Seeing and Critical Focus

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Abstract

While attempting to perform imaging at a sub-arcsecond per pixel resolution under less than optimal seeing the author experienced some significant issues in trying to achieve critical focus. Under these conditions focus moves that were above the typically computed critical focus zone (CFZ) size did not appear to have any impact on the FWHM of the observed candidate stars.

This lead to a post session analysis of the issues experienced and the causes and effects of the observed behavior.

This paper takes a critical look at seeing and the effect of telescope resolution with an emphasis on the impacts for imaging.

Contents

1 Diffraction Limited Optics

We will begin by looking at the theoretical basis for diffraction limited optics. From there we will look at the impact this has on the key sampling parameter of interest to astrophotographers. Name, how this relates to image scale measured in arc-seconds per pixel and the corresponding size of the stars.

1.1 The Airy Disk

The study of the nature of the in-focus diffraction pattern of a star was first studied by Professior Goerge Biddell Airy of Trinity College,University of Cambridge. His work was first read at the November 24^{th}, 1834 meeting of the Cambridge Philosophical Society. The intensity of the Airy diffraction rings are given by the following formula:

$$I(\theta) = I_0 \left(\frac{2J_1(kasin\theta)}{kasin(\theta)} \right)^2 = I_0 \left(\frac{2J_1(x)}{x} \right)^2 \tag{1}$$

where

$x = kasin\theta$

I_0 is the maximum intensity of the disk

J_1 is the Bessel function of the first kind of order 1[1]

Further:

$k = \frac{2\pi}{\lambda}$

$a =$radius of the aperture

θ is the angle from the centerline to the point the intensity is measured.

This is a complex function to understand and a picture will tell a thousand words. Figure 1 shows the plot of the diffraction pattern intensity in both 2-D and 3-D.

The value of $I(\theta)$ will be zero when the value of $J_1(x) = 0$. It can be seen that the first root appears at $x \approx 3.83$.

Thus:

[1] See Appendix A

$x = kasin\theta \approx 3.83$

$sin\theta \approx \frac{3.83}{ka} = \frac{3.83\lambda}{2\pi a}$

$$sin\theta = 1.22\frac{\lambda}{d} \approx \theta \tag{2}$$

More usually this is expressed in terms of the full diameter of the disk rather than the radius:

$$D_{Airy} = 2.44\frac{\lambda}{d} \tag{3}$$

This is the more usual form of the expression for the Airy disk, where d is the diameter of the objective. It also forms the basis for a number of the expressions for the limits of resolution of a telescope.

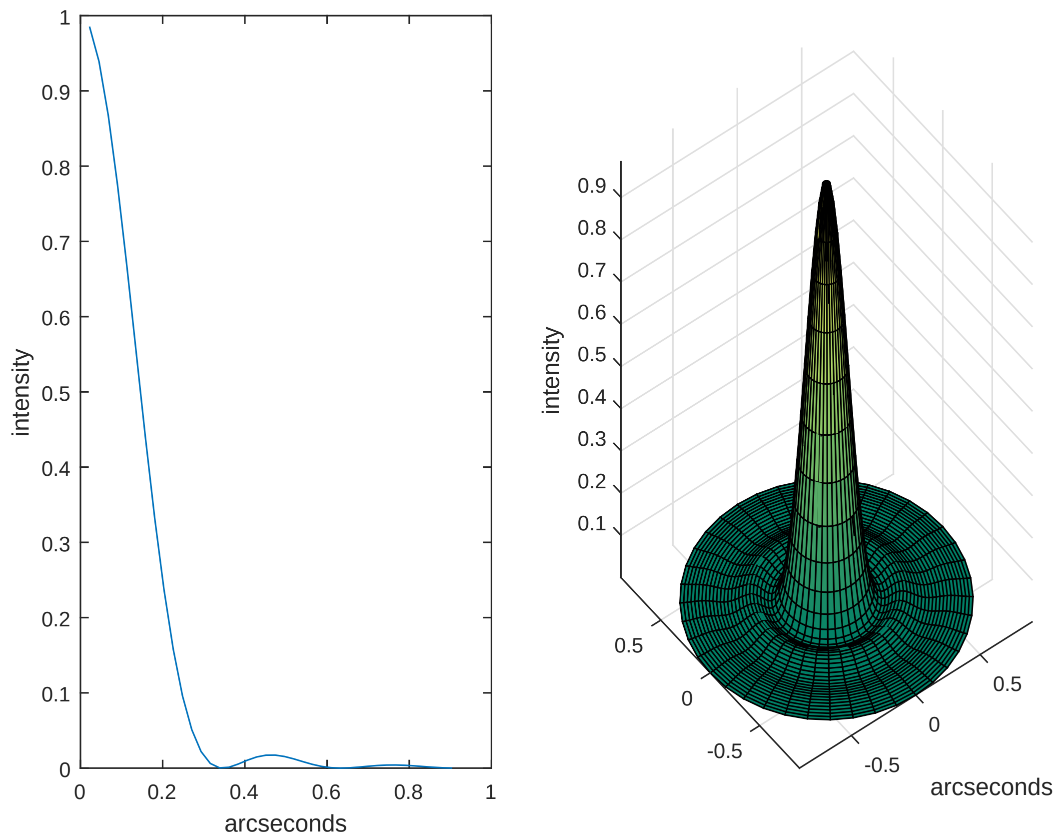

Figure 1: Airy Disk Intensity for 200 mm unobstructed telescope

1.2 Effect of aperture on resolution

It can be readily seen that the angular size of the Airy disk is a function of the wavelength of the light and the aperture of the scope only. For visual use it is common to take the wavelength of yellow light as the reference, since yellow light is is where the eye is most sensitive. This is likely an evolutionary link to the predominant color of the sun. The wavelength is given as 550 nM, or 550×10^{-9} M

What is perhaps surprising is the dramatic increase in resolution going from a 50mm aperture to a 100mm aperture. The improvements from 100mm to 200mm and 300mm apertures are not nearly so apparent. Indeed, beyond about 400mm the improvements in angular resolution become very slight indeed.

This might then provide some indication of why refractor telescopes of around

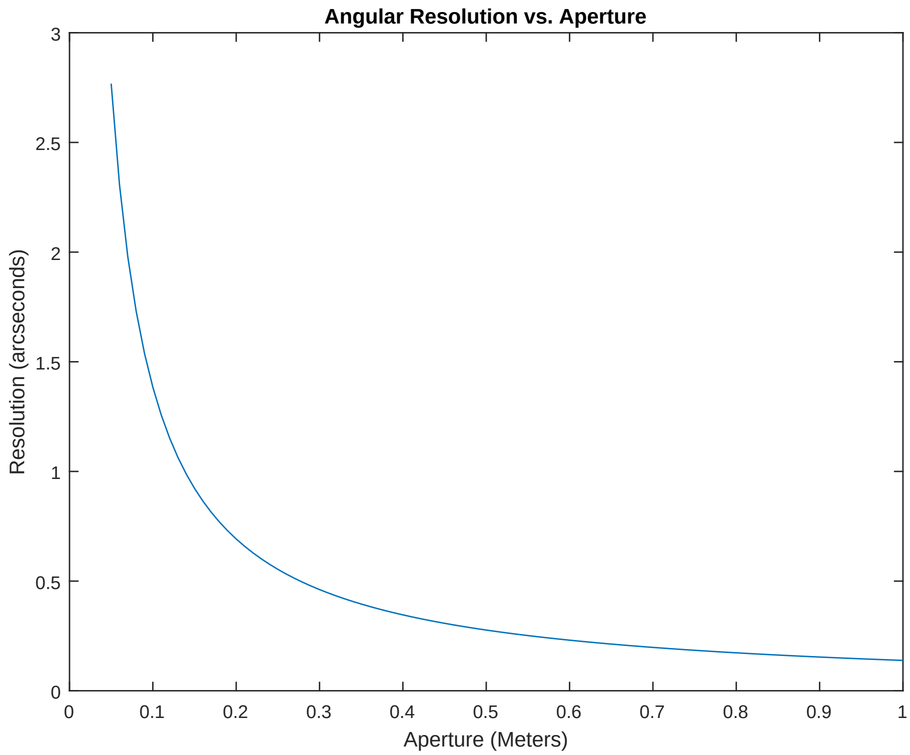

Figure 2: Angular Resolution

120mm aperture are seen as the optimum instruments for visual observation of stellar objects. At this point most of the benefits of improved angular have been realized and we at the limits imposed by atmospheric seeing.

Diffuse objects such as galaxies and nebulae are not constrained by angular resolution, but by the raw light gathering power of the instrument. Thus for those classes of objects befits will continue to be realized with increased aperture.

1.3 Spot Size

So far we have dealt with the size of the Airy disk purely in angular terms. However, when dealing with imaging the physical size of the spot as it appears on the sensor is key. The physical size of the spot may be calculated from the geometry of the telescope.

The Airy disk will appear to be formed at the objective lens of the telescope. Since we know the angular size of the spot its physical dimensions may be calculated by simple trigonometry. With reference to figure 3:

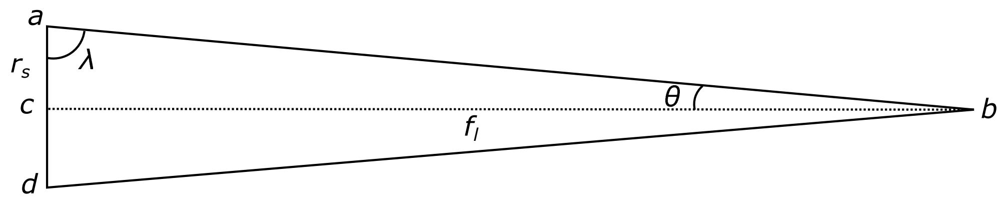

Figure 3: Airy Disk Geometry

Here θ is the angle subtended by the radius of the Airy disk, f_l is the focal length of the telescope and r_s is the radius of the Airy disk.

The triangle abc is an right angle triangle. From this, and from the sin rule for triangles we may say that:

$$\frac{sin(\theta)}{r_s} = \frac{sin(\lambda)}{f_l}$$

But from the sum of internal angles in a triangle

$$\lambda + \theta + 90 = 180 \implies \lambda = 90 - \theta$$

$$\frac{sin(\theta)}{r_s} = \frac{sin(90-\theta)}{f_l}$$

$$\frac{sin(\theta)}{r_s} = \frac{cos(\theta)}{f_l}$$

$$r_s = \frac{sin(\theta)}{cos(\theta)} f_l$$

But for small values of θ, $sin(\theta) \approx \theta$ and $cos(\theta) \approx 1$ thus:

$$r_s = \theta f_l$$

Substituting the value for θ derived in eqn 2 we find that:

$$r_s = \frac{1.22\lambda f_l}{d}$$

$$r_s = 1.22\lambda f_r \implies d_s = 2.44\lambda f_r \tag{4}$$

Where f_r is the focal ratio of the instrument and d_s is the spot diameter.

This is actually quite a surprising result. The spot size for an optically perfect instrument is governed by the focal ratio of the instrument. Thus a small refractor with a focal ratio of f_r will produce a spot size the same as a large instrument of the same focal ratio.

At first blush this appears to run counter to the notion that larger aperture instruments have finer angular resolution, but a simple diagram will illustrate this finding is in fact consistent.

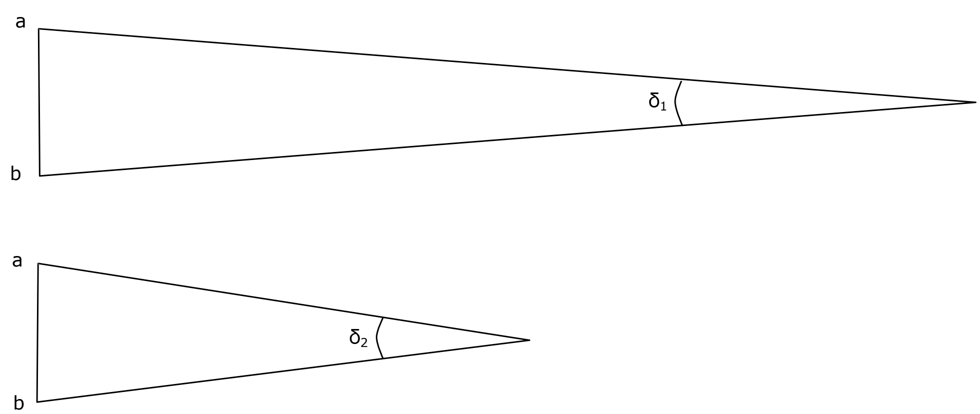

Figure 4: Spot Size and Resolution

Let the points a and b be the centers of 2 Airy disks that are spaced at the limit of resolution in 2 instruments of similar focal ratio but with different apertures. Simply by observation it may be seen that δ_1 is a smaller angle than δ_2. Thus the larger instrument with the longer focal length indeed has better angular resolution even though it has the same focal ratio.

1.4 The zone of critical focus

Now that we have established the spot size we can move on to look at the critical focus zone (CFZ). Focus in a telescope is established by moving the plane of the detector back and forth until the point of best focus is reached. We have established that there is a point at which the spot reaches its minimum diameter. This is shown in Figure 5

Here we see the light cone of the instrument as it extends beyond the zone of focus. The dashed line at F_1 indicates size of the out of focus disk on the sensor at point F_1.

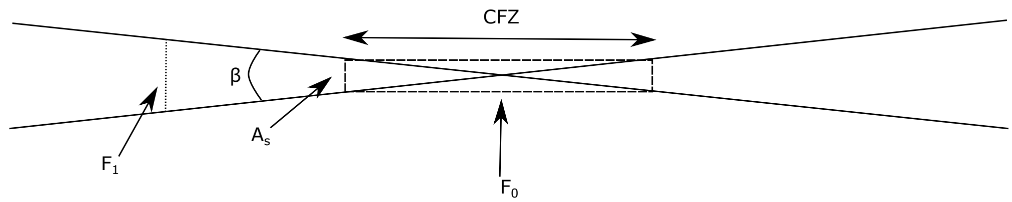

Figure 5: Spot Size and Resolution

As F_1 moves towards F_0, the theoretical point of convergence of the light cone, the size of the out of focus image will reach the minimum Airy spot size indicated by A_s.

This minimum spot size will remain unchanged on the sensor for the distance indicated by CFZ. Beyond this the spot size on the sensor will then begin to increase again.

It should be clear then, that any change to the position of the sensor in the range given by CFZ will result in no change in the appearance of the star. The aim then is to place the sensor within the bounds of this dead zone.

The cone angle β is a function of the focal ratio, f_r, of the instrument. From simple trigonometry it can be shown that β is given by:

$\beta = 2 \times \arctan(\frac{1}{2f_r})$

To compute the physical size of the CFZ we may decompose the problem into a set of right angle triangles.

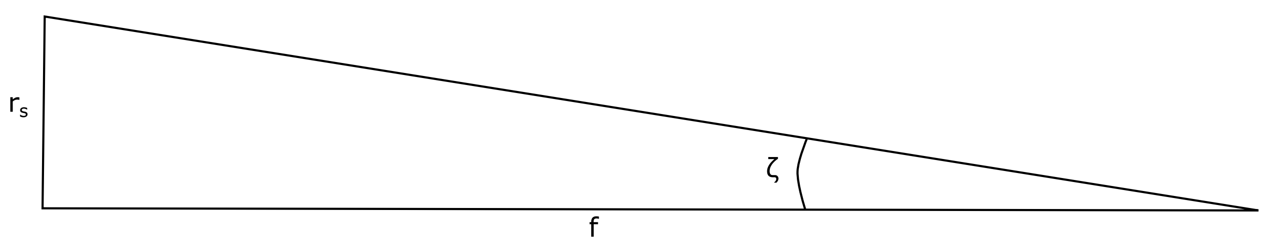

Figure 6: Spot Size and Resolution

r_s is the Airy radius as given by equation 2

f is half the CFZ

ζ is half the cone angle, $\zeta = arctan(\frac{1}{2f_r})$

Because ζ is small:

$\zeta \approx \frac{1}{2f_r} \approx \frac{r_s}{f} \implies f = 2r_s f_r$

Substitute Eqn 2 for r_s:

$f = 2.44\lambda f_r^2 \implies$

$$CFZ = 4.88\lambda f_r^2 \tag{5}$$

Thus we may now plot the critical focus zone for yellow light for a range of focal ratios:

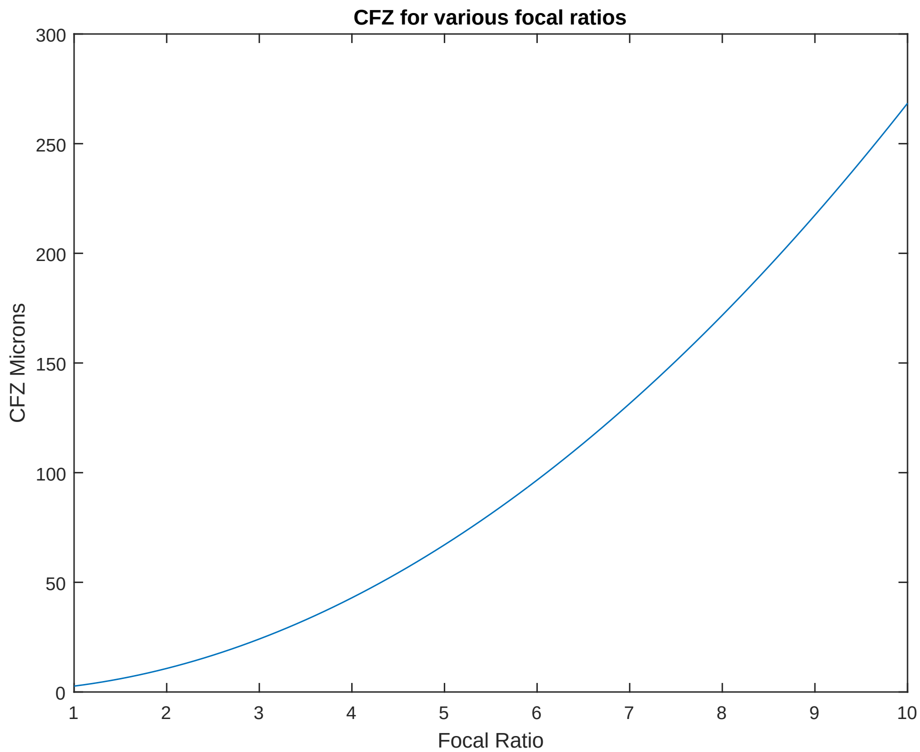

Figure 7: CFZ against Focal Ratio

One thing to observe is the size of the CFZ in comparison to the claimed step size of a number of commercial motorized focusing systems. It may readily be seen that the claimed resolution of many of these devices is far, far smaller than is required to achieve the optimum focus.

1.5 The effect of a central obstruction

The effect of adding a central obstruction of the instrument is described by a modification to eqn 1 as follows:

$$I(\theta) = \frac{I_0}{(1-\epsilon^2)^2} \left(\frac{2J_1(x)}{x} - \frac{2\epsilon J_1(\epsilon x)}{x} \right)^2 \tag{6}$$

where ϵ is the fractional size of the obstruction. For a typical amateur 200mm RCOS scope this value is about 0.40.

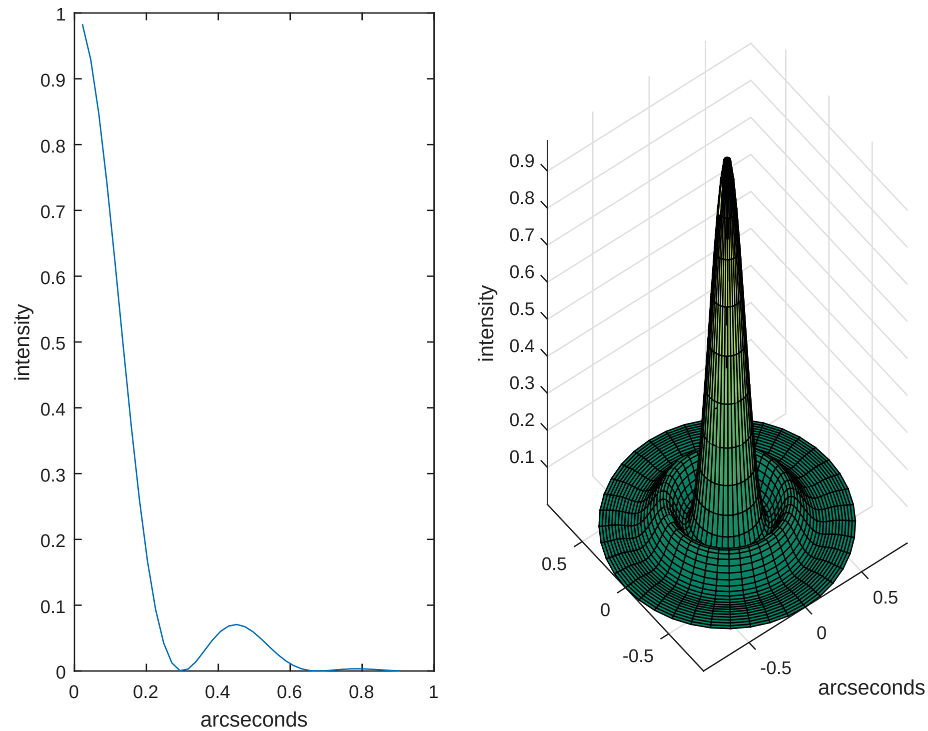

Figure 8: Obstructed Airy Diffraction Pattern

If we compare figure 1 to figure 8 we can see that the effect of the obstruction is make the central diffraction peak sharper at the cost of making the first diffraction ring more intense. This has the effect of lowering the contrast between two adjacent stars.

2 The effects of Seeing and Sampling

2.1 What seeing does to diffraction limits

Seeing is a way of describing the turbulence in the atmosphere. Whilst seeing effects both visual observers and astrophotographers, it is the latter that are effected most.

The human eye can adapt rapidly to the movement of the star and can grasp moments of clarity that the imaging sensor cannot. The human eye will form an image in a fraction of a second. An imaging sensor may take many minutes to form a usable image.

The effect for an imaging sensor is to integrate all the atmospherically induced movements with the result that the stellar image is smeared across a larger area. In poor seeing the stars will appear larger, and the fine detail of deep space objects will be lost.

This is seen in the shape of the ideal intensity plots for stars for good seeing and bad seeing as shown in figure 9.

Over given period of time the total amount of photons received from the star does not change. The photon flux may be considered invariant. When the seeing is bad the total amount of light received is spread across a wider area than when the seeing is good. Overall, in bad seeing, the image of the star is dimmer and wider.

The width of the star is given in terms of the Full Width half Maximum value (FWHM). The FWHM is the width of the peak at the point where the intensity is half the peak value. The half maximum lines for good seeing and bad seeing are shown in figure 9.

The size of the FWHM for good seeing is considerably narrower then for bad seeing. This will equate to stars that appear to more pin-point like and brighter in the image.

When seeing is described by astrophotographers as being 2 arcseconds or 4 arcseconds, it is the size of this FWHM value that they are referring to.

2.2 Real World Critical Focus

For the purposes of imaging at least it is very rare indeed that the performance of the optical train will be at the diffraction limit. Rather, the performance will be limited by seeing.

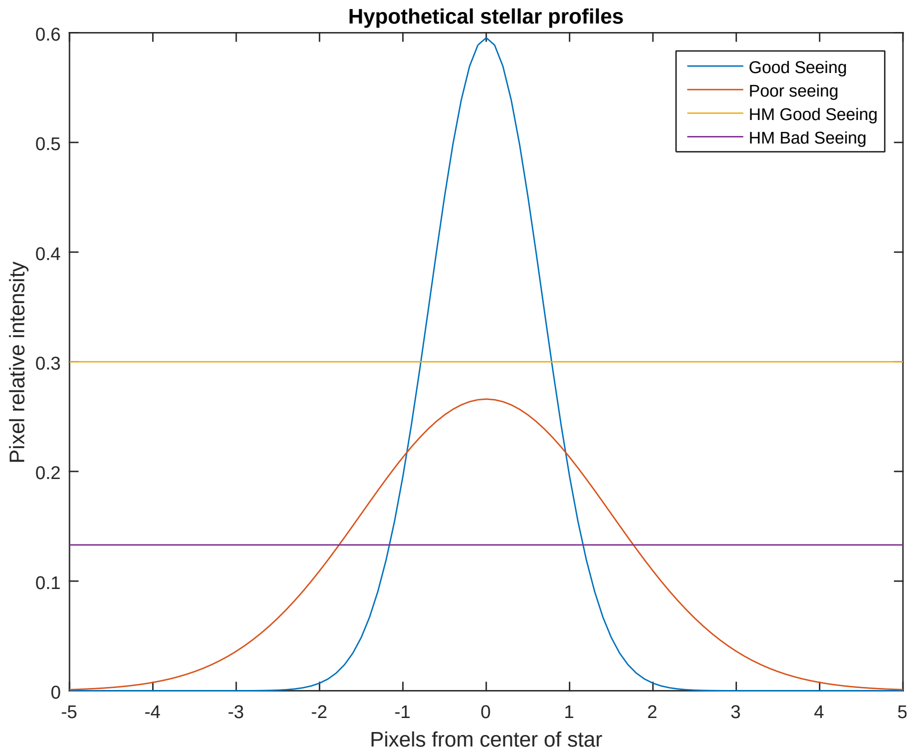

Figure 9: Stellar Profiles

How then does this effect the CFZ? In this case the smallest possible spot size will not be given by the diffraction limited value derived in equation 4, but instead will be limited by the FWHM value.

2.2.1 Critical Focus Zone for various systems

Appendices

Appendix A

The contents...

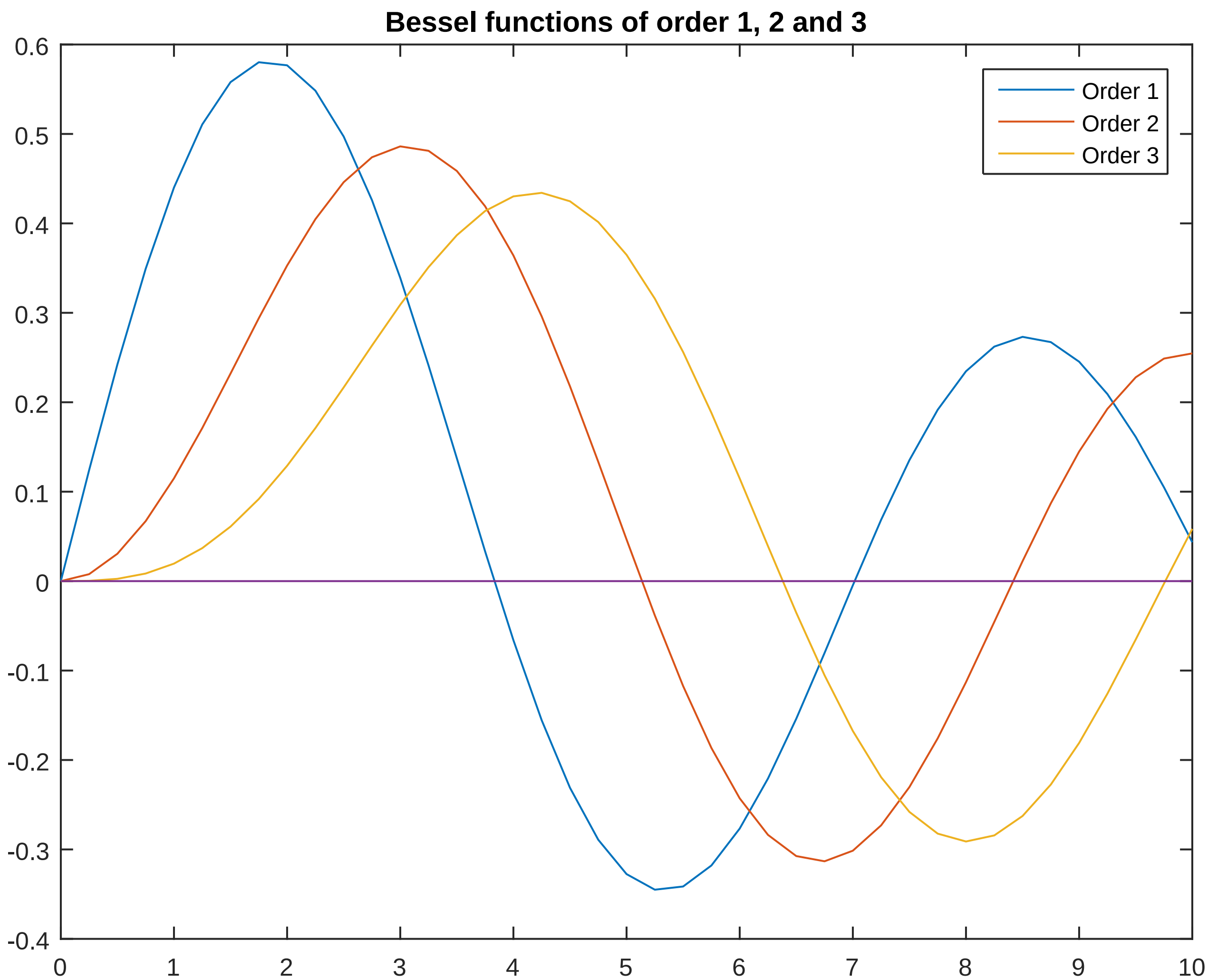

Figure 10: Bessel Functions