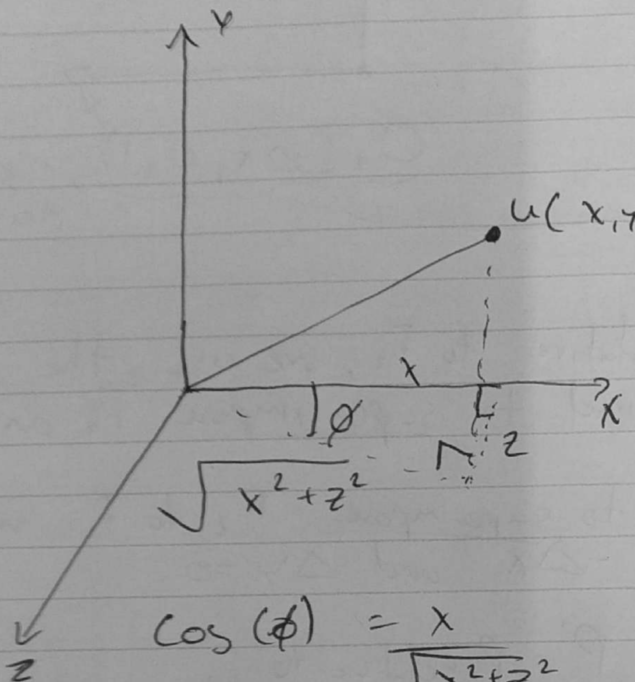
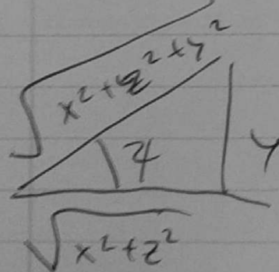
3D Rotation CS 488

How can we represent a rotation (θ), with respect to an arbitrary axis?



$$\cos(\phi) = \frac{x}{\sqrt{x^2+z^2}}$$

$$\sin(\phi) = \frac{z}{\sqrt{x^2+z^2}}$$



$$\cos\psi = \frac{\sqrt{x^2+z^2}}{\sqrt{x^2+z^2+y^2}}$$

$$\sin\psi = \frac{y}{\sqrt{x^2+z^2+y^2}}$$

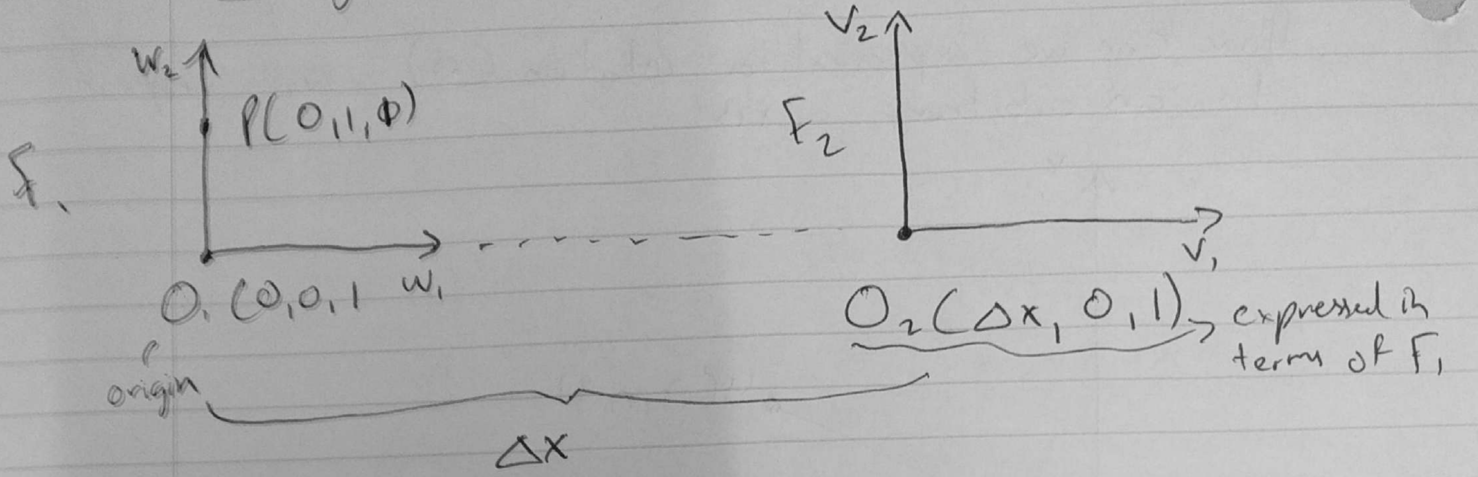
if $|\vec{a}| = 1$ $\quad \sqrt{x^2+z^2+y^2} = 1 \quad \cos\psi = \sqrt{x^2+z^2}$

$\sin\psi = y$

$$R(\theta, \hat{\omega}) = R_y(\phi) R_z(\psi) R_x(\theta) R_z(-\psi) R_y(-\phi)$$

$\longrightarrow$

Change of Base



- To represent P relative to F_2, we use the same transformation required to super impose F_2 on F_1

- In this example to superimpose F_2 to F_1 we translate F_2 by $-\Delta x$ and $\Delta y = 0$

$$T = \begin{bmatrix} 1 & 0 & -\Delta x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$P' = P$ relative to F_2

$$p' = TP = \begin{bmatrix} 1 & 0 & -\Delta x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -\Delta x \\ 1 \\ 1 \end{bmatrix}$$

- Recall the graphics pipeline.

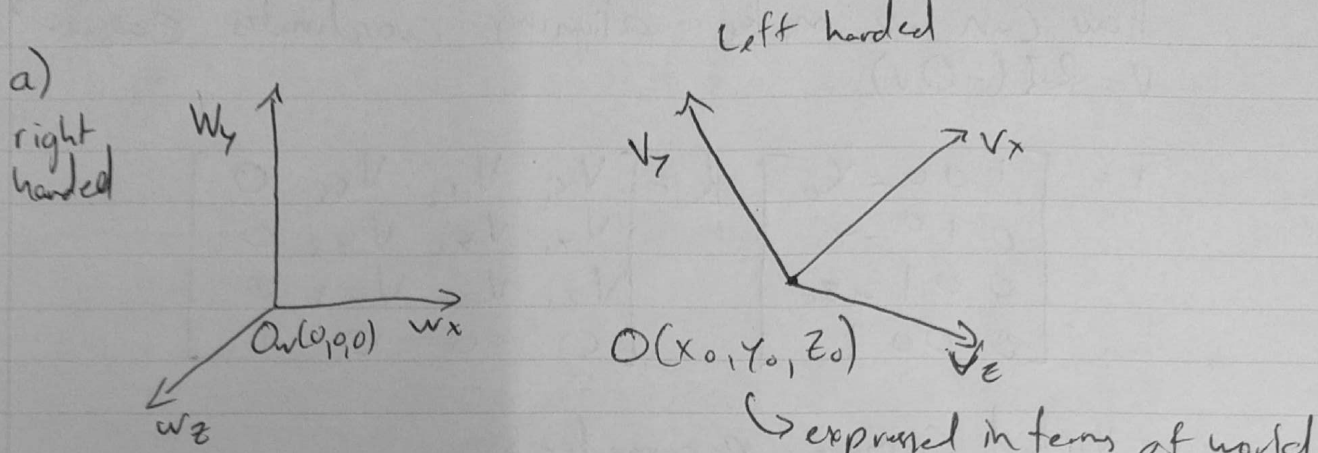
point $p' = P \cdot V \cdot M \cdot p$

Projection, world to view, modelling

→ in order to transform from world to viewing we superimpose the viewing frame (V_x, V_y, V_z, Q) onto the world frame (W_x, W_y, W_z, O_w).

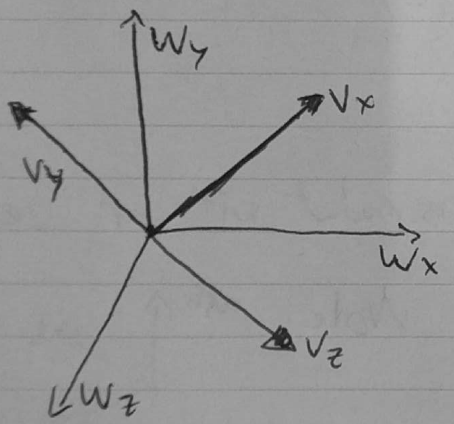
CS488 Transformations

a)

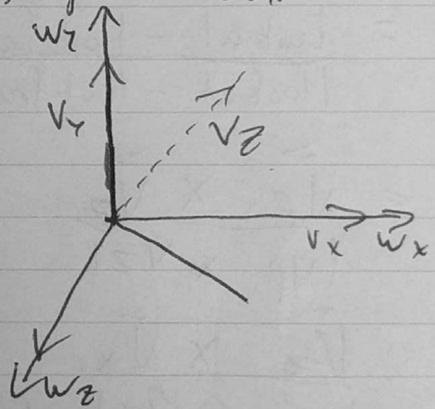


first we make the origins coincidenet

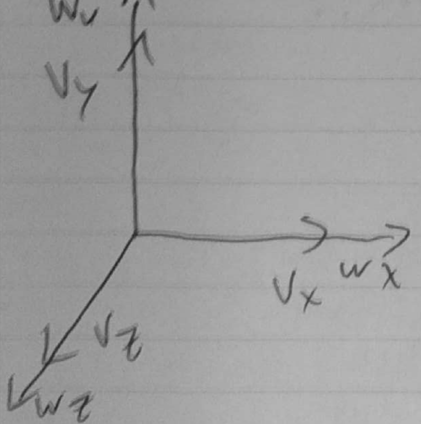
b)



c) Align the axis



d) since we are flipped hands for view vs world we need to flip one of them (reflection)



Translation + Rotation + Reflection

How can we make aligning coordinates easier?

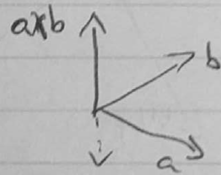
$V = R \cdot T(-O_v)$

$$T = \begin{bmatrix} 1 & 0 & 0 & -x_0 \\ 0 & 1 & 0 & -y_0 \\ 0 & 0 & 1 & -z_0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad R = \begin{bmatrix} V_{x_1} & V_{x_2} & V_{x_3} & 0 \\ V_{y_1} & V_{y_2} & V_{y_3} & 0 \\ V_{z_1} & V_{z_2} & V_{z_3} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- Virtual Camera parameters
  - we know location of the camera (called look-from)
  - we know where the camera is viewing (called look-at)
  - up vector $P = (0,1,0)$

$$\vec{V}_z = \frac{\text{Lookat} - \text{Lookfrom}}{|\text{lookat} - \text{lookfrom}|}$$

$$\vec{V}_x = \frac{\vec{V}_P \times \vec{V}_z}{|V_P \times V_z|}$$ → cross product with up vector

Note: 

$$\vec{V}_y = \frac{\vec{V}_z \times \vec{V}_x}{\cancel{|\vec{V}_z \times \vec{V}_x|}}$$ - do we normalize? probably not

$b \times a = -a \times b$

- asks this on exams often!! ☆ ☆ ☆

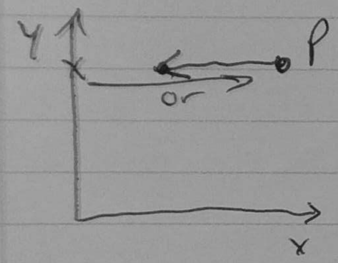
CS488 Transformation

What if Up is parallel to lookat - lookfrom?
= end up with a divide by zero

Defined parameters of view.

near (n) → closest view
far (f) → farthest view
field of view ϕ → angle we can see.
aspect ratio

Remark on Transformations



- we want to translate P to the X
- can translate the point T
- can translate the frame T^{-1}

→ when we apply a transformation to a point, use
$p' = VMT_p$

→ when we apply a transformation to a transformation, we use $p' = T^{-1}VM_p$