

# Hierarchical Bayesian Small Area Estimation Using Weakly Informative Priors in the Interior Western US

Grayson White<sup>1</sup>, Kelly McConville<sup>2</sup>,  
Gretchen Moisen<sup>3</sup>, Tracey Frescino<sup>3</sup>

<sup>1</sup> RedCastle Resources Inc., <sup>2</sup> Reed College, <sup>3</sup> USDA Forest Service

# The Forest Inventory and Analysis Program (FIA)



Orcas Island, Washington

# FIA Overview

- Responsible for reporting status and trends of the nation's forests.
- Collects inventory data on and computes estimates for many forest attributes to monitor the status and trends of the nation's forests.
- Sampling design is meant for sufficient precision to provide state-level estimates through post-stratified estimation.



*The Nation's Forest Census*

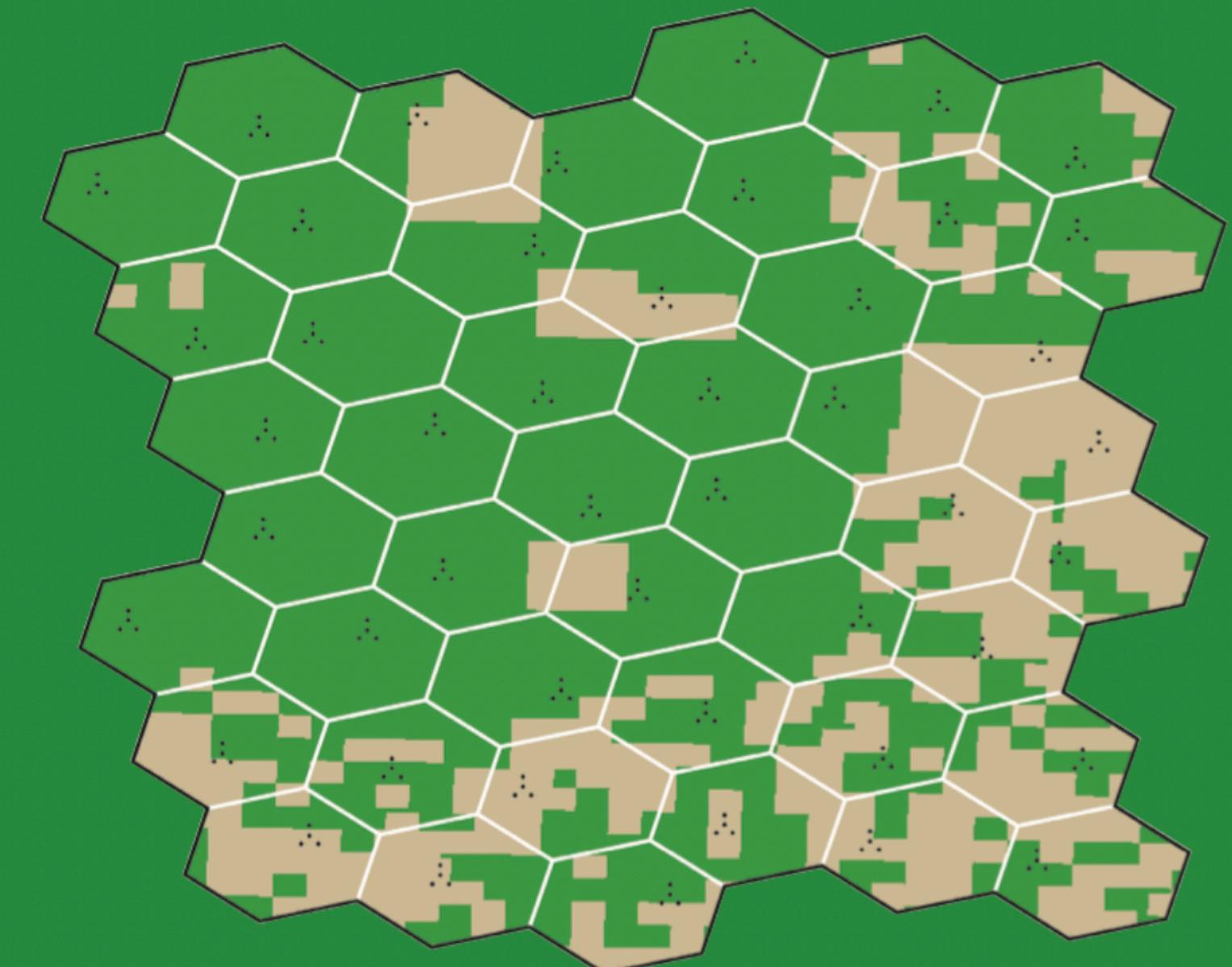
# FIA's Sampling Design

- Overlay hexagons with area of 6000 acres across the United States. Randomly select a location in each hexagon for a FIA plot.
- A FIA crew visits each plot once every 10 years to remeasure the plot.
- A 10 year timeframe where each plot has been measured is referred to as an “evaluation”.

United States  
Department of  
Agriculture  
  
Forest Service  
  
  
Southern  
Research Station  
  
General Technical  
Report SRS-80

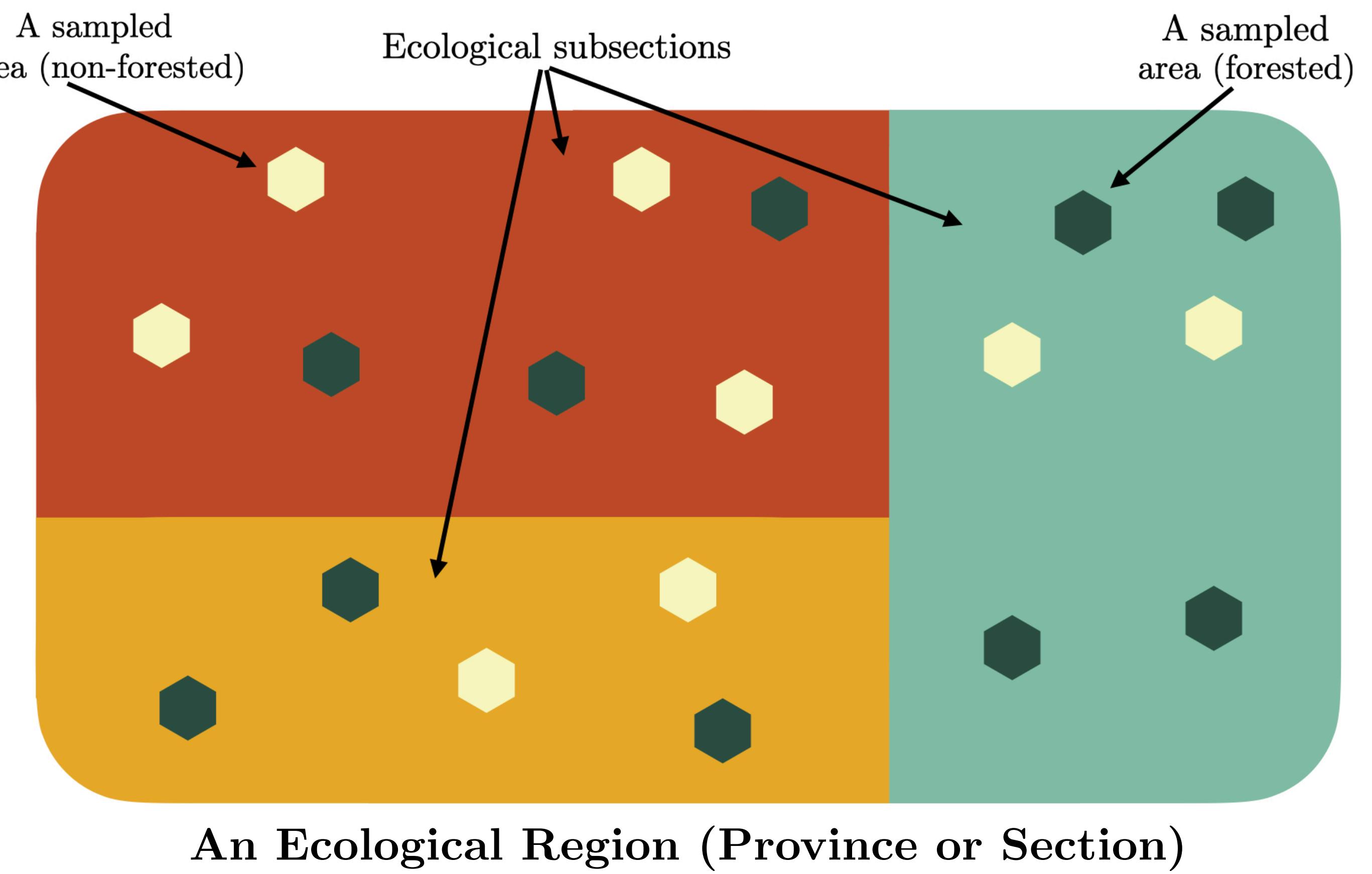
The Enhanced Forest  
Inventory and Analysis  
Program—National  
Sampling Design and  
Estimation Procedures

William A. Bechtold and  
Paul L. Patterson, Editors



# FIA's Small Area Estimation Needs

- FIA has a growing need for statistically defensible estimates over smaller geographic regions such as:
  - Counties
  - Watersheds
  - Ecologically defined regions
  - Recently burned areas
  - And more!



# The Data



Washington Highway 410, edge of Mt. Rainier National Park

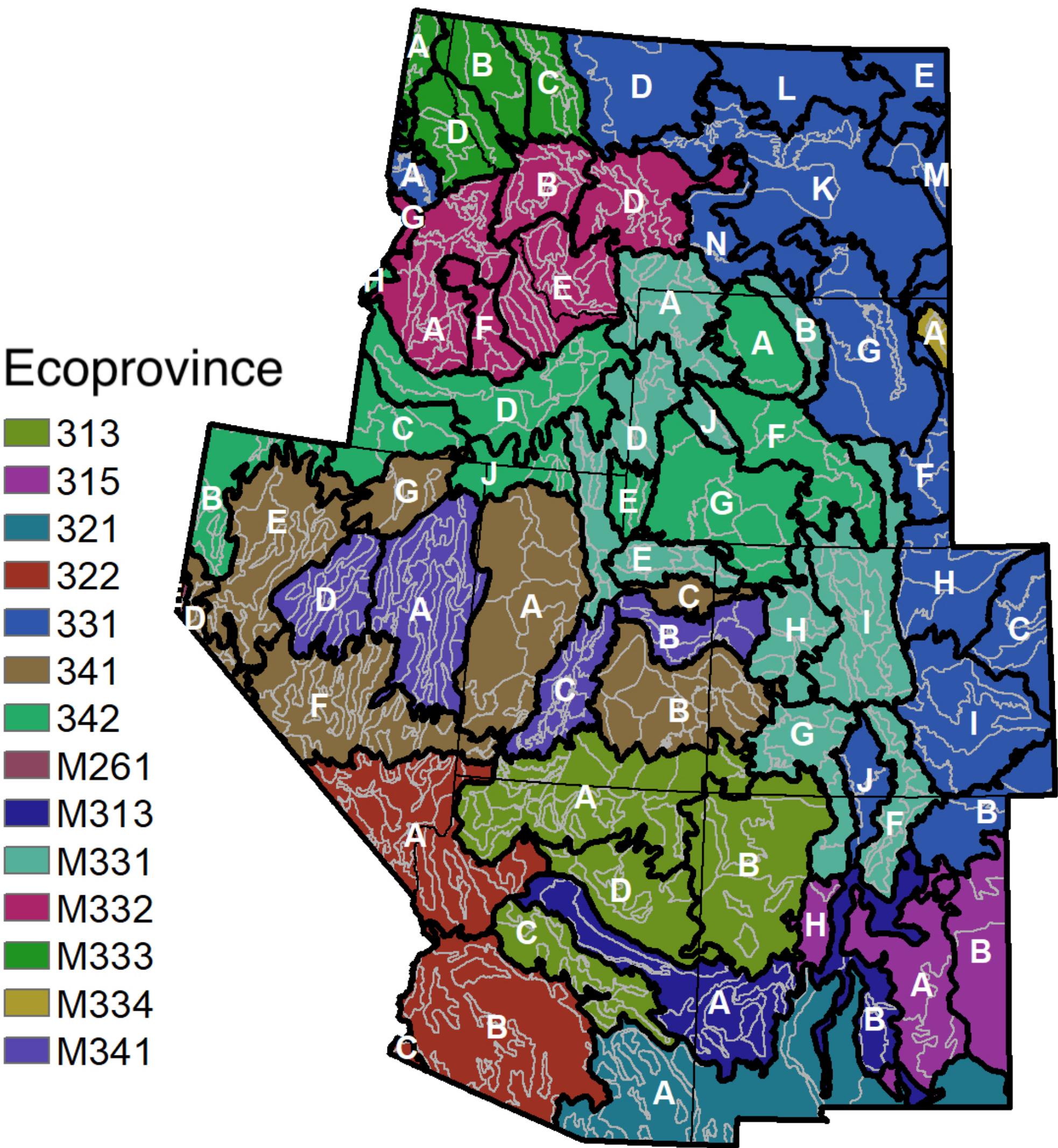
# FIA Data Overview

- We used FIESTA to retrieve a FIA evaluation of data from the Interior Western region of the United States from 2007 to 2017, which included 86,065 plots.
- FIESTA is FIA's custom estimation tool which allows for easy access to FIA data and many estimators, all in R.



# Auxiliary Data Overview

- Remotely-sensed explanatory variable at pixel level: tree canopy cover.
- Forest/non-forest strata indicator variable for each sampled plot.
- Ecologically-defined regions with three levels of hierarchically (largest to smallest): provinces, sections, subsections.



# Methods



Cape Flattery, the most northwest part of the continental United States

# The Big Idea

- Combine FIA data and auxiliary data to fit and compare multiple area-level small area models to the industry-standard post-stratified estimator for four FIA variables of interest.
- Predict means of these FIA variables (basal area, biomass, trees/acre, and net volume) of interest in ecological subsections.
- Understand the benefits of fitting hierarchical Bayesian small area models in this context: how can we leverage prior information in this specific scenario? Is it helpful to do so? When? Why?
- Compare the small area models with each other, and with the post-stratified estimator.

# The Post-Stratified Estimator

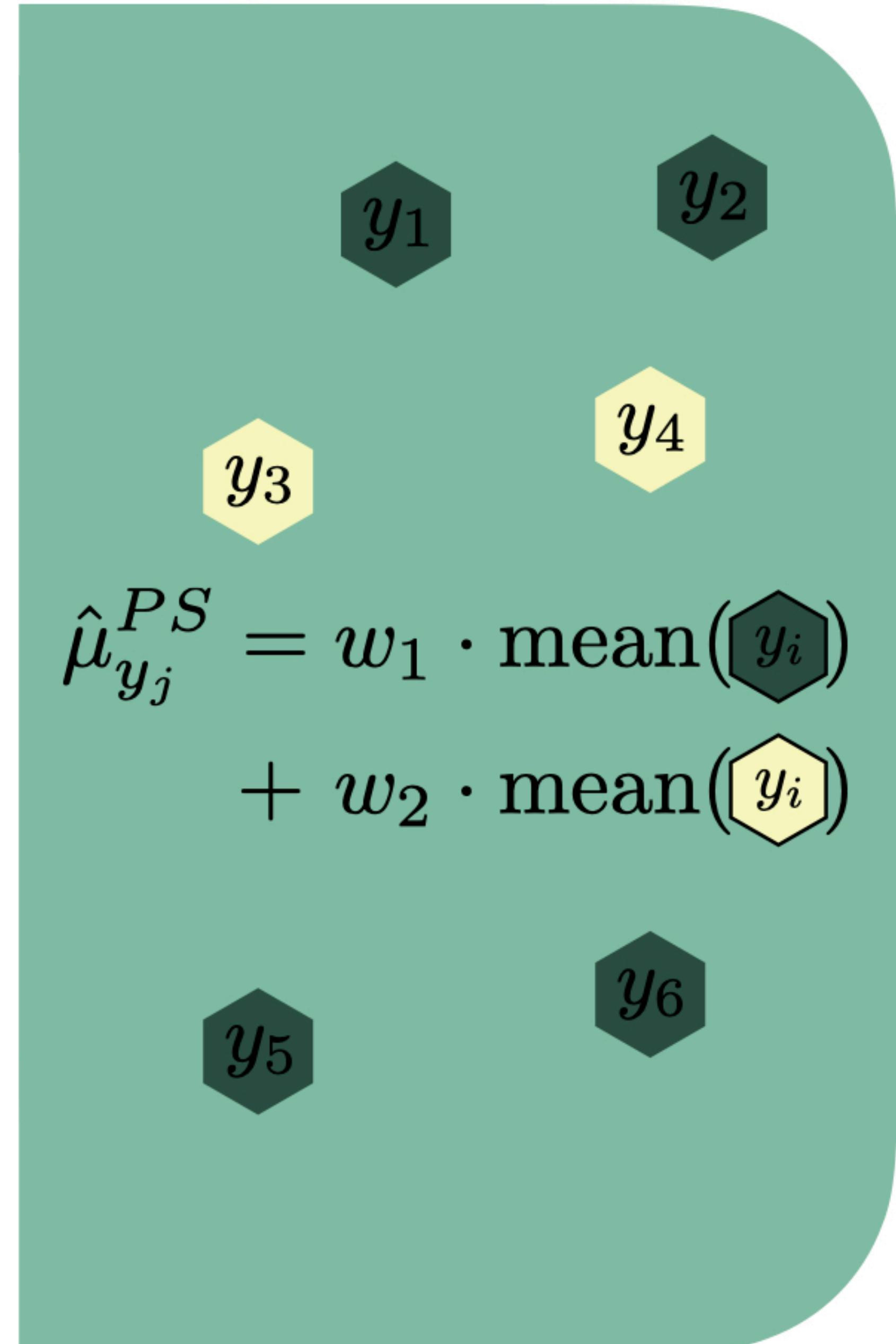
## A Direct Estimator

Estimate:

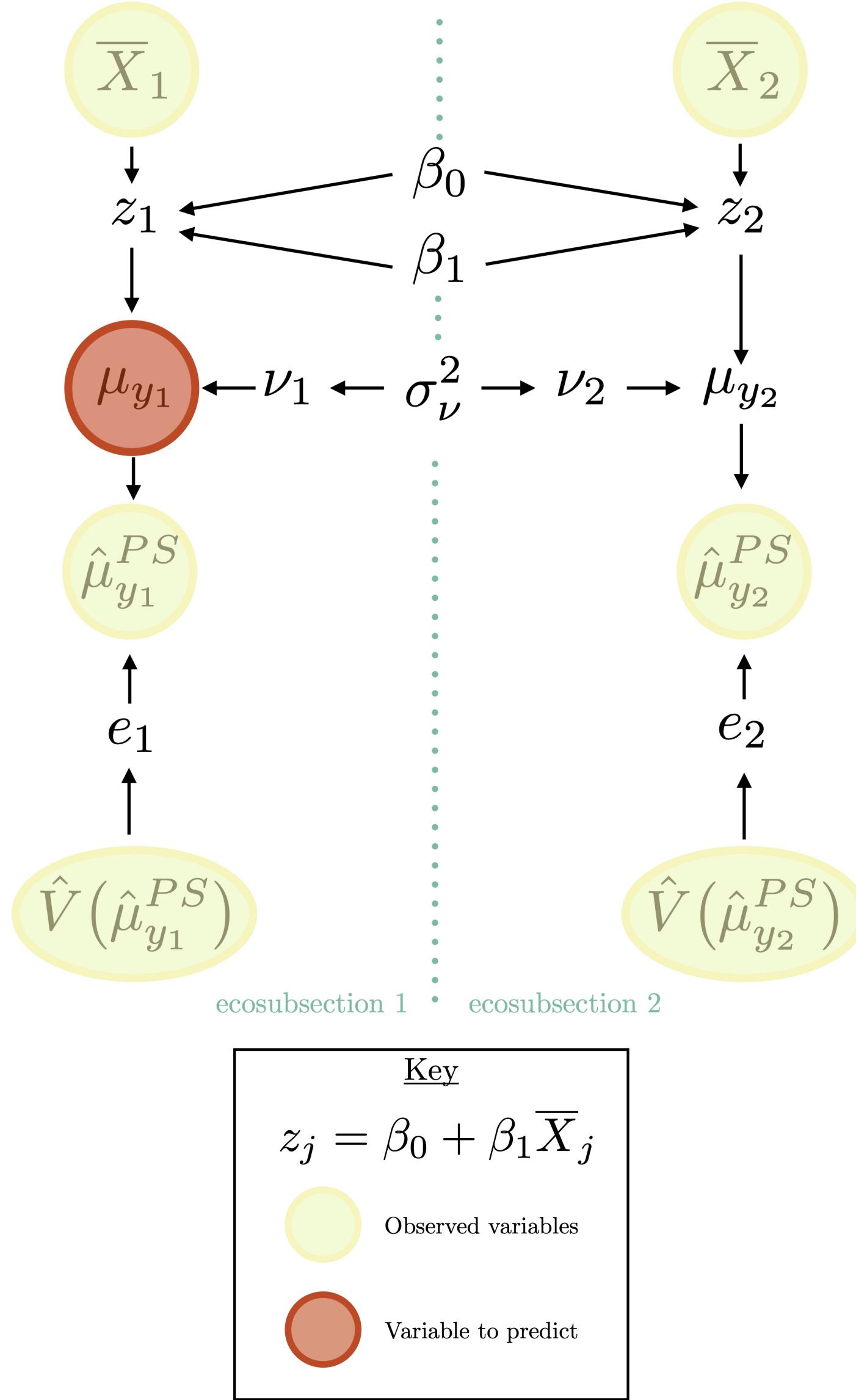
$$\hat{\mu}_{y_j}^{PS} = \sum_{k=1}^2 w_{jk} \cdot \hat{\mu}_{y_{jk}}^{HT}$$

Variance:

$$\hat{V}\left(\hat{\mu}_{y_j}^{PS}\right) = \frac{1}{n_j} \left( \sum_{k=1}^2 w_{jk} n_{jk} \hat{V}\left(\hat{\mu}_{y_{jk}}^{HT}\right) + \sum_{k=1}^2 (1 - w_{jk}) \frac{n_{jk}}{n_j} \hat{V}\left(\hat{\mu}_{y_{jk}}^{HT}\right) \right)$$



# Our Fay-Herriot Models



- Six variations of the Fay-Herriot model, all which can be represented by this Probabilistic Graphical Model (PGM).

Short Name	Fitting Method	Borrow Strength to...	Prior on $\sigma_\nu^2$
EBLUP Section	REML	Section	NA
EBLUP Province	REML	Province	NA
HB Section Flat Prior	MCMC	Section	$f(\sigma_\nu^2) \propto 1$
HB Province Flat Prior	MCMC	Province	$f(\sigma_\nu^2) \propto 1$
HB Section half Cauchy Prior	MCMC	Section	$\sigma_\nu \sim \text{half-Cauchy}(\text{scale} = 1)$
HB Province half Cauchy Prior	MCMC	Province	$\sigma_\nu \sim \text{half-Cauchy}(\text{scale} = 1)$

# Results

Note: All results shown are for the basal area response variable

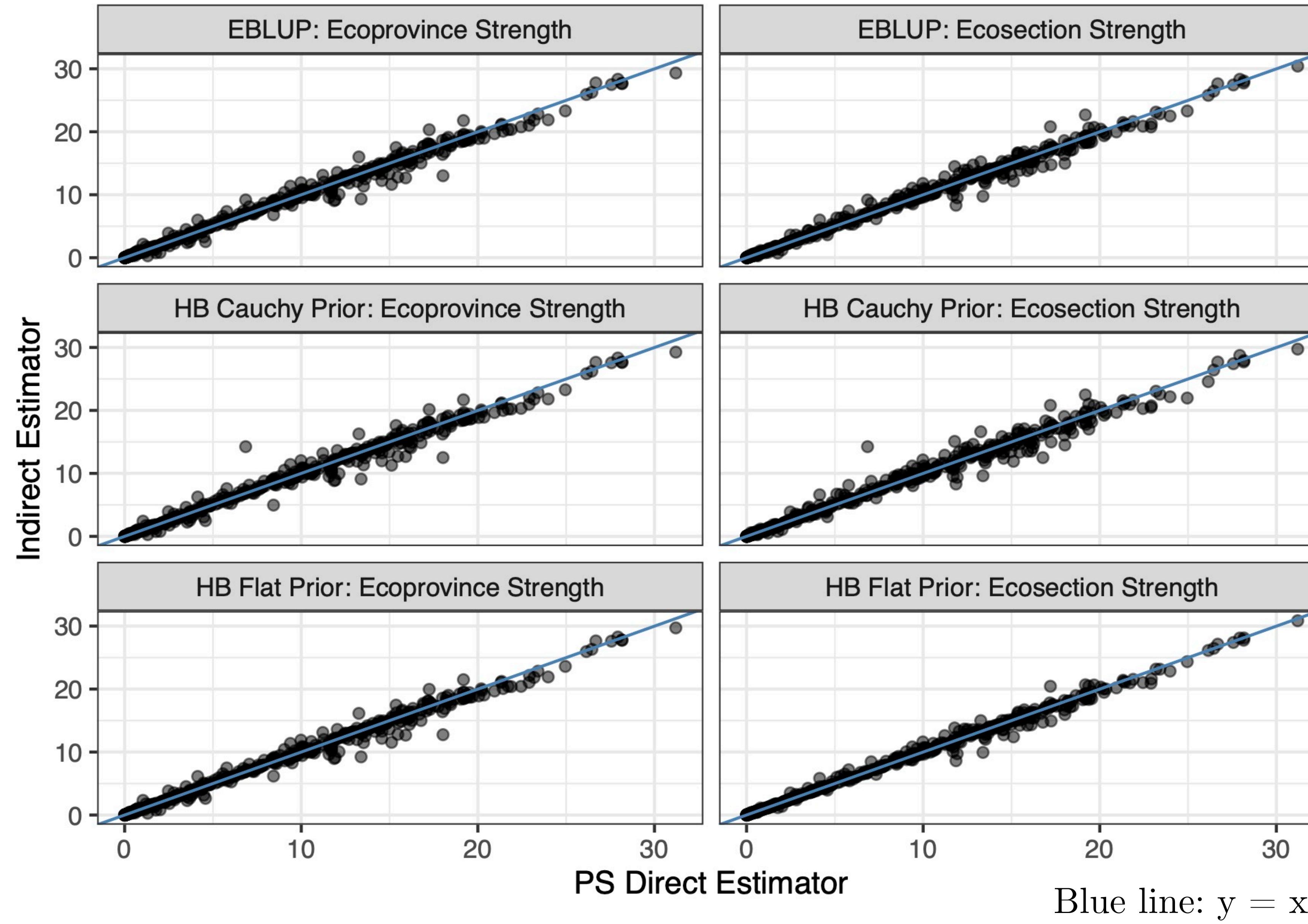


Mt. Hood, Oregon

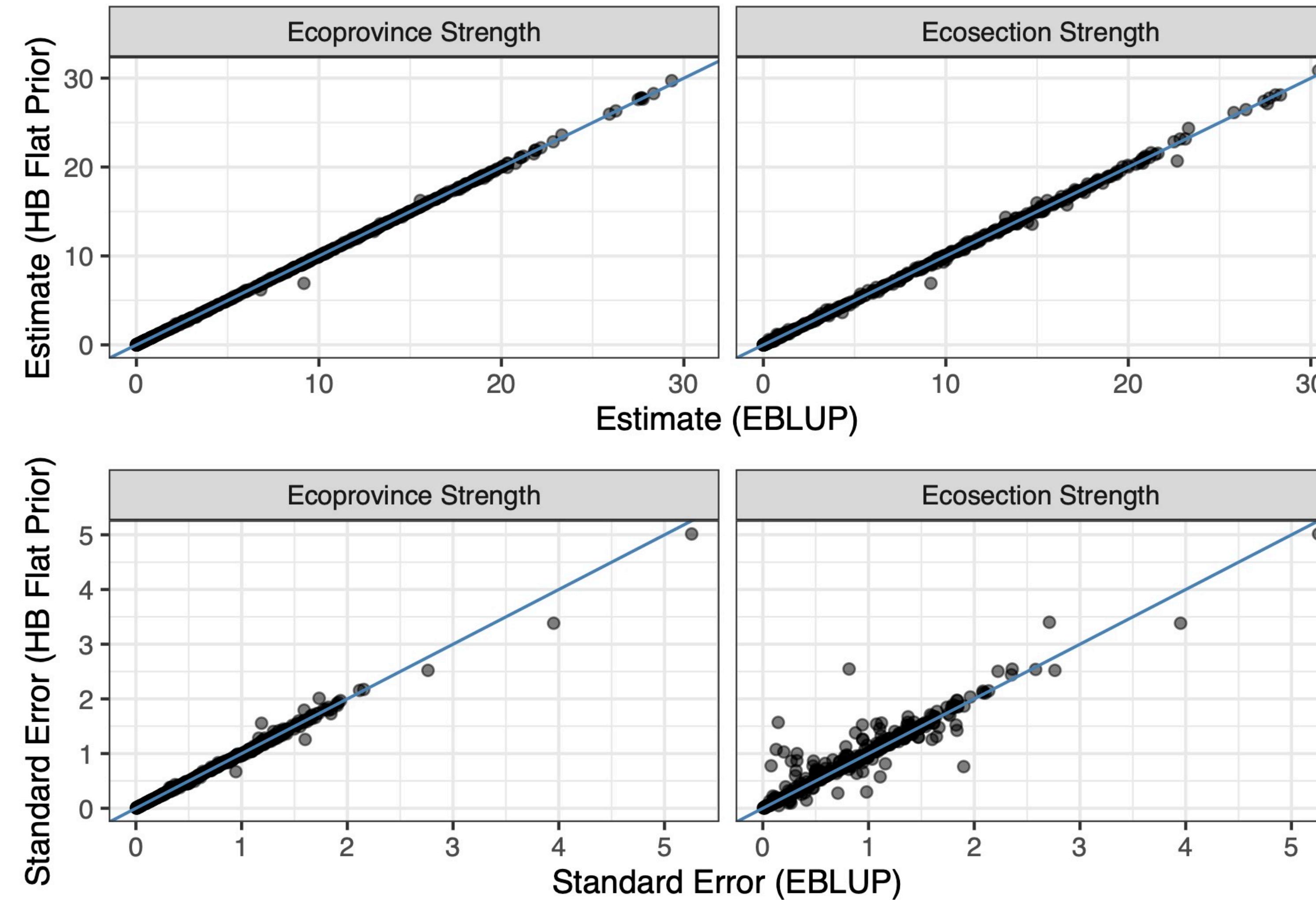


Valley of the Giants, Oregon

# Assessing Bias via Agreement with Post-Stratified Estimator

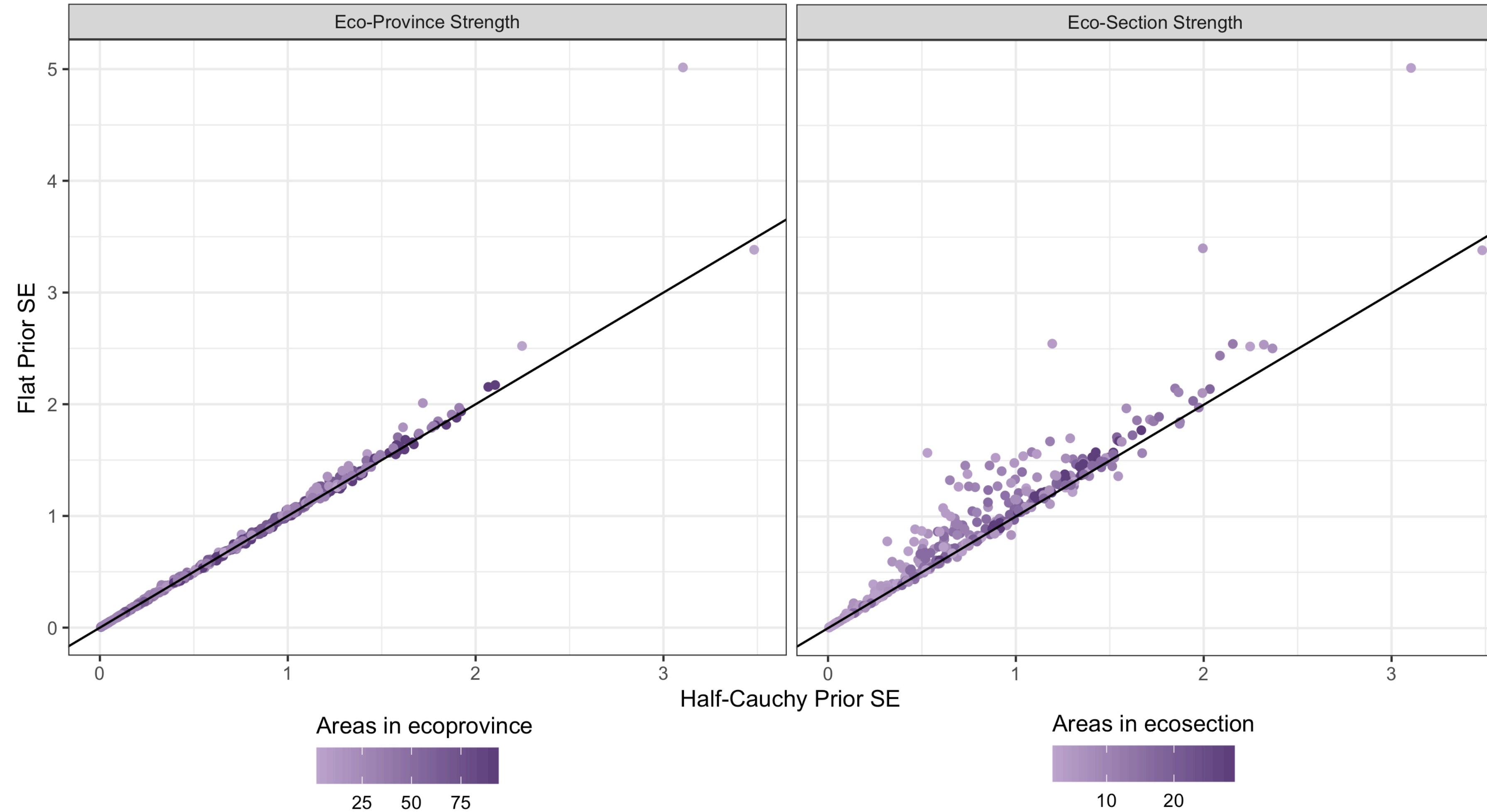


# Comparing the HB Flat Prior and EBLUP Estimates (top) and Standard Errors (bottom)



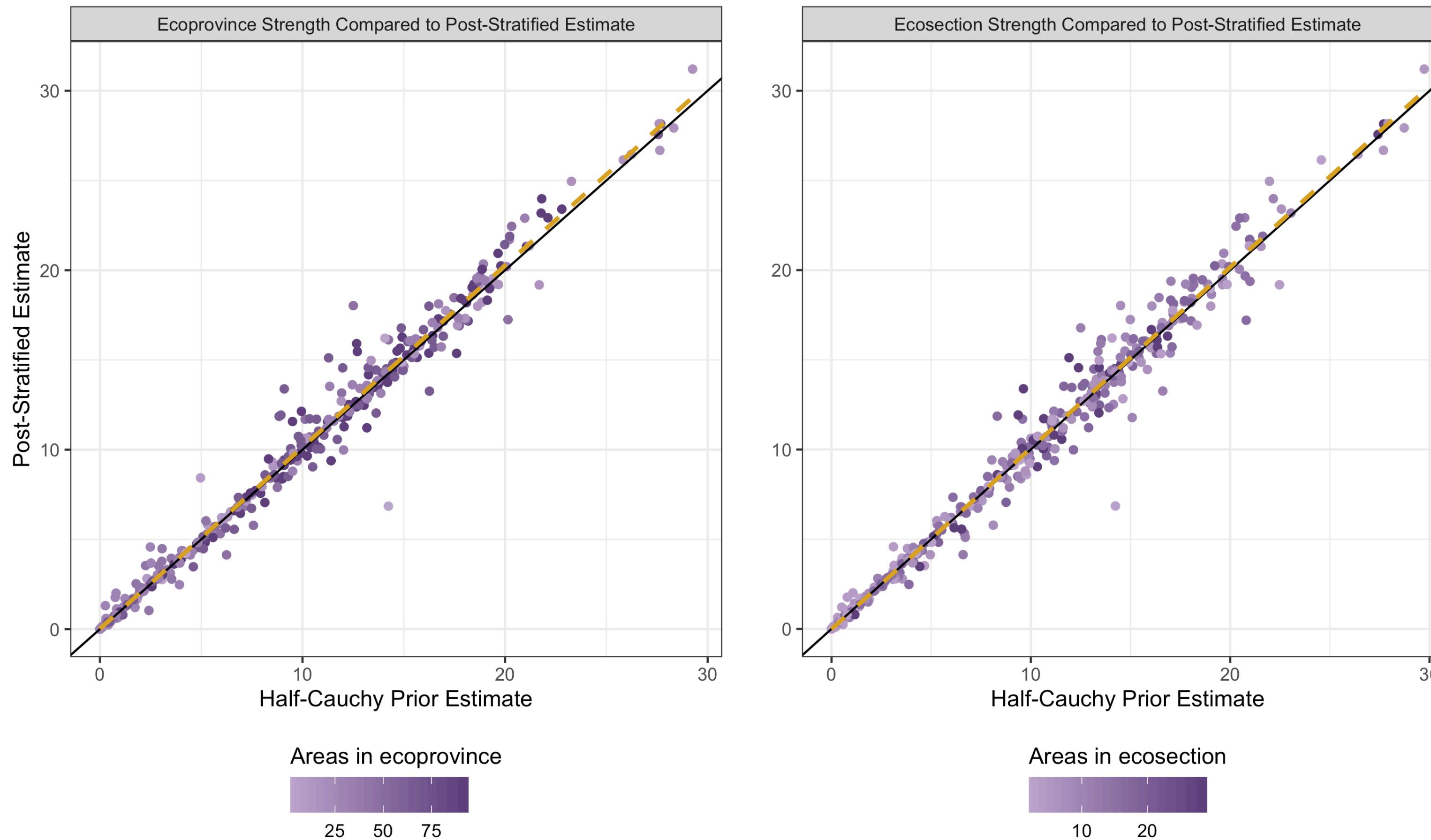
Blue line:  $y = x$ .

# Standard Error differences in HB models



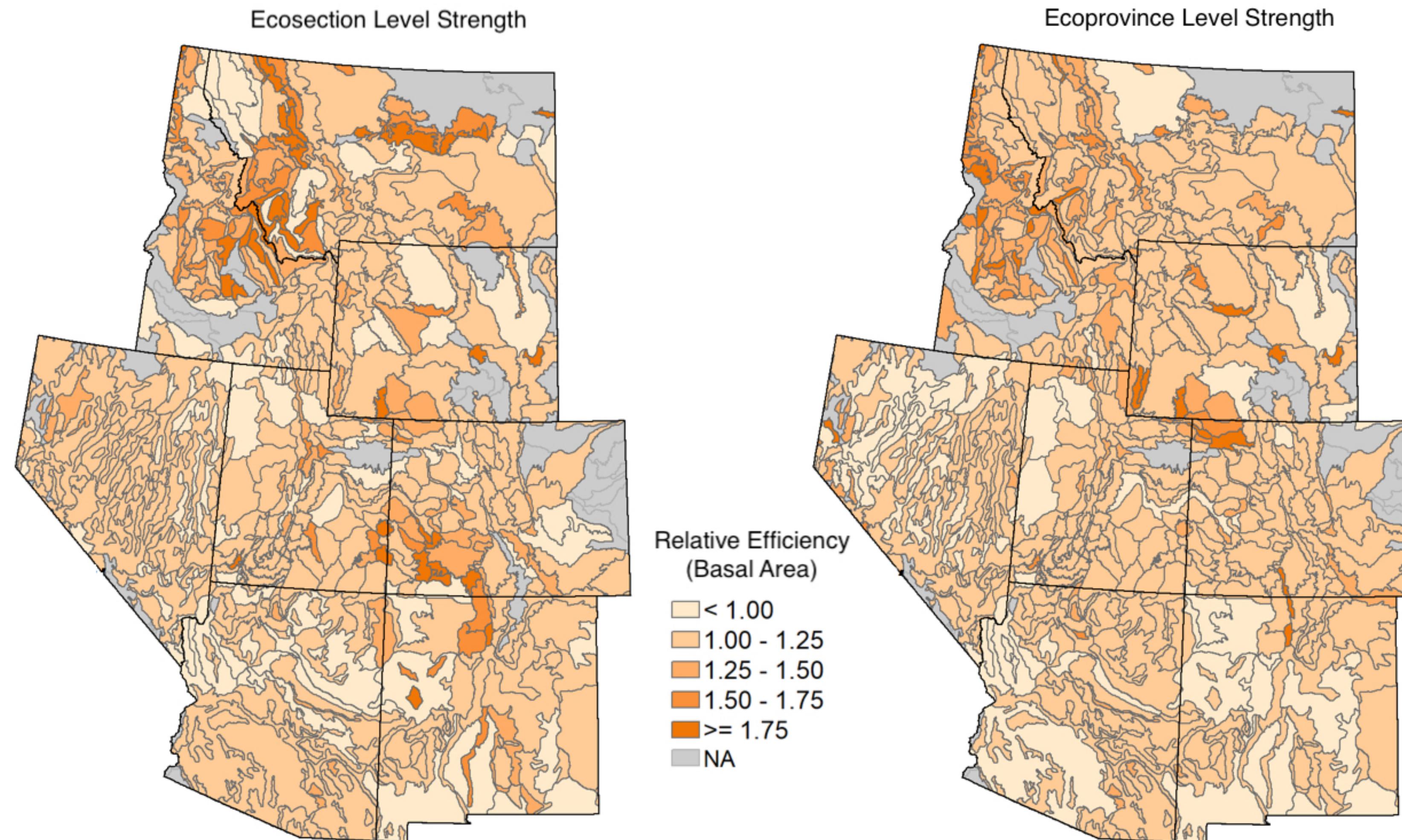
- Black line:  $y = x$ .
- Purple hue:  
amount of areas  
strength is  
borrowed from.

# HB half Cauchy Prior: Provincial vs. Sectional Borrowing Estimates



- Black line:  $y = x$ .
- Yellow dashed line: OLS best fit.
- Purple hue: amount of areas strength is borrowed from.

# Relative Efficiency of HB half Cauchy Prior Estimators and Post-Stratified Estimator



# Future Work



Death Valley, California

- Applying these methods across the rest of the United States.
- Unit-level small area models: zero-inflated models.
- A simulation study to understand potential bias.
- Multivariate small area models for many FIA variables of interest.

# Thank you!