

Air Quality Monitoring Wireless System

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Advection-diffusion-reaction model equation: second order pde equation

$$\frac{\partial x}{\partial t} - \lambda \nabla^2 x + \mathbf{v}^T \nabla x = f \quad \text{in } \mathbb{R}^2 \quad (1)$$

where:

$x(\mathbf{p}, t)$ is the space-time dependent pollutant concentration field, defined over the space-time domain with an initial condition $x(\mathbf{p}, 0) = x_0$;

$\mathbf{p} \in \mathbb{R}^2$ denotes the 2-dimensional position vector;

$t \in \mathbb{R}^+$ denotes time;

λ is the diffusion coefficient;

$\mathbf{v}(\mathbf{p}, t)$ is the advection velocity vector;

$f(\mathbf{p}, t) = s(t)F(\mathbf{p})$ represents the internal sources of pollution, defined as the product of the signal wavelength $s(t)$ and the position-depending base function $F(\mathbf{p})$

We need to truncate the physical domain to solve numerically the problem by using the finite element method (FEM).

variational formulation of the problem:

$$\int_{\Omega} \frac{\partial x}{\partial t} \varphi \, d\mathbf{p} - \lambda \int_{\Omega} \nabla^2 x \, \varphi \, d\mathbf{p} + \int_{\Omega} \mathbf{v}^T \nabla x \, \varphi \, d\mathbf{p} = \int_{\Omega} f \varphi \, d\mathbf{p}$$

where $\varphi(\mathbf{p})$ is a generic space-dependent weight function.

Green's identity substitution

$$\varphi \nabla^2 x = \nabla \cdot (\varphi \nabla x) - \nabla \varphi \cdot \nabla x$$

$$\int_{\Omega} \frac{\partial x}{\partial t} \varphi \, d\mathbf{p} - \lambda \int_{\Omega} \nabla \cdot (\varphi \nabla x) \, d\mathbf{p} + \lambda \int_{\Omega} \nabla \varphi \cdot \nabla x \, d\mathbf{p} + \int_{\Omega} \mathbf{v}^T \nabla x \, \varphi \, d\mathbf{p} = \int_{\Omega} f \varphi \, d\mathbf{p}$$

$$\int_{\Omega} \frac{\partial x}{\partial t} \varphi \, d\mathbf{p} - \lambda \int_{\partial \Omega} \varphi \nabla x \cdot \mathbf{n} \, d\mathbf{p} + \lambda \int_{\Omega} \nabla \varphi \cdot \nabla x \, d\mathbf{p} + \int_{\Omega} \mathbf{v}^T \nabla x \, \varphi \, d\mathbf{p} = s(t) \int_{\Omega} F(\mathbf{p}) \varphi \, d\mathbf{p}$$

FEM approximation:

$$x(p, t) = \sum_{j=1}^n \phi_j(p) x_j(t) = \phi^T(p) x(t)$$

FEM derivative model: a set of n inhomogeneous differential equations

$$\begin{aligned} & \underbrace{\left[\int_{\Omega} \phi(p) \phi^T(p) dp \right]}_M \dot{x}(t) + \underbrace{\left[\lambda \int_{\Omega} \nabla \phi(p) \nabla \phi^T(p) dp \right]}_{S_{\lambda}} x(t) \\ & + \underbrace{\left[\int_{\Omega} \phi(p) v^T(p) \nabla \phi^T(p) dp \right]}_G x(t) + \underbrace{\left[\lambda \int_{\partial\Omega} \beta(p) \phi(p) \phi^T(p) dp \right]}_{Q_{\beta}} x(t) = \\ & \underbrace{\left[\int_{\Omega} \phi(p) F(p) dp \right]}_{Q_f} s(t) \quad (2) \end{aligned}$$

example1:

$$F(p) = \delta(p - p_s)$$

$$\int_{\Omega} \phi(p) F(p) dp = \int_{\Omega} \phi(p) \delta(p - p_s) dp = \phi(p_s)$$

example2:

multiple point source

$$f(p, t) = \sum_{s=1}^{N_s} s_s(t) \delta(p - p_s)$$

$$\int_{\Omega} \phi(p) f(p, t) dp = \sum_{s=1}^{N_s} s_s(t) \int_{\Omega} \phi(p) \delta(p - p_s) dp = \sum_{s=1}^{N_s} s_s(t) \phi(p_s)$$

Advection Diffusion Model:

advection velocity vector = $[5 \ 5]$ m/sec.

diffusion coefficient = 5 ppm/m^2















