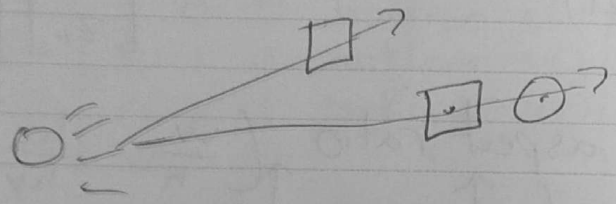
Graphics Raytracing

Pg 1 June 12th

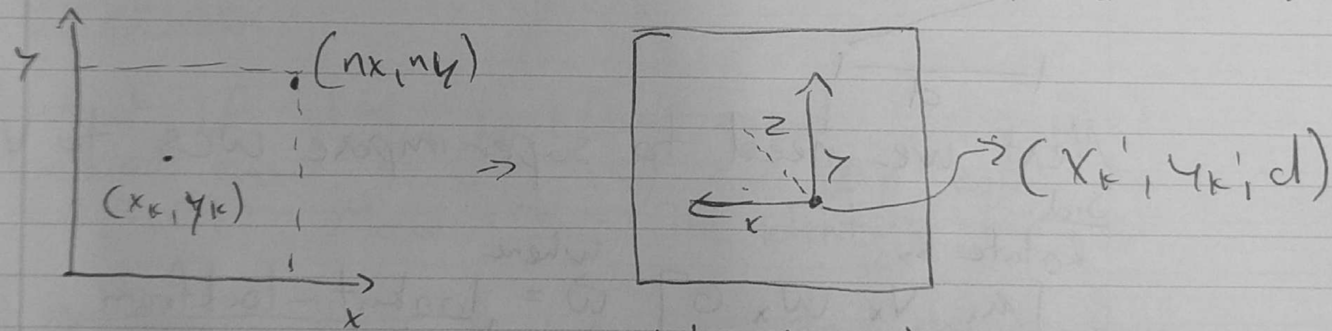
1960's Raycasting



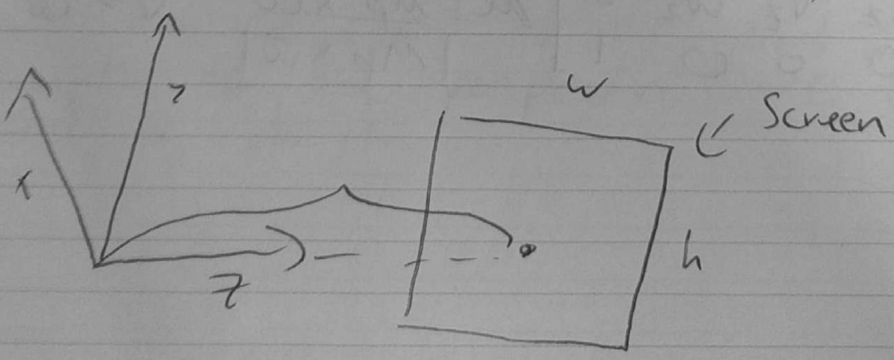
1980's Raytracing
- Turner Witted

- go from each screen pixel from device position to world position.
  → go backwards down the pipeline.

In order to determine the color of pixel k (p_k) we first determine the world coordinates of the pixel. Find P_{world} given (x_k, y_k) on $[0, n_x] \times [0, n_y]$

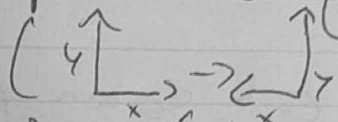


$(\frac{w}{2}, \frac{-h}{2}, d)$



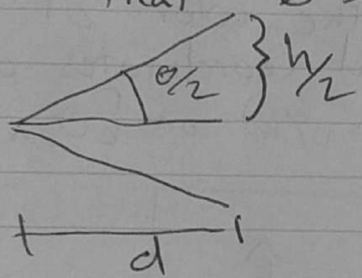
Steps

1. make $z_k = 0$ (already zero), translate by $\left(-\frac{n_x}{2}, \frac{n_y}{2}, d\right)$ $\begin{bmatrix} x_k \\ y_k \\ z_k \end{bmatrix}$

2. to preserve the aspect ratio $\left(\frac{w}{h} = \frac{n_x}{n_y}\right)$
and correct the sign 

Scale by: $\left(\frac{-w}{n_x}, \frac{h}{n_y}, 1\right) \equiv \left(\frac{-h}{n_y}, \frac{h}{n_y}, 1\right)$

Note that θ = field of view



$w = \frac{n_x}{n_y} h$ $\quad h = 2d \tan\frac{\theta}{2}$

Next we need to superimpose WCS to VCS

3rd.
Rotate by:

$$R = \begin{bmatrix} u_x & v_x & w_x & 0 \\ u_y & v_y & w_y & 0 \\ u_z & v_z & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

where

$$\vec{w} = \frac{lookat - lookfrom}{|lookat - lookfrom|}$$

$$u = \frac{\vec{up} \times \vec{w}}{|\vec{up} \times \vec{w}|} \qquad \vec{v} = \vec{w} \times \vec{u}.$$

Raytracing Part 2

Steps continued

4.

Translate by:

$$T = \begin{bmatrix} 1 & 0 & 0 & lookfrom.x() \\ 0 & 1 & 0 & lookfrom.y() \\ 0 & 0 & 1 & lookfrom.z() \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$P_{world} = \underbrace{T_4 R_3}_{M_{vw}} S_2 T_1 P_K$$

ray origin = Lookfrom (WCS)
ray direction = Pworld − lookfrom

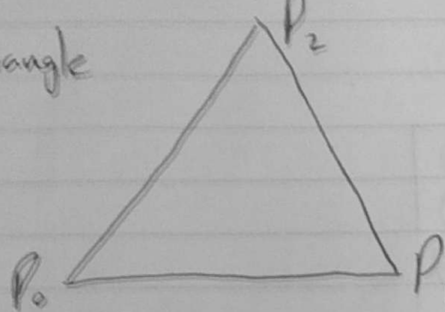
Exercise:

What if d=1, lookfrom equal (0,0,0,1) lookat equal to (0,0,d) and up equal to (0,1,0)? Mvw = ? → The identity matrix!

Final exam: Why do we not do perspective transformation? Since we are going from the eyepoint to the pixel, it already accounts for the perspective, we do not need to do it.

Ray Tracing Intersections

Triangle



A point is on Δ if it is on the plane
$p = \alpha P_0 + \beta P_1 + \gamma P_2$

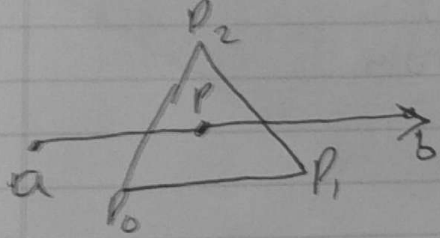
if $\beta \geq 0$ and $\gamma \geq 0$ and $\beta + \gamma \leq 1$ the point is on the Δ.

Parametric Eq. for the triangle. $P = P_0 + \beta(P_1 - P_0) + \gamma(P_2 - P_0)$

Parametric Eq. for the ray: $p(t) = a + t(b-a)$

a ↓ origin, $(b-a)$ ↓ direction

$b = P_{world}$

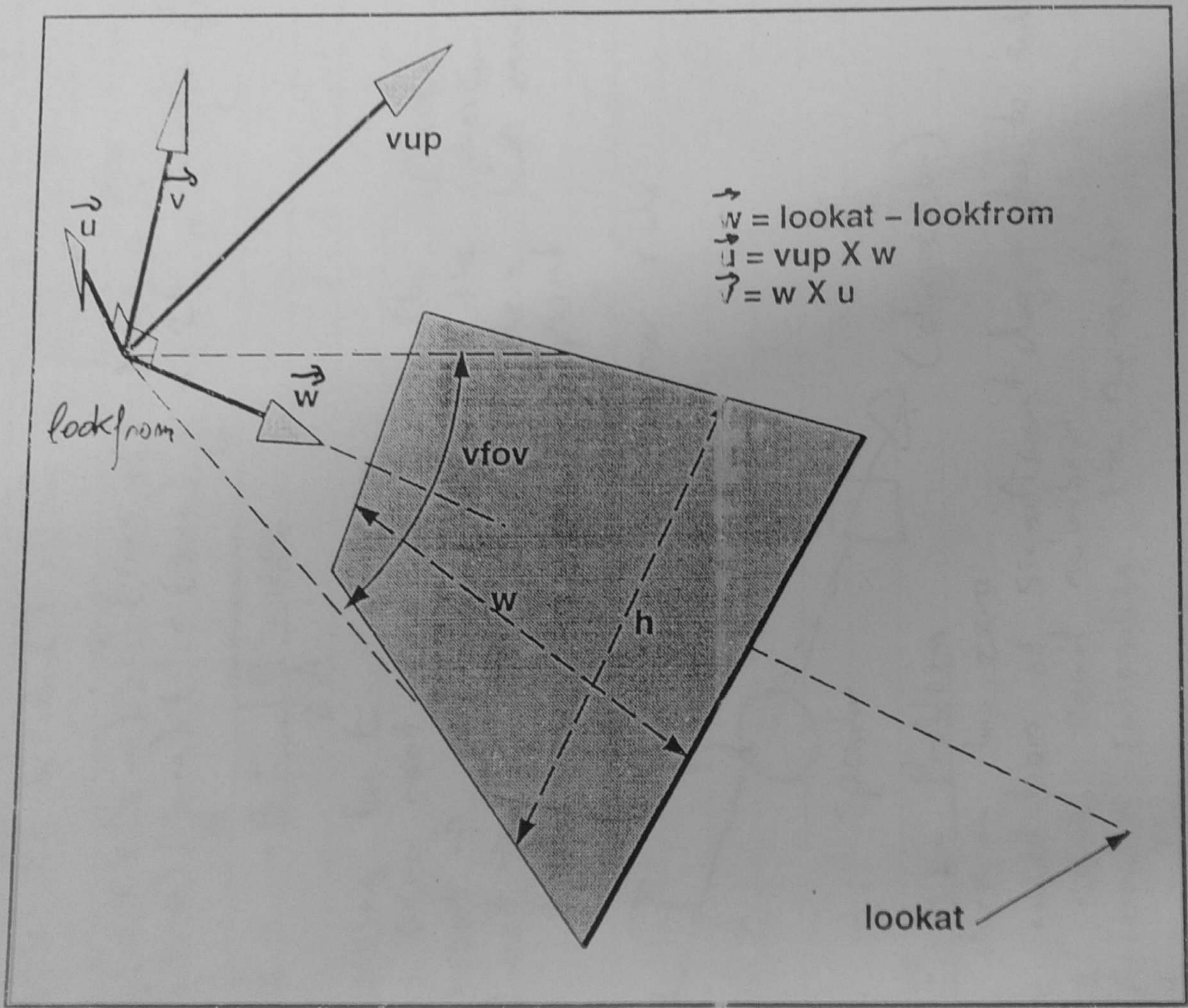


To find the intersection of a ray with a triangle.

$$a + t(b-a) = P_0 + \beta(P_1 - P_0) + \gamma(P_2 - P_0)$$

- each coordinate has 3 coordinates, so we have 3 equations & 3 unknowns.

Scanned by CamScanner


vup
v
u
$\vec{w}$ = lookat – lookfrom
$\vec{u}$ = vup X w
$\vec{v}$ = w X u
$\vec{w}$
lookfrom
vfov
w
h
lookat

Ray Tracing Intersections

Isolate independent terms

$$a - P_0 = (P_1 - P_0)\beta + (P_2 - P_0)\delta - (b-a)t$$

$$a.x - P_0.x = (P_1.x - P_0.x)\beta + (P_2.x - P_0.x)\delta - (b.x - a.x)t$$

(also for .z & .y.

$$a.y - P_0.y = (P_1.y - P_0.y)\beta + (P_2.y - P_0.y)\delta - (b.y - a.y)t$$

$$a.z - P_0.z = (P_1.z - P_0.z)\beta + (P_2.z - P_0.z)\delta - (b.z - a.z)t$$

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \end{bmatrix} = \begin{bmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{bmatrix} \begin{bmatrix} \beta \\ \delta \\ t \end{bmatrix}$$

Apply kramer's rule $D = \det \begin{bmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{bmatrix}$

$$D_1 = \begin{bmatrix} R_1 & X_2 & X_3 \\ R_2 & Y_2 & Y_3 \\ R_3 & Z_2 & Z_3 \end{bmatrix} \qquad \beta = \frac{D_1}{D}$$

Similarily with $\delta = \frac{D_2}{D}$ and $t = \frac{D_3}{D}$

Ray Tracing Intersections Sphere

$(P - c)(P - c) = R^2$ c = center, R = radius

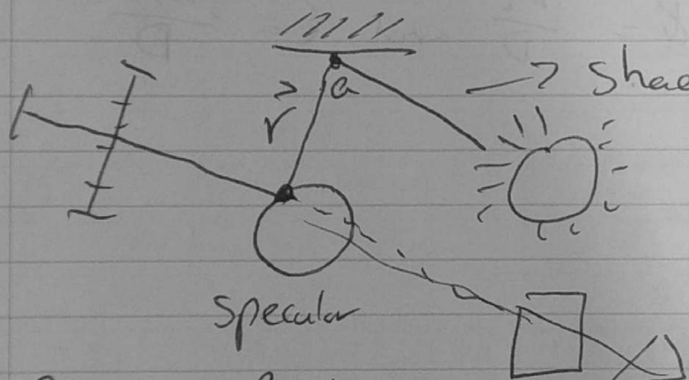
ray: $P = a + t(b-a)$

$((a + t(b-a) - c)((a + t(b-a) - c) = R^2$

$\underbrace{(b-a)(b-a)}_{A} t^2 + \underbrace{(b-a)(a-c)2}_{B} t + \underbrace{(a-c)(a-c) - R^2}_{C} = 0$

$t = \dfrac{-B \pm \sqrt{B^2 - 4AC}}{2A}$

Solving for t:
- zero root → ray misses the sphere
- 1 root → ray is tangent to the sphere
- 2 root → ray hits the sphere but must test for the closest point.



Possible Problems

(digression)

- division by zero
- cancellation of significant digit due to subtraction of nearly equal number

Alternative: rationalize the numerator.

$t = \dfrac{-2C}{B \pm \sqrt{B^2 - 4AC}}$

$t = t_i$??