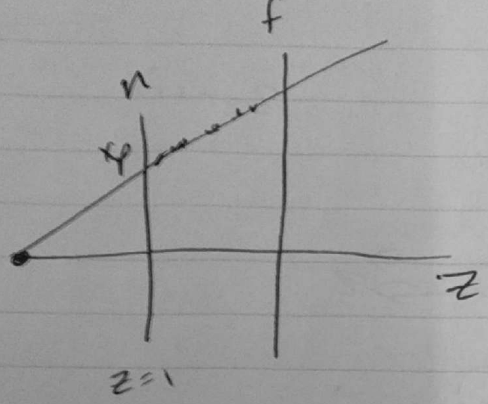
Graphics Mapping?

May 17^{th}, 2018



n = near
f = far

$x - x_p$
$1 - z$

$$x_p = \frac{x}{z} n \qquad n=1$$

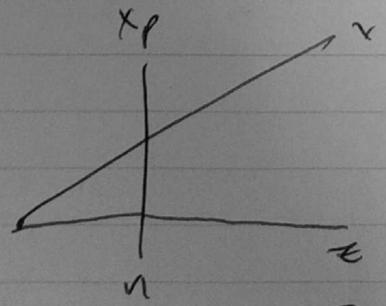
$$x_p = \frac{x}{z} \cdot 1$$

Alternative maxtrix without $z \to z'[-1, 1]$ mapping.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{n+f}{n} & -f \\ 0 & 0 & \frac{1}{n} & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ \frac{(n+f)}{n} z - f \\ \frac{z}{n} \end{bmatrix}$$

After normalization

$$\begin{bmatrix} \frac{nx}{z} \\ \frac{ny}{z} \\ n+f - \frac{fn}{z} \\ 1 \end{bmatrix}$$

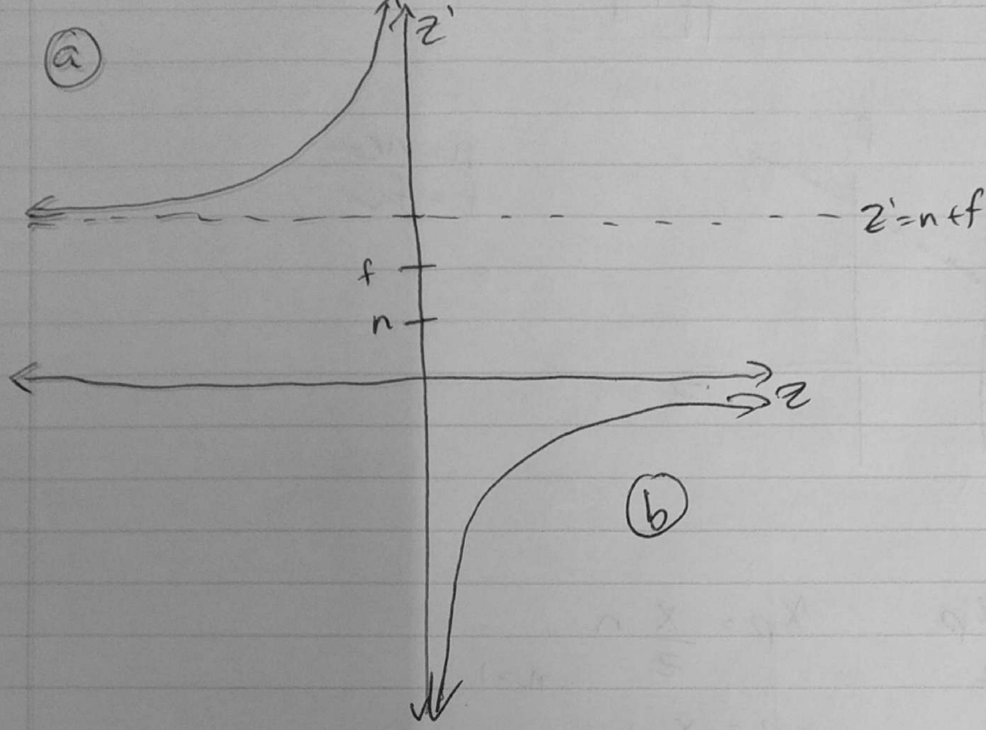


$$\frac{x_p}{n} = \frac{x}{z} \qquad x_p = \frac{xn}{z}$$

$$z' = n+f - \frac{fn}{z}$$

$z \to -\infty \quad z' \to n+f$

$\to z \to \infty \quad z' \to n+f$



$z' = n + f - \frac{fn}{z}$ $n = 1$ $f = 10$ $z' = 11 - \frac{10}{z}$ a)

$z = -0.1$ $z' = 111$

a) stuff behind the head may be mapped in front.

$z = 0.1$ $z' = -89$ b

↳ stuff in front mapped way behind!

Scanned by CamScanner

Graphics Mappings

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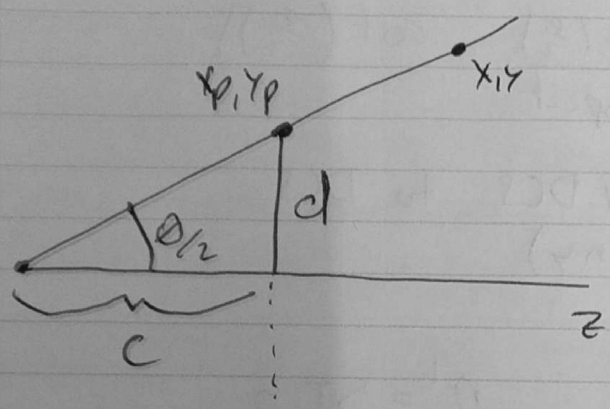
Map to NDC (scaling)



$$y_p = \frac{c \cdot y}{z}$$

scaling y_p

$$y_p = \left(\frac{cy}{\cancel{z}}\right) \cdot \frac{1}{d} = \frac{y}{z} \frac{c}{d}$$

$$\frac{y_p}{c} = \frac{y}{z}$$

angle $\frac{\theta}{2} \rightarrow \frac{c}{d} = \cot\left(\frac{\theta}{2}\right)$

$$y_p = \frac{y}{z} \cot\left(\frac{\theta}{2}\right)$$ → given by the projection

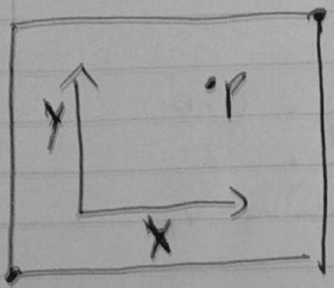
$$\text{aspect} = \frac{w}{h} \ ; \ h = 2d \ ; \ \frac{w}{2} = \text{aspect} \cdot d$$

$$x_p = \frac{x}{z} \frac{c}{(w/2)} = \frac{x}{z} \frac{\cot(w/\theta)}{\frac{w}{2} \cdot \frac{1}{d}}$$

$$x_p = \frac{x}{z} \frac{\cot(\theta/2)}{\text{aspect}}$$ → given by the projection.

$$\begin{bmatrix} \frac{\cot(\theta/2)}{\text{aspect}} & 0 & 0 & 0 \\ 0 & \cot(\theta/2) & 0 & 0 \\ 0 & 0 & \frac{n+f}{n} & -f \\ 0 & 0 & \frac{1}{n} & 0 \end{bmatrix}$$

Final Mapping

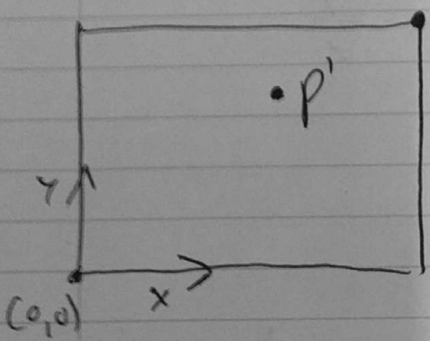


$$\left(\frac{\cot\left(\frac{\theta}{2}\right)}{\text{aspect}},\ \cot\left(\frac{\theta}{2}\right)\right)$$

$$\left(-\frac{\cot\left(\frac{\theta}{2}\right)}{\text{aspect}},\ \cot\left(\frac{\theta}{2}\right)\right)$$

$\downarrow$ maps from NDCS to DCS

(nx, ny)



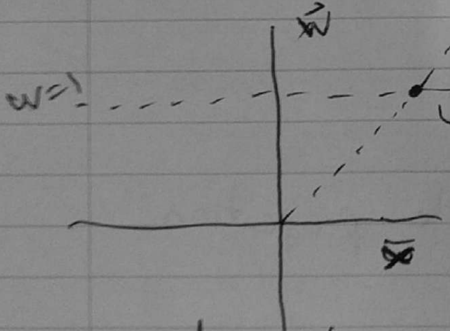
$$p' = ST\,p$$

ie $p' = \text{scale} \times \text{Transform} \times p$

Exercise to find scale + transform

<u>Homogenous clipping</u>

$(\bar{x}, \bar{y}, \bar{z}, \bar{w})$



$\left(\frac{\bar{x}}{\bar{w}}, \frac{\bar{y}}{\bar{w}}, \frac{\bar{z}}{\bar{w}}, 1\right)$

Can do clipping before or after normalization.

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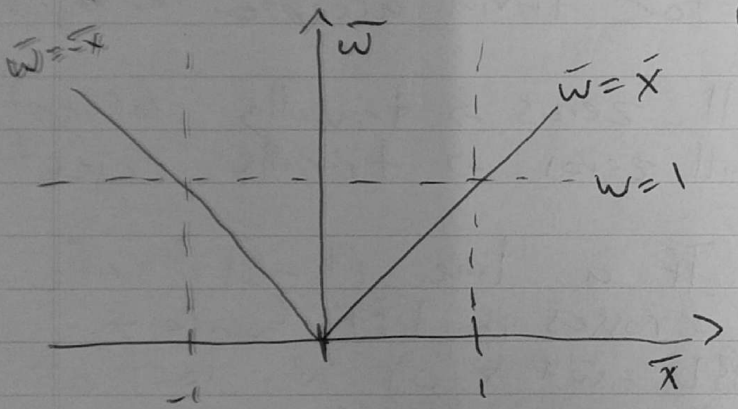
Graphics Homogeneous Clipping

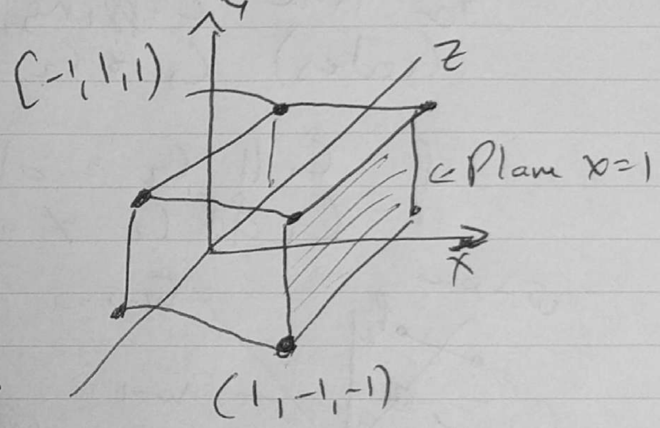
Reference: "Clipping using homogeneous Coordinates" by JF Blinn and M.E Newell

Computer Graphics Vol 2, N13, 1978, pg 281 - 287.

Homogeneous Plane

Assumming a normalized symmetrical canonical cube obtained after the multiplication by the perspective matrix





- Assuming that we are clipping against the boundaries of the geometric cube, a point is inside this normalized viewing volume if the project coordinates satisfy

$$-1 \leq \frac{\bar{x}}{\bar{w}} \leq 1, \quad -1 \leq \frac{\bar{y}}{\bar{w}} \leq 1, \quad -1 \leq \frac{\bar{z}}{\bar{w}} \leq 1$$

There is a boundary coordinate for each clipping boundary. (assuming $w > 0$)

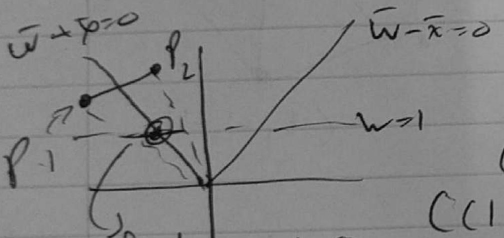
1st bit: $BL = \bar{w} + \bar{x} < 0$, left (of the left plane)
2nd bit: $BR = \bar{w} - x < 0$, right
3rd bit: $BB = \bar{w} + \bar{y} < 0$, bottom
4th bit: $BT = \bar{w} - \bar{y} < 0$, top
5th bit: $BN = \bar{w} + \bar{z} < 0$, near
6th bit: $BF = \bar{w} - \bar{z} < 0$, far

yes = 1

no = 0

For line clipping, we first test the endpoints (codes) C_1 & C_2 for trivial acceptance or rejection.

if $C_1 \,||\, C_2$ = all zeros → trivially accept
if $C_1 \,\&\&\, C_2 \neq$ all zeros → trivially reject.



find point ⓐ

If a line $(1-a)P_1 + aP_2$ crosses the left boundary ($BL = \bar{w} + \bar{x} = 0$) it does at

$$((1-a)w_1 + aw_2) + ((1-a)x_1 + ax_2) = 0$$

$$ⓐ = \frac{w_1 + x_1}{(w_1 + x_1) - (w_2 + x_2)} = \frac{BL_1}{BL_1 - BL_2}$$