroup data

Medium:-

Arrange in order & take exact middl value of an array.

$= \left(\frac{n+1}{2}\right)^{th}$ obs of an array.

Mode: Most repeated / highest / occured value in any array.

↳ if there is no mode in any array then we can say that <u>no mode exist</u>.

Quantiles:- ÷ the # line in equal part

Qualtiles: ↘

4 equal parts and there are 3 qualtiles.

deciles

10 equal parts

Deceals Percentile

100 equal parts

Q:-

20, 12, 17, 20, 14, 11, 13, 18, 10,

Ascending:-

= 10, 11, 12, 13, 14, 17, 18, 20, 20

Median:-

$$\tilde{X} = \left(\frac{n+1}{2}\right)^{th}$$

$$= \left(\frac{10+1}{2}\right)^{th} = \frac{11}{2} = 5.5$$

$$\tilde{X} = \frac{13+14}{2} = 13.5$$

Mode:- This dataset have 2 mode

Mode 1 = 20

Mode 2 = 11

Quartiles :- $\left(\frac{n+1}{4}\right)^{th}$

$$= \frac{10+1}{4} = \frac{11}{4} = 2.75$$

$$Q_1 = \frac{11+11}{2} = 11$$

$Q_2 = D_5 = P_{50} = \text{Median}$

Q_2 = Median :-

$$D_1 = \left(\frac{n+1}{10}\right)^{th}$$

$$D_5 = 5\left(\frac{n+1}{10}\right)^{th} = \text{Median}$$

$$Q_3 = 3\left(\frac{n+1}{4}\right)$$

$$= \frac{3(10)+1}{4}$$

$$= \frac{31}{4} = 7.75$$

$$= \frac{17+18}{2} = 17.5$$

$$D_1 = \left(\frac{10+1}{10}\right)^{th} = \frac{11}{10} = 1.1$$

$$D_5 = 5\left(\frac{10+1}{10}\right) = \frac{51}{10} = 5.1$$

$$= \frac{13+14}{2} = 13.5$$

$$D_2 = 2\left(\frac{10+1}{10}\right) = \frac{21}{10} = 2.1$$

$$= \frac{11+11}{2} = 11$$

Measures of central Tendency for group dataset

$$A.M = \bar{x} = \frac{\Sigma fx}{\Sigma f}$$

$f \rightarrow$ frequency

$x \rightarrow$ class Mark / class Midpoint

↳ Take average ÷(2)

classes	f	x	fx	$1/x$	$f(1/x)$

$$H.M = \frac{\Sigma f}{\Sigma f\left(\frac{1}{x}\right)}$$

$$G.M = \text{Antilog}\left[\frac{\Sigma f \log x}{\Sigma f}\right]$$

classes	f	x	log x	f log x

→ $\frac{n}{2}$ (sum of freq) in cumulative value → check in c.f column where it stand sour kay class is Median c

Median:- $l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

l → lower class boundary of median class
h → class interval " " "
f → frequency " "
c → cumulative freq " "

The class which contains $\frac{n}{2}$ in F is known as Median class

Mode:- $l + \left\{\frac{f_m - f_1}{(f_m - f_1) + (f_m - f_2)}\right\} \times h$

l → lower class boundary of Model class
f_m → frequency of Model class
f_1 → frequency above "
f_2 → " below " "
h → class interval of Model class

class highest frequency → Model class
pehli class f_1 =

Quartiles:-

$$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - c\right)$$

$l \rightarrow$ lower class boundary of Q_1 class

$h \rightarrow$ class interval of Q_1 class

$f \rightarrow$ frequency of " "

$n \rightarrow$ Total frequency

$c \rightarrow$ cumulative freq. preceding to Q_1 class.

The class which contains $\frac{n}{4}$ is F is known as Q_1 class

$$Q_2 = l + \frac{h}{f}\left(\frac{2n}{4} - c\right)$$

$l \rightarrow$ lower class boundary of Q_2 class.

$h \rightarrow$ class interval " " "

$f \rightarrow$ frequency

$c \rightarrow$ cumulative freq. preceding in Q_2 class

The class contain $\frac{2n}{4}$ in F. in Q_2 class

$$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - c\right)$$

$l \rightarrow$ lower class boundary of Q_3 class

$h \rightarrow$ class interval " " "

$f \rightarrow$ frequency

deciles:-

$$D_1 = l + \frac{h}{f}\left(\frac{n}{10} - c\right)$$

$$D_7 = l + \frac{h}{f}\left(\frac{7n}{10} - c\right)$$

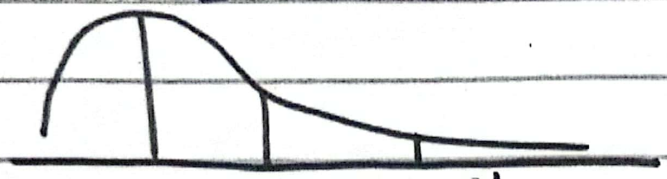
percentiles:-

$$P_1 = l + \frac{h}{f}\left(\frac{n}{100} - c\right)$$

Note: $Q_2 = D_5 = P_{50} = \text{Median}$

Skewness:- Types:- → aik tail ki taraf zaida hota hai

(i) Case - I



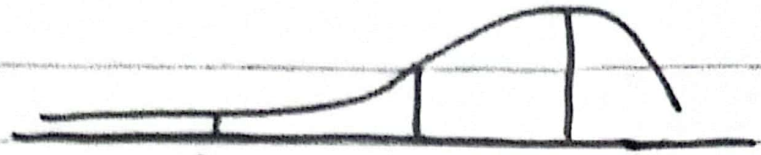
if $Q_3 - Q_2 > Q_2 - Q_1$

Mean > Median > Mode

Skewness is +ve.

or the distribution is +vly skewed.

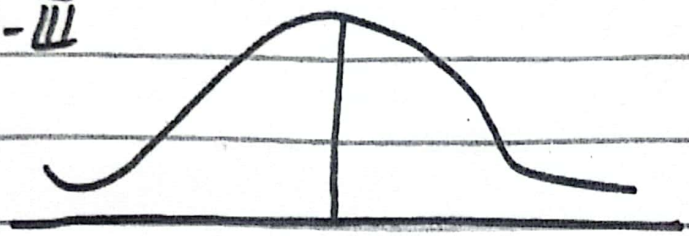
case II



Mean < Median < Mode

Skewness is -ve or the distribution is -ve skewed.

case-III



Mean = Medium = Mode

Normal

No skewed.

Grouped dataset.

classes	freq	(x Midpoint)	fx	1/x	f(1/x)	Log x
10-16	3	13	39	0.07	0.2307	1.113
17-23	8	20	160	0.05	0.4	1.301
24-30	8	27	216	0.03	0.296	1.43
31-37	5	34	170	0.029	0.145	1.531
38-44	10	41	410	0.024	0.24	1.612
45-51	2	48	96	0.020	0.04	1.681
52-58	3	52	156	0.019	0.057	1.716
59-65	1	62	62	0.016	0.016	1.792

$\Sigma = 40$ $\Sigma fx = 1309$ $\Sigma f(1/x) = 1.4277$

f log x	C.B	C.F
3.339	9.5-16.5	3
10.408	16.5-23.5	11
11.44	23.5-30.5	19
7.655	30.5-37.5	24
16.12	37.5-44.5	34
3.362	44.5-51.5	36
5.148	52.5-58.5	39
1.792	58.5-65.5	40

$\Sigma f = 59.285$

$$A.M = \frac{\Sigma f x}{\Sigma f} = \frac{1309}{40} = 32.7 \text{ Marks}$$

$$H.M = \frac{\Sigma f}{\Sigma f(1/x)} = \frac{40}{1.4277} = 28.01 \text{ Marks}$$

$$G.M = \text{antilog}\left[\frac{\Sigma f \log x}{\Sigma f}\right] = \text{antilog}\left[\frac{59.285}{40}\right]$$

$$= \text{antilog}[1.482] = 30.34 \text{ Marks.}$$

Median: $l + \frac{h}{f}\left(\frac{n}{2} - c\right)$

$= \frac{40}{2} = 20$ ∴ underline class is Median class

$$= 30.5 + \frac{7}{5}\left(\frac{40}{20} - 19\right) = 30.5 + \frac{7}{5}(1)$$

$$= 31.9$$

Mode $= l + \frac{(f_m - f_1)}{(f_m - f_1) + (f_m - f_2)} \times h$

highest freq = 10 bwe → Model class

$=$ Mode $=$

$$= 37.5 + \left[\frac{(10-5)}{(10-5)+(10-2)}\right] \times 7$$

$= 40.17$ Marks.

$$Q_1 = l + \frac{h}{f}\left(\frac{n}{4} - c\right)$$

$$= \frac{n}{4} = \frac{40}{4} = 10$$

lies in second

$$= 16.5 + \frac{7}{8}\left(10 - 3\right)$$

$$= 22.625$$

$$Q_3 = l + \frac{h}{f}\left(\frac{3n}{4} - c\right)$$

$$= \frac{3(40)}{4} = 30$$

$$= 37.5 + \frac{7}{10}\left(30 - 24\right)$$

$$= 37.5 + \frac{7}{10}(6) = 41.7$$

percentile:

$$P_1 = l + \frac{h}{f}\left(\frac{n}{100} - c\right)$$

$$\frac{80n}{100} = 32$$

$$= 37.5 + \frac{7}{10}\left(32 - 24\right)$$

42.1

decile: $l + \frac{h}{f}\left(\frac{7n}{10} - c\right)$

$\frac{7(40)}{10} = 28$

$= 37.5 + \frac{7}{10}(28 - 24)$

$= 40.3.$

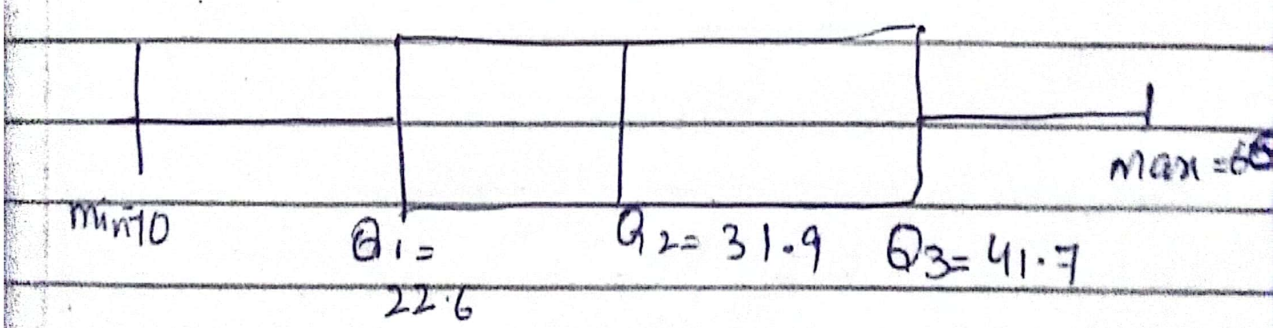
Skewness:-

Box & wisker.

$X_{min} = 10$ $X_{max} = 65$, $Q_1 = 22.6$

$Q_2 = 31.9$ $Q_3 = 41.7$

$Q_2 - Q_1 = 9.3$ $Q_3 - Q_2 = 9.8$



+ve skewed.

Mean = 32.7 Median = 31.9 mode=40

Measure of Dispersions.

↳ spread / varies of dataset

↳ Variation of data batata hai

absolute measure

- contains the same unit as dataset
- → range
- → Variance
- → Standard deviation

relative measure

- Unit free term
- → Coefficients of variation

Range:-

gives idea about the variation of observations..

ungroup:- $X_{max} - X_{min}$

group:- $R = U.C.L \text{ (Last Class)} - L.C.L \text{ (1st class)}$

(+ unit) attached.

Variance: spread of the values batata hai from their average points.

→ measure of variability which is sum of square deviations of the values from their average point

σ^2 = population s^2 = sample data

ungrouped:-

$$\sigma^2 = s^2 = \frac{\Sigma(x-\bar{x})^2}{n} \text{ or } \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

grouped:-

$$s^2 = \frac{\Sigma f(x-\bar{x})^2}{\Sigma f} \text{ or } \frac{\Sigma f x^2}{\Sigma f} - \left(\frac{\Sigma f(x)}{\Sigma f}\right)^2$$

Standard deviation:-

Take sq. root of variance

$$s = \sqrt{s^2} = \sqrt{\Sigma\left(x-\bar{x}\right)^2/n}$$

co-effient of variation:-

$$C.V = \frac{s}{\bar{x}} \times 100$$

where $\bar{x} = \frac{\Sigma x}{n}$ or $\frac{\Sigma f x}{\Sigma f}$

جس کے ساتھ freq ہوتی ہے وہ grap data ہوتا ہے اور نہ
ہو تو dataset ungroup ہوتا ہے -

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n}} \quad \text{or} \quad \sqrt{\frac{\Sigma f(x-\bar{x})^2}{\Sigma f}}$$

↳ ungroup ↳ group

For ungrouped

Calculate range, standard, deviation, C.V.

20, 12, 17, 20, 14, 11, 13, 18, 10, 11

Range:-

$X_{max} - X_{min}$

$= 20 - 10 = 10$

Variance:-

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{n}$$

$$\bar{x} = \frac{\Sigma x}{n} = \frac{146}{10} = 14.6$$

$$(x-\bar{x})^2 = (20-14.6)^2 =$$

$$S^2 = \frac{\Sigma(x-\bar{x})^2}{n} = \frac{132.4}{10} = 13.24$$

$Mark^2$

x	$(x-\bar{x})^2$	x^2
20	$(20-14.6)^2 = 29.16$	400
12	6.76	144
17	5.76	289
20	29.16	400
14	0.36	196
11	12.96	121
13	2.56	169
18	11.56	324
10	21.16	100
11	12.96	121
$\bar{x}=14.6$	$\Sigma f = 132.4$	$\Sigma x^2 = 2264$

Standard deviation:-

$$S = \sqrt{S^2} = \sqrt{132.4} = 3.63 \text{ Marks}$$

$$C.V :- \frac{S}{\bar{x}} \times 100 = \frac{3.63}{14.6} \times 100$$

$$= 24.86\%$$

Solving for another q.:-

Variance second Formula:-

$$S^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$$

$$= \frac{2264}{10} - \left(\frac{146}{10}\right)^2$$

$$= 226.4 - (14.6)^2$$

$$= 13.24 \text{ Ans.}$$

For Grouped dataset

Classes	freq.	Mid point (x)	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
10-16	3	13	39	388.87	1166.61
17-23	8	20	160	161.79	1294.32
24-30	8	27	216	32.71	261.68
31-37	5	34	170	1.63	8.15
38-44	10	41	410	68.55	685.5
45-51	2	48	96	233.47	466.94
52-58	3	52	156	371.71	1115.13
59-65	1	62	62	857.31	857.31
	$\Sigma f = 40$	$\Sigma x =$ 291	$\Sigma fx =$ 1309	$\Sigma f \pm 2116.14$	$= 5855.64$

range = U.C.L (Last class) — L.C.L (1st class)

$= 65 - 10 = 55$

variance :-

$= s^2 = \frac{\Sigma f(x - \bar{x})^2}{\Sigma f}$

$\bar{x} = \frac{\Sigma fx}{fx}$

$\bar{x} = \frac{1309}{40} = 32.72$

$= \frac{5855.64}{40} = 146.39$

St.dev.

$s = \sqrt{s^2} = \sqrt{146.39}$

$= 12.09$

$C.V = \frac{s}{\bar{x}} \times 100 = \frac{12.09}{32.72} \times 100$

$= 36.94\%$

$= 13.36$

Ungrouped Question:-

Mins Variation.

15, 28, 25, 48, 22, 43, 49, 34, 22, 33,
27, 25, 22, 20, 39

Grouped Question:-

# of order	f
10-12	4
13-15	12
16-18	20
19-21	14

Ungrouped Q

ungrouped question solution:-

Range = $X_{max} - X_{min}$

= 49 - 15

= 34 mint

Variance:-

$s^2 = \frac{\Sigma (X - \bar{X})^2}{n}$

$\bar{X} = \frac{\Sigma X}{n} = \frac{452}{15}$

= 30.13

X	$(X-\bar{X})^2$	X^2 → = 228.91
15	$(15 - 30.13)^2$	225
28	4.53	784
25	26.31	625
48	319.33	2304
22	66.09	484
43	165.63	1849
49	356.07	2401
34	14.97	456
22	66.09	484
33	8.23	1089
27	9.79	729
25	26.31	625
22	66.09	484
20	102.61	400
39	78.67	1521
ΣX = 452	Σ = 1539.63	

$s^2 = \frac{\Sigma (X - \bar{X})^2}{n}$

$= \frac{1539.63}{15}$

$s^2 = 102.642$ min^2

Standard deviation:-

$$S = \sqrt{S^2} = \sqrt{102.64}$$

$$= 10.13 \quad \text{minutes}$$

$$C.V = \frac{S}{\bar{x}} \times 100$$

$$= \frac{10.13}{30.13} \times 100$$

$$= 33.62\%$$

Grouped dataset

Classes	f	Midpoint	fx	$(x-\bar{x})^2$	$f(x-\bar{x})^2$
10-12	4	11	44	31.80	127.2
13-15	12	14	168	6.96	83.52
16-18	20	17	340	0.12	2.4
19-21	14	20	280	11.28	158.05
	50		=832	=50.16	371.17

range = U.C.L − L.C.L
Last 1st

= 21 − 10 = 11 mark.

variance =

$$s^2 = \frac{\Sigma f (x - \bar{x})^2}{\Sigma f}$$

$$\bar{x} = \frac{\Sigma f x}{\Sigma f} = \frac{832}{50} = 16.64$$

$$= \frac{374.17}{50} = 7.423 \text{ Mark}^2$$

St. deviation

$$s = \sqrt{s^2} = \sqrt{7.423}$$

= 2.72 Mark

C.V =

$$= \frac{s}{\bar{x}} \times 100$$

$$= \frac{2.72}{16.64} \times 100$$

= 16.34 /.

Probability

↳ chances that something that might

Measure of uncertainity — degree of belief

Range - 0 to 1

If prob of any event is 0 → impossible
" " " 1 → sure/certain
" " b/w 0 to 1 → uncertain

$$\text{probability} = \frac{\text{\# of favorable outcomes}}{\text{Total \# of outcomes}}$$

Dictory

(1) Sample space:-

A complete list of all possible outcomes
If we denote with 'S' and close elements with cruly braces then it becomes statistic sample space.

Types:
discrete
countinous

$S = \{H, T\}$

Discrete:

Each & every element is writeable in sampl space

Countinuous:-

The samplespace that contain non-writeabl or not easily writeable

example:

- If we toss a fair coin once, its sample is head & tail. (C)
- if we have bag containing # of balls, and total balls are 50. (discrete)
- The temperature reading b/w 45° to 90° (C)
- No of calls arriving callcentre during error in the system (countinuous)
- Time interval of message arrival same network. (C)

- Write a sample space of butter toast experi

$S = \{$ butter slideup, slidedown $\}$.

(2) Element / outcomes:

Entries of the sample space = outcome or elements.

→

(3) event:- (something special)

outcome of the intrest (جس کو ہم جاننا چاہتے ہیں) in which outcome we are intrested.

(4) Experiment:-

To perform something to get the outcom is known as experiment.

(5) Tree diagram:

(diagram) dramgramitical representation of sample space.

Q:- Write an example by spliting (stateting) all dictionary of probability:-

1- In ps19 shahdaab got

In psl 9 islamabadunited got 8 victories and 4 defeat.

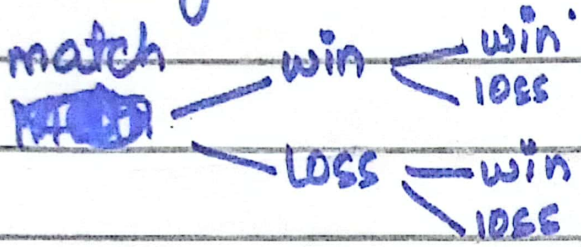
sample space:- S = {Win, Loss--}

Elements:- {4, {3,7}, Win, Loss --

Event:- victory

Experiment:- playing

Tree diagram:-



Rules of counting:--

permutation | combination | multiplica

permutation:- when order matters.
combination:- doesn't matter order
multiplication:- numerical problem.

permutation :- $^{n}P_{r} = \frac{n!}{(n-r)!}$

combination:- $^{n}C_{r} = \frac{n!}{r!(n-r)!}$

Multiplication:-

$$n_1 \times n_2 \times \cdots \times n_k$$

where n = Total $r \rightarrow$ selected,

Q: From a group of 10 people in how many ways 2 officers can be selected if

(a) one president & 1 treasure
(b) one chief editor & 1 editor

(d) two managers.

(a):- Total = 10 $r = 2$

$${}^{n}P_{r} = \frac{n!}{(n-r)!} =$$

$$= 90$$

$${}^{10}P_{1} =$$

(2) 1 CE & 1E

$${}^{10}P_{2} = \frac{10!}{(10-2)!} = 90$$

(3) 2 E:

$${}^{10}C_{2} = \frac{10!}{2!\,(10-2)!} = \frac{10!}{2!\,8!} = 45$$

(4) 2 M:

$${}^{10}C_{2} = \frac{10!}{2!\,(10-2)!} = \frac{10!}{2!\,8!} = 45$$

Q: A policeman wants to do challan of vehicle LHE5__. he forget the last two digits of plate but sure that all digits are different. In how many ways he can find the missing digits.

Total = 10 digits.
5 is already present

L HE 5 _ _ _

pechay reh gay 9. ab aik dawfa jo agaya woh dobara nahi aiy ga

L HE 5 _ _ _ = 72
9 8 ↳ 72 possible outcomes.

Probability (applicable for only one variable)

probability = $\frac{\#\text{ of favourable outcome}}{\text{Total all \# of outcome}}$

range = 0 to 1.

if prob is 0 → Event is impossible

1 → Event is sure/certain

b/w 0 & 1 → uncertain

Rules of probability: more than one event

Additive → 'OR' ∪

identification = neither, nor

Multiplicative = and ∩ (identity = both)

conditional if, ready occur, that give '1'

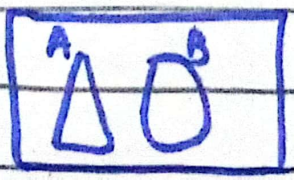
Types of additive

1- **Mutually exclusive:** Koi common point na ho.

2- **Non-mutually exclusive:** common point ho.

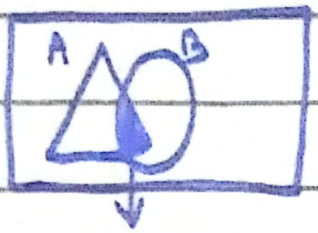
$P(A \cup B) = P(A) + P(B)$

agar prob given nahi tou then pehlay prob find karin gay.



Non-Mutually exclusive:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



common

Addictive: Multiplicative Type.

independent: if the occuance of oneevent doesn't effect the occurance of other event then it is independent. (w.R.P with replacement)

dependent:

→ does effect the occuance of other (without)

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = P(A) \times P\left(\frac{B}{A}\right)$$

example.

ایک کلاس میں 56 Student ہے ان میں سے ایک ایک بچے کو لوگے تو prob $\frac{1}{56}$ ہوگی ۔ یہ independ ہوگا لیکن اگر کوئی کلاس سے چلا گیا تو prob $\frac{1}{55}$ ہوگی تو وہ depend ہوگی

conditional:- $P(\text{Find} \mid \text{Give}) = \frac{P(\text{find} \cap \text{give})}{P(\text{give})}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ or $P(B|A) = \frac{P(A \cap B)}{P(A)}$

Question:

Roll a dice 2 times

(a) Make a sample space of it.

(b) represent it in tree diagram

(c) What is the probability that sum of the outcome is 7.

(d) outcome is 11. (event)

(e) outcome is even,

(f) outcome is odd,

(g) outcome 7 or 11.

(h) outcome is 7 & 11

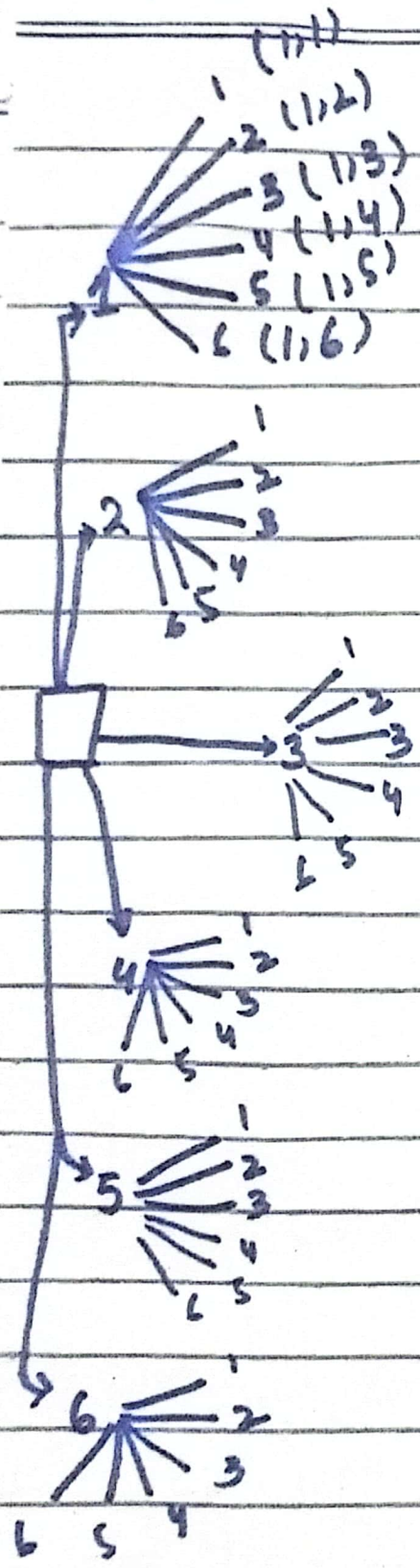
(i) outcome is 7 given that outcome is odd.

Answer.

$S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)\ (2,1)\ (2,2)$
$(2,3)\ (2,4)\ (2,5)\ (2,6)\ (3,1)\ (3,2)\ (3,3)\ (3,4)$
$(3,5)\ (3,6)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)\ (6,1)$
$(6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$\Rightarrow 6^2 = 36$

Part #b



Sum of the outcom
appas mai add ka
like (1+1) = 2

part # c
let A be the event th
sum of the outcome is 7

A = {(1,6)(2,5)(3,4),(4,
(5,2)(6,1)}

n(A) = 6

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

part # d
outcome is 11

A = {(5,6) (6,5)}

$$P(A) = \frac{2}{36} = \frac{1}{18}$$

part 'e' outcome is even. (3,1)

A = {(1,1) (1,3) (1,5) (2,2) (2,4) (2,6) (3,3)
(3,5) (4,2) (4,4) (4,6) (5,1) (5,3)
(5,5) (6,2) (6,4) (6,6)}

$$P(A) = \frac{18}{36} = \frac{1}{2}$$

part 'f' odd

$$P(A) = \frac{18}{36} = 1/2$$

part 'gh' 7 and 11

$$A = \{(1,6), (2,5) (3,4) (4,3) (5,2) (5,6) (6,1) (6,5)\}$$

$$\Rightarrow A = \frac{8}{36} = \frac{2}{9}$$

part 'g' 7 or 11

$$P(A \cup B) = P(A) + P(B)$$

$$= \frac{6}{36} + \frac{2}{36} = \frac{8}{36} = \frac{2}{9}$$

part 'h' 7 and 11

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{6}{36} \times \frac{2}{36} = \frac{1}{108}$$

(c) outcome is 7 given that outcome is odd.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{\quad}{\quad} =$$

$$= \frac{\quad}{1/8}$$

$$P(A \cap B) = P(A) \times P(B)$$

$$= \frac{1}{6} \times \frac{1}{18} = \frac{1}{108}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$A = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$$

$$B = \{(1,2)(1,4)(1,6)(2,1)(2,3)(2,5)(3,2)(3,4)$$
$$(3,6)(4,1)(4,3)(4,5)(5,2)(5,4)(5,6)(6,1)$$
$$(6,3)(6,5)\}$$

$$AND = \{(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)\}$$

$$= \frac{6/36}{18/36} = \frac{6}{18} = \frac{1}{3}$$

Question :-

20 bulb in a box in which 5 are fused if a retrailer wants to select 2 bulbs without replacement what is probability that both are used.

Total = 20 fused → 5

$$P(A) = \frac{5}{20} \qquad P(B/A) = \frac{4}{19}$$

$$P(A \cap B) = \frac{5}{\cancel{20}_{5}} \times \frac{\cancel{4}^{1}}{19} = \frac{5}{95} = 1/19$$

Question:-

Toss a coin 2 times, what is the probability that nohead occur

(b) no tail occur

(c) one head 1 tail occur

(d)

Q #04:

3 horses in a race the prob that horse A wins is twice as likely 2 horse B. The probability horse B win is twice likely horse C wins.

what is the prob horse A or horse B wins

Q#05

The probability that

July First is the rainy day is 0.24.

The prob that july 2nd is the rainy day = 0

" " 1 & 2nd rainy day is = 0.16

what is the prob that 1st july is rainy given that the july 2nd is rainy

1st july $P(A) = 0.24$ $P(B) = 0.34$

$P(A \cap B) = 0.16$

$$P(A \cap B) = \frac{0.16}{0.34} = \frac{8}{17} = 0.47$$

Q:- Coin:

Head — Head
Head — Tail
Tail — Head
Tail — Tail

= (H,H) (H,T) (T,H) (T,T)

$n(A) = 4$

no-head occur = 1/4

no tail occur = 1/4

1 head 1 tail = 2/4 = 1/2

Q:- Horse:-

Let → Horse C has the probability = x

Horse B has the prob = $2x$

Horse A has the pro = $2(2x) = 4x$

Total Prob = $4x + 2x + x = 7x$

Horse A or horse B wins

$P(A) = \frac{4}{7}$ $\quad P(B) = \frac{2}{7}$

$P(A \cup B) = \frac{4}{7} + \frac{2}{7} = \frac{6}{7}$ ✗ $\left[\frac{4}{7} \times \frac{2}{7} = \frac{8}{49}\right]$ ✗

$P(A \cup B) = \frac{4}{7} + \frac{2}{7} - \frac{8}{49}$ ✗

$= \frac{34}{49}$