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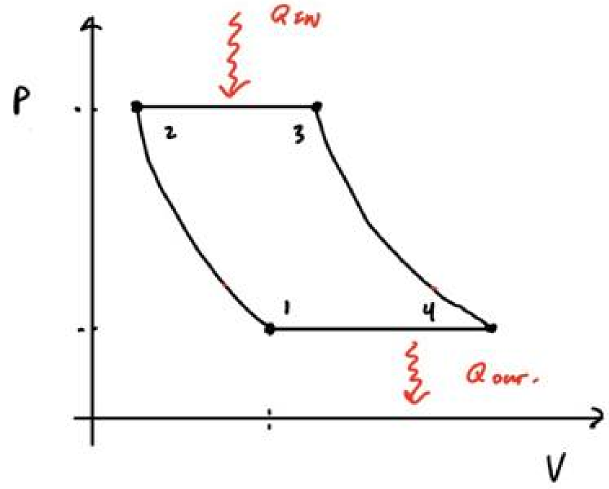
High Performance Brayton Cycle: Reheating, Intercooling and Regeneration

Introduction to the Brayton Cycle

The Brayton Cycle is a heat engine characterized by constant pressure heat addition and rejection processes. Connecting the two fixed pressure states are compressive and expansive adiabats. The combination of these processes results in a practical heat engine, with many modern applications.



In general, a heat engine can be characterized as any process which seeks to convert heat into work. The rapid compression and expansion present in the Brayton Cycle makes it an ideal candidate for a machine that can accommodate swift heat addition and dissipation. This would allow the cycle to run at a higher frequency, in turn producing a greater power output.

A worthy machine for this task is one which contains a turbine and compressor in combination. Turbines, like those found in power plants and jet engines, are able to convert the rapid expansion of the working fluid in the Brayton Cycle into rotational energy. In the case of the jet engine, this rotational energy sourced from the working fluid culminates in thrust. Since jet engines and power plant turbines both operate in the open air, heat dissipation is solved simply by expelling the heated working fluid from the cycle, and sucking in new fresh fluid at a lower temperature. Rapid heat addition is taken care of with a combustion step, providing the working fluid with the energy necessary to rapidly expand through the turbine.

Efficiency in Heat Engines

Efficiency in heat engines is characterized by the amount of work that can be obtained from a given cycle in proportion to how much heat the cycle requires. Getting more work out of a cycle for a given amount of heat supplied to the cycle corresponds to an increase in efficiency.

Heat losses and work are related by the first law of thermodynamics, which means efficiency can also be expressed in terms of the heat added to a process and the waste heat that is produced by the process. One way to think about minimizing the heat losses is through minimizing additional entropy creation (corresponding to energy that cannot be converted to work). This can be achieved by taking the change in entropy of the universe to be 0. This derivation can be seen below.

$$\Delta S_{universe} = \Delta S_{sys} + \Delta S_{env.}$$

$$\Delta S_{universe} = 0 \quad \therefore \text{ NO EXTRA ENTROPY CREATION}$$

$$\left[\frac{\partial S}{\partial U} = \frac{1}{T}\right]_{V,N} \longrightarrow dS = \frac{dU}{T} = \frac{\delta Q}{T}$$

$$dU_{sys} = -dU_{env.}$$

$$\frac{\delta Q_h}{T_h} = \frac{\delta Q_{sys}}{T_{sys}}$$

SAME CAN BE APPLIED FOR T_c

$\boxed{\therefore \text{ LET } T_h = T_{sys} \text{ FOR NO NEW ENTROPY CREATION WHEN ADDING HEAT...}}$

As a result of the system and environment having an equal and opposite change in entropy, their temperatures must be equal. This result shows that maximum work output is achieved when a heat flow occurs as the system is in equilibrium with the thermal reservoir (isothermal expansion or compression).

Further investigation of heat flow during an isothermal expansion or compression leads to a new equation for efficiency, which represents a maximum for an engine operating between two temperatures. This is known as the carnot efficiency. For other cycles to approach the carnot cycle's efficiency different efficiency boosting strategies can be employed. For the Brayton Cycle, this comes in the form of reheating, intercooling and regeneration.

$\Delta U = Q + W \quad \therefore W = \Delta U - Q$

$$\boxed{e = \frac{W}{Q_{hot.}} = \frac{Q_H - Q_C}{Q_H} = \left[1 - \frac{Q_C}{Q_H}\right]} \longrightarrow e \to 1 \text{ WHEN } Q_C = 0$$

$$\boxed{e_{max} = 1 - \frac{T_C}{T_H}}$$

The Ericsson Cycle and Efficiency

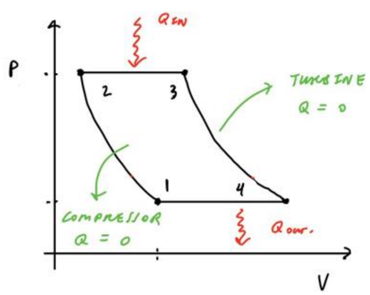
Another heat engine that operates between two isobaric set-points is the Ericsson Cycle. This differs from the Brayton Cycle in its compression and expansion stages. In the Ericsson Cycle these come in the form of isotherms as opposed to adiabats. The majority of heat flow in the Ericsson Cycle occurs as the working fluid is at thermal equilibrium with the hot or cold reservoirs, however the isobars that work between the two isotherms do still expose the cycle to non-isothermal heat flows. To counteract the extra entropy creation from these processes a technique called regeneration is applied. Regeneration involves capturing the waste heat from one end of the cycle and using it as a source of heat at the other end of the process.

In the case of the Ericsson Cycle, regeneration is used to capture heat from the isobaric compression and repurpose it to aid in heating the working fluid during the isobaric expansion. In an ideal case, this regeneration can be assumed to be helpful enough such that there are no heat additions through differential temperatures throughout the entire process. This means that the Ericsson Cycle will approach the carnot efficiency when effective regeneration is used.

Although this is a notable result, the Ericsson and Carnot Cycles themselves are far from practical heat engines. The main limiting factor for these cycles are the isothermal expansion and compression steps. Equilibrating a working fluid to a thermal reservoir at each point in an expansion or compression is a very slow process. This means that the frequency at which these cycles can be run is not great enough to produce any significant power output.

Efficiency in the Brayton Cycle

In order to analyze Brayton Cycle, which is typically open to the atmosphere, on a closed TS or PV diagram, certain assumptions must be made. The adiabatic compression and expansion stages are maintained whether the cycle is assumed to be open or closed, but the stages where working fluid is expelled or sucked into the cycle need to be modelled to form a closed process. To achieve this, isobaric heating and cooling processes are added. Step 1-4 on the diagram below approximates the expulsion of heated working fluid from the cycle after it is expanded through the turbine, and step 2-3 approximates the heating of the combustion step.



$$Q_H = c_p \, dT_1 = c_p \left[T_3 - T_2\right]$$

$$Q_c = c_p \cdot dT_2 = c_p \left[T_4 - T_1\right]$$

$$\therefore \; e = 1 - \frac{c_p\left[T_4 - T_1\right]}{c_p\left[T_3 - T_2\right]}$$

$$= 1 - \frac{T_1}{T_2}\left[\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right]$$

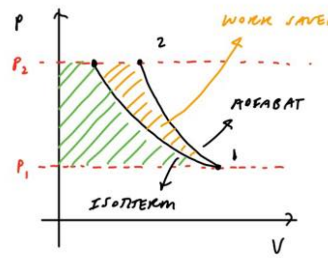
$$= 1 - \left[\frac{P_1}{P_2}\right]^{\frac{\gamma-1}{\gamma}}$$

$$\boxed{e = 1 - \left[\frac{1}{r^{\frac{\gamma-1}{\gamma}}}\right]}$$

Unlike the Ericsson cycle, regeneration alone cannot make the Brayton Cycle approach carnot efficiency. Instead, the efficiency of the Brayton Cycle can be expressed in terms of its pressure ratio. From this expression we can see that the efficiency of the Brayton Cycle will increase monotonically with pressure ratio, which is bounded by the maximum temperature that components of the turbine can handle. Again, this efficiency is strictly less than the carnot efficiency between any two maximum and minimum operating temperatures. For the 747's PW4000 jet engine, turbines can experience temperatures upwards of 1500K during the rapid expansion of hot pressurized air. Another way of looking at the efficiency of the brayton cycle is through the ratio of the temperatures of the working fluid at the inlet and after the compressor step (positions 1 and 2). This comes from the relation between pressure and temperature in adiabatic processes, and gives a more intuitive sense of why the Brayton cycle's efficiency is always less than the Carnot efficiency (position 1 represents a minimum temperature, but the temperature at 2 is less than the maximum temperature experienced in the cycle).

Boosting the Performance of the Brayton Cycle

Like in the Ericsson cycle, regeneration between the two isobaric heat addition steps can be used to repurpose heat expelled from the cycle, thus increasing its efficiency. Another way to boost the performance of the Brayton cycle is to increase it's net work output. This can be done by analysing the net work achieved in different expansion and compression processes. More specifically, comparing the net work achieved through adiabatic and isothermal expansion and compression can give insights as to how the Brayton Cycle could be optimized.



$$W = \int -P \cdot dV \longrightarrow d(PV) = PdV + dPV$$

$$\therefore \quad dV = -\frac{V}{P} \cdot dP$$

$$\boxed{W = \int V \cdot dP}$$

ISOTHERMAL :

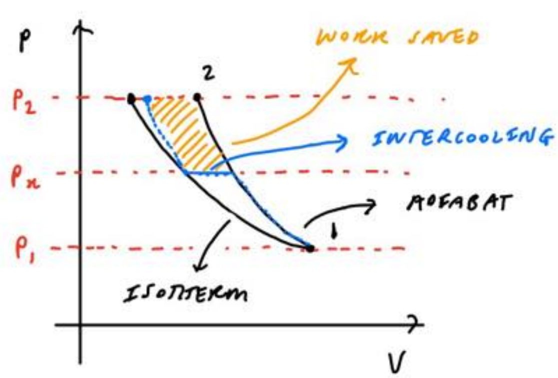
ADIABATIC :

$$W = \int V \cdot dP = \int_1^2 \frac{Nk_BT}{P} \cdot dP \qquad W = \int V \cdot dP = \int_1^2 c \cdot P^{-\gamma} \cdot dP$$

$$\boxed{W = Nk_B T_1 \ln\left(\frac{P_2}{P_1}\right)} < \boxed{W = Nk_B T_1 \left[\frac{\gamma}{\gamma - 1}\right] \cdot \left[\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]}$$

From the above derivation and sketch it becomes clear that, given a high and low pressure bound, compression from the low pressure to the high pressure can be achieved with less input work in an isothermal process compared to an adiabatic process. A similar result can be realized from the analysis of an expansion from the high pressure bound to the low pressure bound. In this case the isothermal process would output a greater amount of work as compared to the adiabatic process.

From this result, it is clear that the net work output of the Brayton Cycle can be increased if the adiabatic expansion and compression processes were to approximate isothermal expansion and compression. One way to achieve this is by employing techniques called intercooling and reheating. In the case of this diagram, the working fluid can be adiabatically compressed in stages, where each stage of compression is preceded by an isobaric cooling period. This allows the working fluid to constantly return to its starting temperature from low pressure point

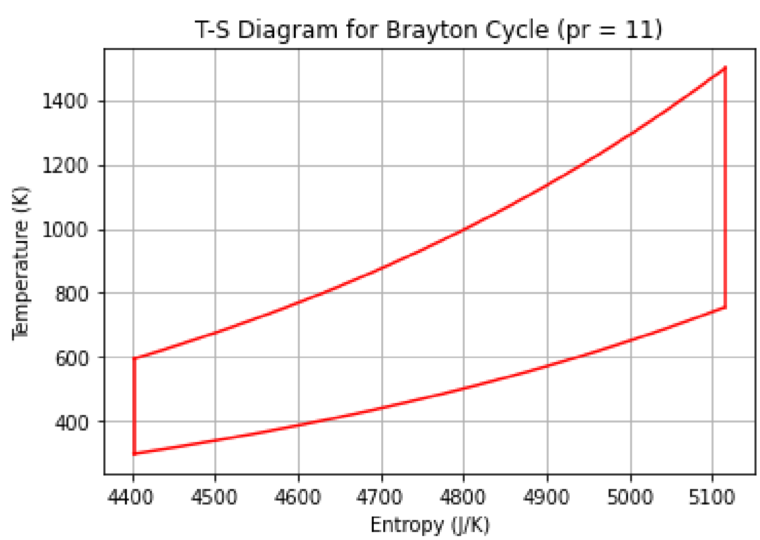


1. This roughly approximates an isothermal compression process. This staged compression can be practically achieved by daisy chaining multiple compressors-intercooler pairs together. To this effect, each compressor is doing less work individually and the sum of these individual works will be less than that of a single compression stage with no intercooling. It is important to note that extra heat is lost in this staged expansion. This means that employing intercooling by itself would most definitely decrease the efficiency of a Brayton Cycle. However, effective regeneration can be coupled with the intercooling to reuse lost heat and counteract these efficiency losses.

In the case of expansion of the working fluid, reheating achieves an analogous result to intercooling. The isobaric stages work to return the air to the cycle's max temperature, and the staged adiabatic expansion can be seen to approximate an isothermal expansion. In practice, this means daisy chaining multiple turbine-reheater pairs together and utilizing regeneration to counteract the efficiency losses of adding extra heat to the system.

Another important consideration is related to what pressure values in the intermediate cooling or heating steps should occur at. The optimal pressures turn out to be those which maintain the same pressure ratio between each of the intermediate steps.

Modelling the Brayton Cycle

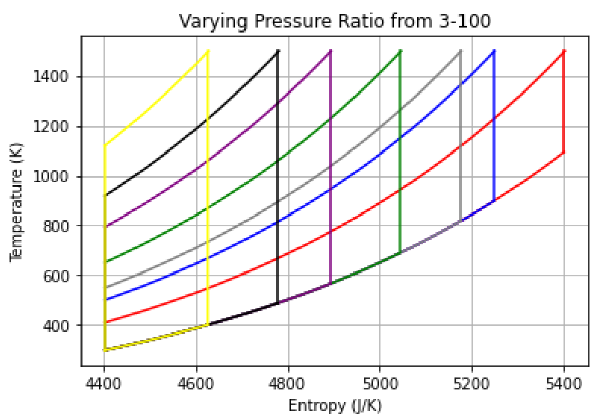


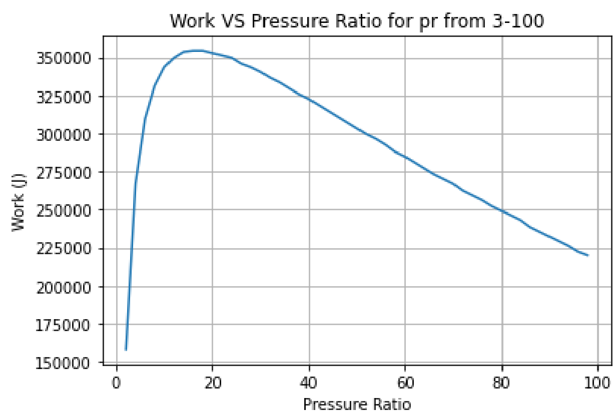
To further investigate the Brayton Cycle, python can be leveraged to plot TS diagrams. TS plots are the natural choice for modeling a Brayton Cycle as it is operationally bounded by pressures and temperatures rather than volumes. Entropy arises quite naturally for isobaric processes as well: represented as the natural log of temperature multiplied by the constant pressure specific heat.

Taking the pressure ratio of the cycle as a free parameter, one can numerically investigate its implications on work output and efficiency for a given operational temperature range. Also, a simulation can be constructed to illustrate the transformation of the Brayton Cycle over many steps of intercooling and reheating. This model will be constructed around a set of estimated operational parameters for a 747 jet engine. This includes an inlet volume of approximately $2.2m^3$, a maximum cycle temperature of *1500K* (occurring prior to adiabatic expansion) and an inlet temperature and pressure of *300K* and *100kPa*.

Starting off with the TS diagram for the regular ideal brayton cycle, the adiabatic compression and expansion stages will appear as vertical lines as no heat is entering or leaving the working fluid in these steps. The isobaric regions appear as exponentials as expected (the inverse of the entropy vs temperature). The efficiency of this cycle can be approximated by taking 1 minus the division integrals of the two isobaric steps. Numerically these integrals can be taken by summing over all temperature steps the change in entropy between two temperatures multiplied by the average of the two temperatures. Working through this process for the above Brayton Cycle yields an efficiency of 0.497. Drawing on our previous discussion of efficiency it can be seen that the equation relating the pressure ratio to efficiency gives a nearly identical value of 0.496.

As the pressure ratio is increased from 11, it can be seen that the efficiency of the cycle continues to increase. Also it can be seen that the region enclosed by the TS diagram changes. This means that the net-work of the cycle is changing. The pressure ratio for maximum work can be obtained by plotting these two parameters and taking a maximum. For the working temperature bounds of the 747 engine (300K to 1500K) the ideal pressure ratio is approximately 17. This achieves an efficiency of 0.57 and a maximum net work.

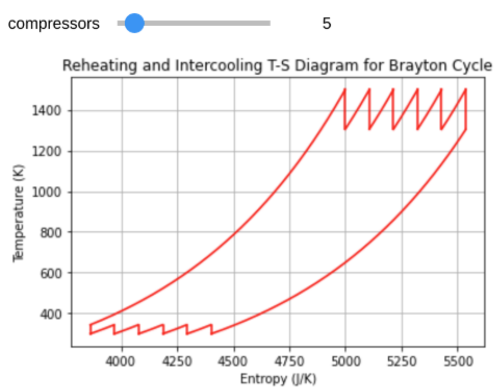




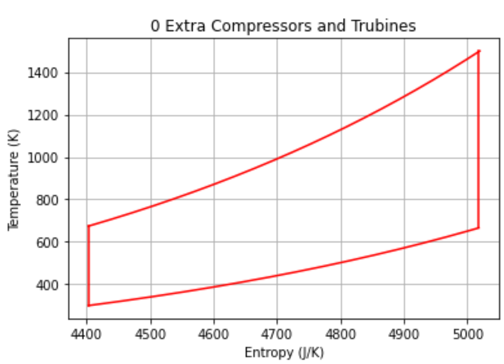
To further optimize the brayton cycle, intercooling and reheating steps can be added. The first challenge involved with adding this to the python simulation is determining what the intermediate pressure values will be. From the earlier discussion of the intermediate pressures, we know that the maximum net work gains can be achieved when the pressure ratio of each intermediate expansion is the same. In the absence of an elegant way to generate these pressures, one can establish an initial list of evenly spaced values staged between the high and low pressures of the cycle and then iteratively check each pressure ratio and adjust values accordingly. After a number of iterative steps a list of intermediate pressures will be spit out, where the ratio of any value with its neighbour is uniform throughout.

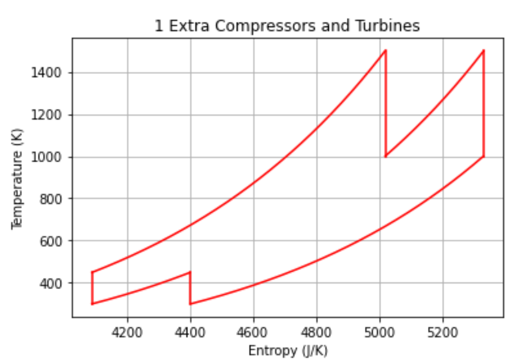
From this list of intermediate pressures and using the adiabatic relation between pressure and temperature, the uniform temperature ratio of the intermediate expansions can be determined. This value is particularly critical to modeling the process on the TS diagram, as multiplying the inlet temperature of 300K by the temperature ratio will give the maximum intermediate temperature that cycle will approach in its staged compression before cooling back to the inlet temperature. The same technique can be applied for the staged expansion step, where the minimum staged expansion temperature will be the maximum temperature of the cycle divided by the temperature ratio. Finally, we can take the number of these intercooling or preheating steps to be a parameter on a slider, which can be used to illustrate how the cycle evolves.

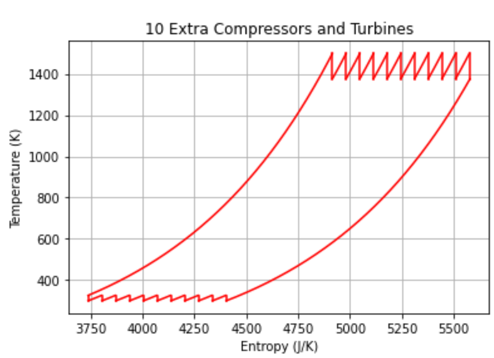
```
import ipywidgets as widgets
from __future__ import print_function
from ipywidgets import interact, interactive, fixed, interact_manual

interact(plotCycle, compressors = (0,50))
```

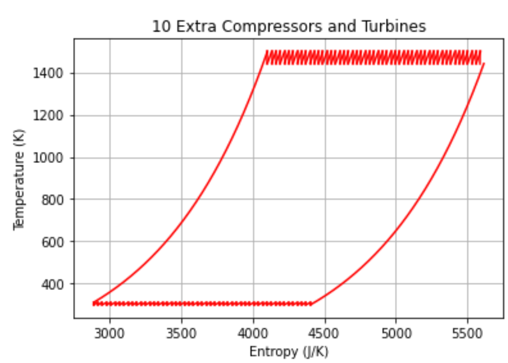


Ranging from a single additional compressor and turbine pair to 50 additional compressors and turbines, one can see that the previously adiabatic processes in the Brayton Cycle begin to resemble the isothermal processes. This emergent behaviour remains in-line with the prior discussion of the goals and effects of intercooling and reheating. As the number of the compressor and turbine pairs added to the process gets large, the uniform pressure ratio and thus temperature ratio for the intermediate expansion and compression become small. This means that the temperature of the working fluid is only able to increase or decrease an infinitesimal amount during expansion or compression before an intercooling or reheating device restores it to the inlet or maximum temperature.









Finally, it can be seen that taking the number of compressors and turbines in the Brayton Cycle to infinity causes its efficiency to approach that of the Ericsson Cycle. Assuming that regeneration is used in conjunction with intercooling and reheating, the efficiency of the Brayton Cycle will approach Carnot efficiency.

Limitations of Intercooling, Reheating, and Regeneration

Obviously it is impractical to expect that an infinite number of reheaters, intercoolers, compressors, and turbines could be combined into a single Brayton Cycle. One of the more important considerations for a heat engine that has so many practical use cases, is the cost of its components and its compactness. Although the efficiency of the cycle may benefit from extensive intercooling and reheating steps, the cost effectiveness and size of the device using this heat engine would be negatively impacted. Ultimately, the task of balancing these considerations to form an optimized heat engine is the role of the engineers that design things like jet engines and power plants.

Video of Simulation

https://youtu.be/XosEx4ilnxA

Link to Simulation Code

https://github.com/graysonk546/phys203-final-project

Note that the certain libraries will need to be downloaded to properly render the widgets for the intercooling and reheating simulation. Jupyter should throw errors that indicate any missing libraries.

This stack overflow post may also be helpful if any widget errors come up:

https://stackoverflow.com/questions/36351109/ipython-notebook-ipywidgets-does-not-show

Citations

Quattrochi, Douglas. “3.7 The Brayton Cycle.” *Massachusetts Institute of Technology*, 8 June 2006, web.mit.edu/16.unified/www/SPRING/propulsion/notes/node27.html.

ÇEngel, Yunus A., and Michael A. Boles. *Thermodynamics : An Engineering Approach*. 8th ed., MC GRAW HILL INDIA, 2021.

“Thermodynamics : Brayton Cycle with Regeneration, Brayton Cycle with Intercooling (32 of 51).” *YouTube*, uploaded by CPPMechEngTutorials, 26 Jan. 2019, www.youtube.com/watch?v=L3GEwPEqAdw&t=488s.