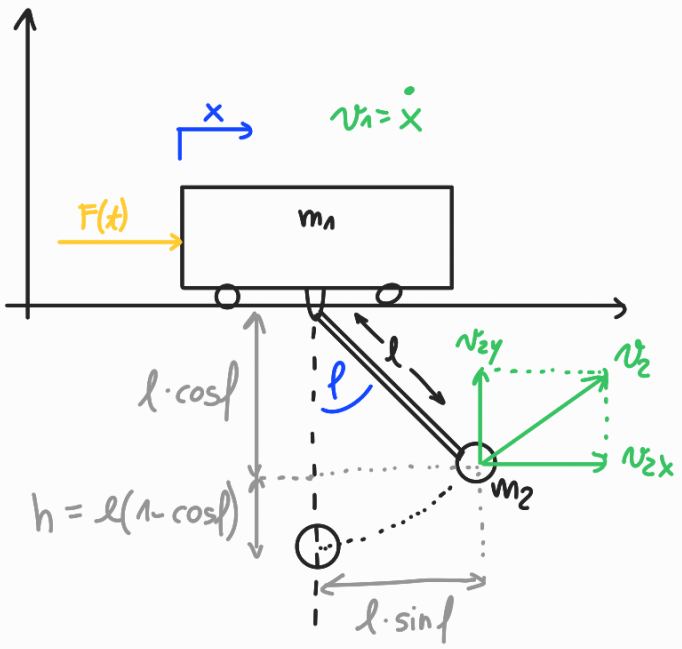
ŽERJAV – izpeljava modela



vhod:
- $F(t)$ – sila na vozičku

izhod:
- $x(t)$ – premik vozička
- $\varphi(t)$ – kot nihala

] posplošene koord.

1) $T_k = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$ → kinetična energija

↓ zapis z posplošenimi koordinatami

- $v_1 = \dot{x}$
- $v_2^2 = v_{2x}^2 + v_{2y}^2 =$

↳ odvod premika v x smeri:

$$= \left[\frac{d}{dt}(x + l\sin\varphi)\right]^2 + \left[\frac{d}{dt}(l(1-\cos\varphi))\right]^2 =$$

odvod po času:
$\frac{d}{dt}\sin\varphi = \cos(\varphi)\cdot\dot{\varphi}$
$\frac{d}{dt}\cos\varphi = -\sin(\varphi)\cdot\dot{\varphi}$

$$= \left[\dot{x} + l\cdot\cos(\varphi)\cdot\dot{\varphi}\right]^2 + \left[+l\cdot\sin(\varphi)\cdot\dot{\varphi}\right]^2 =$$

$$= \dot{x}^2 + 2\dot{x}l\cos(\varphi)\dot{\varphi} + \underline{l^2\cos(\varphi)^2\cdot\dot{\varphi}^2} + \underline{l^2\sin(\varphi)^2\dot{\varphi}^2} =$$

$$= \dot{x}^2 + 2l\dot{x}\cos(\varphi)\dot{\varphi} + l^2\cdot\dot{\varphi}^2$$

$$T_k = \frac{1}{2} m_1 \dot{x}^2 + \frac{1}{2} m_2 \left(\dot{x}^2 + 2l\dot{x}\cos(\varphi)\dot{\varphi} + l^2\cdot\dot{\varphi}^2\right)$$

$$T_k = \frac{1}{2}(m_1+m_2)\dot{x}^2 + m_2 l\dot{x}\cos(\varphi)\dot{\varphi} + \frac{1}{2} m_2 l^2\dot{\varphi}^2$$

2) $V_p = mgh = mgl(1-\cos(\varphi))$ → potencialna energija

3) $P = 0$ → sistem nima izgub

4) Lagrangeva funkcija

$$L = T_k - V_p = \frac{1}{2}(m_1+m_2)\dot{x}^2 + m_2 l\dot{x}\cos(\varphi)\dot{\varphi} + \frac{1}{2} m_2 l^2\dot{\varphi}^2 - m_2 g l(1-\cos(\varphi))$$

5) Lagrangeevi enačba za x (vozička)

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) - \frac{\delta L}{\delta x} + \frac{\partial P}{\delta \dot{x}} = F(t)$$

a) $\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{x}}\right) = \frac{d}{dt}\left[(m_1+m_2)\dot{x} + m_2 l\cos(\varphi)\dot{\varphi}\right] = (m_1+m_2)\ddot{x} - m_2 l\sin(\varphi)\dot{\varphi}^2 + m_2 l\cos(\varphi)\ddot{\varphi}$

$\frac{d}{dt}(\cos\varphi\cdot\dot{\varphi}) = -\sin(\varphi)\dot{\varphi}\cdot\dot{\varphi} + \cos(\varphi)\cdot\ddot{\varphi}$

↓ odvod produkta

b) $\frac{\delta L}{\delta x} = 0$ c) $\frac{\partial P}{\delta \dot{x}} = 0$

$$(m_1+m_2)\ddot{x} - m_2 l\sin(\varphi)\dot{\varphi}^2 + m_2 l\cos(\varphi)\ddot{\varphi} = F(t)$$

6) Lagrangeeva enačba za φ (nihalo)

$$\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\varphi}}\right) - \frac{\delta L}{\delta \varphi} + \frac{\partial P}{\delta \dot{\varphi}} = 0$$

a) $\frac{d}{dt}\left(\frac{\delta L}{\delta \dot{\varphi}}\right) = \frac{d}{dt}\left[m_2 l\overbrace{\dot{x}\cos(\varphi)}^{\text{produkt!}} + m_2 l^2\dot{\varphi}\right] =$

$= m_2 l\ddot{x}\cos(\varphi) - m_2 l\dot{x}\sin(\varphi)\cdot\dot{\varphi} + m_2 l^2\ddot{\varphi}$

b) $\frac{\delta L}{\delta \varphi} = -m_2 l\dot{x}\sin(\varphi)\cdot\dot{\varphi} - m_2 g l\sin(\varphi)$ c) $\frac{\partial P}{\delta \dot{\varphi}} = 0$

$$m_2 l\ddot{x}\cos(\varphi) - \cancel{m_2 l\dot{x}\sin(\varphi)\cdot\dot{\varphi}} + m_2 l^2\ddot{\varphi} + \cancel{m_2 l\dot{x}\sin(\varphi)\dot{\varphi}} + m_2 g l\sin(\varphi) = 0$$

$$m_2 l\ddot{x}\cos(\varphi) + m_2 l^2\ddot{\varphi} + m_2 g l\sin(\varphi) = 0$$

7) Nelinearni model

$$(m_1+m_2)\ddot{x} - m_2 l\sin(\varphi)\dot{\varphi}^2 + m_2 l\cos(\varphi)\ddot{\varphi} = F(t)$$

$$m_2 l\ddot{x}\cos(\varphi) + m_2 l^2\ddot{\varphi} + m_2 g l\sin(\varphi) = 0$$

Izražena $\ddot{x}$ in $\ddot{\varphi}$ za vnos v Matlab

$$(m_1+m_2)\ddot{x} - m_2 l \sin(\varphi)\dot{\varphi}^2 + m_2 l \cos(\varphi)\ddot{\varphi} = F(t)$$

$$m_2 l \ddot{x}\cos(\varphi) + m_2 l^2 \ddot{\varphi} + m_2 g l \sin(\varphi) = 0$$

$$\ddot{\varphi} = \frac{1}{m_2 l^2}\left(-m_2 l \cos(\varphi) - m_2 g l \sin(\varphi)\right)$$

$$(m_1+m_2)\ddot{x} - m_2 l \sin(\varphi)\dot{\varphi}^2 + m_2 l \cos(\varphi)\frac{1}{m_2 l}\left(-m_2 l \cos(\varphi) - m_2 g l \sin(\varphi)\right) = F$$

$$\ddot{x} = \frac{1}{m_1+m_2}\Big(+ m_2 l \sin(\varphi)\dot{\varphi}^2 + m_2 l \cos(\varphi)\frac{1}{m_2 l}\left(+ m_2 l \cos(\varphi) + m_2 g l \sin(\varphi)\right) + F \Big)$$

$$\ddot{x} = \frac{1}{m_1+m_2}\left(F + m_2 l \sin(\varphi)\dot{\varphi}^2 - m_2 l \cos(\varphi)\ddot{\varphi}\right)$$

$$m_2 l \cos(\varphi)\cdot\frac{1}{m_1+m_2}\left(F + m_2 l \sin(\varphi)\dot{\varphi}^2 - m_2 l \cos(\varphi)\ddot{\varphi}\right) + m_2 l^2 \ddot{\varphi} + m_2 g l \sin(\varphi) = 0$$

$$\ddot{\varphi}\left(-m_2^2 l^2 \cos(\varphi)^2 \cdot \frac{1}{m_1+m_2} + m_2 l^2\right) = -m_2 l \cos(\varphi)\cdot\frac{1}{m_1+m_2}\left(F + m_2 l \sin(\varphi)\dot{\varphi}^2\right) - m_2 g l \sin(\varphi)$$

$$\ddot{\varphi} = \frac{1}{-m_2^2 l^2 \cos(\varphi)^2 \cdot \frac{1}{m_1+m_2} + m_2 l^2}\left(-m_2 l \cos(\varphi)\cdot\frac{1}{m_1+m_2}\left(F + m_2 l \sin(\varphi)\dot{\varphi}^2\right) - m_2 g l \sin(\varphi)\right)$$

Linearizacija

$\doteq \varphi$ $\doteq 0$

$$(m_1+m_2)\ddot{x} - m_2 l \sin(\varphi)\dot{\varphi}^2 + m_2 l \cos(\varphi)\ddot{\varphi} = F(t)$$

$$m_2 l \ddot{x}\cos(\varphi) + m_2 l^2 \ddot{\varphi} + m_2 g l \underbrace{\sin(\varphi)}_{\doteq \varphi} = 0$$

Približki za majhne kote:

$\sin\varphi \doteq \varphi$

$\cos\varphi \doteq 1$

$\varphi\dot{\varphi}^2 \doteq 0$

$$(m_1+m_2)\ddot{x} + m_2 l \ddot{\varphi} = F(t)$$

$$m_2 l \ddot{x} + m_2 l^2 \ddot{\varphi} + m_2 g l \varphi = 0$$

Statična karakteristika

⊕ ⟶

U	x_{input}*	Naklon [V/s]
1	/	/
1,5	/	/ (se malo premika)
2	3,1	11,6k
2,5	2	18,2k
3	1,2	25,9k
3,5	1	34,6k
4	0,8	39,5k
4,5	0,7	40,8k
5	0,6	44,2k

⊖ ⟵

U	x_{input}*	Naklon [V/s]
1	/	/
1,5	/	/
2	8	4,4k
2,5	4	11,6k
3	1,3	19,0k
3,5	1	24,0k
4	0,8	30k
4,5	0,7	32,6k
5	0,7	39,0k

Podatki žerjava

- Enkoder od motorja
  22755 pulzov/meter
- Dimenzije & masa
  $m_1 = 2kg$
  $m_2 = 0,21 + 0,1 = 0,31kg$
  $l = 0,5m$

22765/m

	+	−				
A	t_{max}	A_{max}				
0	∞	∞				
0.5	/					
1	/					
1.5						
2	3,23	8,85				
2.5	2	2,9				
3	1,2	1,2	22,62k	16,18k		
3.5	0,9	1,1	25,36k	20,49k		
4	0,8	0,8	19,774		31,32	
4.5	0,7	0,8	18,59	19,134	31,52	27,65
5						

+

se premakne od 18,284V – 20,361

Range 31,77