Monitoring Example CS486

$S \in \{\text{walking}, \text{sitting}\}$ → states

$O \in \{\text{high}, \text{low}\}$ → observables, sensors on phone.

ignoring S_0 for the example

$$\begin{tikzcd}
S_1 \arrow[r] \arrow[d] & S_2 \arrow[r] \arrow[d] & S_3 \arrow[r] \arrow[d] & S_4 \arrow[d] \\
O_1 & O_2 & O_3 & O_4
\end{tikzcd}$$

$$P_r(S_1) = \begin{cases} 0.9 & S_1 = \text{walking} \\ 0.1 & S_t = \text{sitting} \end{cases}$$

$$P_r(S_{t+1} \mid S_t) = \begin{cases} 0.8 & S_{t+1} = w \quad S_t = w \\ 0.2 & S_{t+1} = S \quad S_t = w \\ 0.1 & S_{t+1} = w \quad S_t = S \\ 0.9 & S_{t+1} = S \quad S_t = S \end{cases}$$

$$P(O_t \mid S_t) = \begin{cases} 0.75 & O_t = \text{high} \quad S_t = w \\ 0.25 & O_t = \text{low} \quad S_t = w \\ 0.30 & O_t = \text{high} \quad S_t = S \\ 0.70 & O_t = \text{low} \quad S_t = S \end{cases}$$

$$P(S_1 \mid O_1 \overset{=\text{high}}{}) \propto P(O_1 \mid S_1) P(S_1)$$

$$= \begin{cases} 0.675 \\ 0.030 \end{cases} \propto \begin{cases} 0.9574 & S_1 = w \\ 0.0426 & S_2 = S \end{cases}$$

$$F_1(S_1) = P(O_1 = \text{high} \mid S_1) = \begin{cases} 0.75 & S_1 = w \\ 0.30 & S_1 = S \end{cases}$$

$$P(S_1 \mid O_1 = \text{high}) \propto F_1(S_1) P(S_1)$$

$$P(S_2 \mid O_1 = high, O_2 = high) \propto P(O_2 \mid S_2) \sum_{S_1} P(S_2 \mid S_1) P(S_1 \mid O_1)$$

$$= \begin{cases} 0.5777 \\ 0.0689 \end{cases} \propto \begin{cases} 0.8934 & S_2 = w \\ 0.1066 & S_2 = s \end{cases}$$

$$P(S_3 \mid O_1 = high, O_2 = high, O_3 = low)$$

$$\propto P(O_3 \mid S_3) \sum_{S_2} P(S_3 \mid S_2) P(S_2 \mid O_1 O_2)$$

$$= \begin{cases} 0.1813 \\ 0.1922 \end{cases} \propto \begin{cases} 0.4854 & S_3 = w \\ 0.5146 & S_3 = S \end{cases}$$

Sequence:

$O_1 = high$	$S_1 = ?$	Finding the most likely explanation.
$O_2 = high$	$S_2 = ?$	
$O_3 = low$	$S_3 = ?$	
$O_4 = low$	$S_4 = ?$	

$$F(S_1, S_2) = P(S_2 \mid S_1) P(O_1 \mid S_1) P(S_1) = \begin{cases} 0.540 & S_1 = w & S_2 = w \\ 0.003 & S_1 = S & S_2 = w \\ 0.135 & S_1 = w & S_2 = S \\ 0.027 & S_1 = S & S_2 = S \end{cases}$$

$$\max_{S_1} P(S_1 S_2 \mid O_1 = high) = \max_{S_1} F(S_1, S_2) = f_2(S_2) = \begin{cases} 0.54 & S_2 = w \\ 0.135 & S_2 = S \end{cases}$$

$$f_3(S_2, S_3) = P(S_3 \mid S_2) P(O_2 = high \mid S_2) f_2(S_2)$$

$$= \begin{cases} 0.3240 & S_2 = w & S_3 = w \\ 0.0041 & S_2 = S & S_3 = w \\ 0.0810 & S_2 = w & S_3 = S \\ 0.0365 & S_2 = S & S_3 = S \end{cases}$$

$$f_4(S_3) = \max_{S_2} f_3(S_2, S_3) = \begin{cases} 0.324 & w \\ 0.081 & S \end{cases} \quad S_3$$

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Monitoring Example Contined

Pg 2

$F_5(S_3, S_4) = P(S_4|S_3)P(O_3 = low|S_3)f_4(S_3) = \begin{cases} 0.0648 & w & w \\ 0.0057 & s & w \\ 0.0162 & w & s \\ 0.0510 & s & s \end{cases}$

S3 S4

S_3 = sitting

$F_6(S_4) = \max_{S_3} f_5(S_3, S_4) = \begin{cases} 0.0648 & S_4 = w \\ 0.051 & S_4 = s \end{cases}$

same state

$F_7(S_4) = P(O_4 = low|S_4)f_6(S_4) = \begin{cases} 0.0162 & S_4 = w \\ 0.0357 & S_4 = s \end{cases}$ max

$\max_{S_1S_2S_3S_4} P(S_1S_2S_3S_4|O_1O_2O_3O_4) = \max_{S_4} f_7(S_4) = \underline{0.0357}$

S_1 = Walking
S_2 = Walking
S_3 = sitting
S_4 = Sitting

Working backwards, what was the state that gave the maximum value

Midterm - Lecture 12 → check website

State example

$$V_{n+1}(RU) = \max_a R(RU) + \gamma \sum_{s'} P(s' \mid RU, a) V_n(s')$$

$$= \max \begin{cases} R(RU) + \gamma \sum_{s'} P(s' \mid RU, \text{save}) V_n(s') & s = \text{save} \\ R(RU) + \gamma \sum_{s'} P(s' \mid RU, \text{advertise}) V_n(s') & \text{ad} \end{cases}$$

$$= \max \begin{cases} 10 + 0.9[0.5(10) + 0.5(0)] = 14.5 & \text{save!!} \\ 10 + 0.9[0.5(0) + 0.5(0)] = 10 & \text{ad :(} \end{cases}$$

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