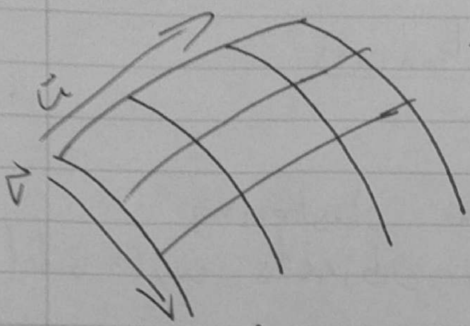
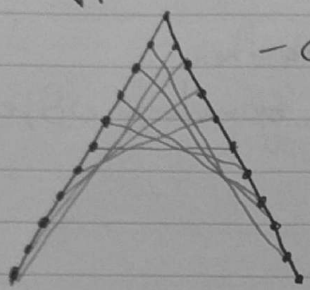
Splines

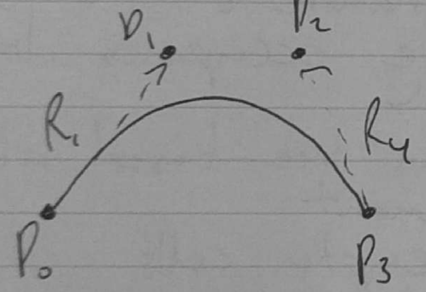


- patches built from splines to create objects.



- creates a nice curve.
  - Bernstien Polynomials
    * possible exam question.

Hermite



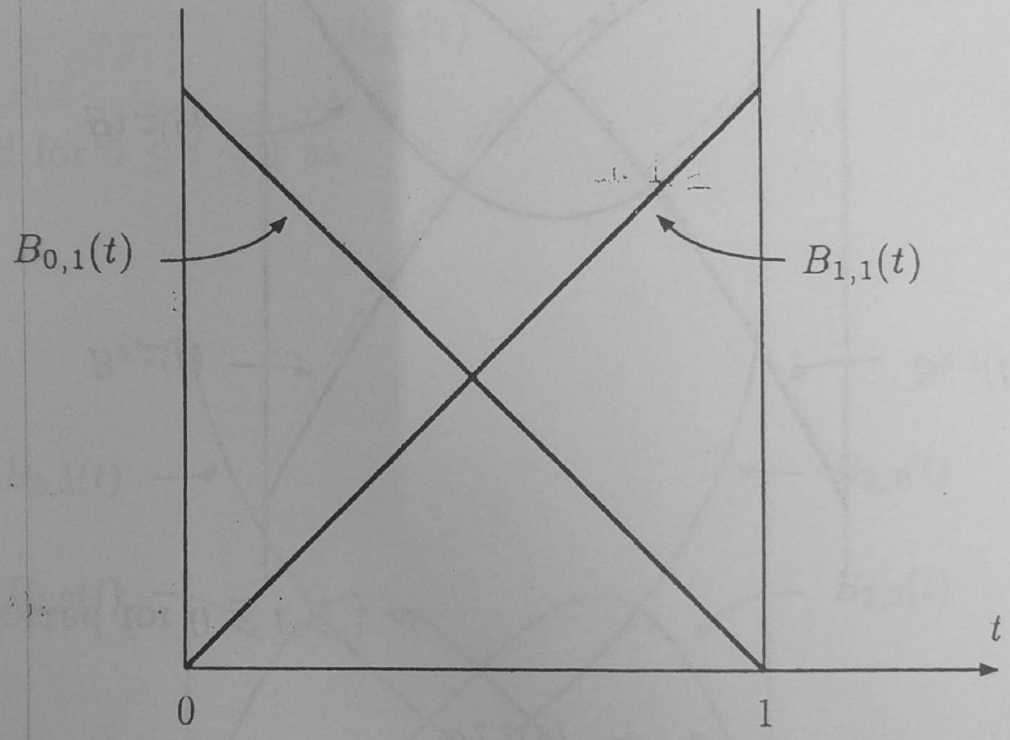
$$Q(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 & 1 \\ -3 & 3 & -2 & -1 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_4 \\ R_1 \\ R_4 \end{bmatrix}$$

- change the vectors R_1 & R_4 to change the shape instead of using the points. (ie ~~Bernstien~~ Bezier)

- The Bernstein polynomials of degree 1 are

$$\begin{aligned} B_{0,1}(t) &= 1-t \\ B_{1,1}(t) &= t \end{aligned}$$

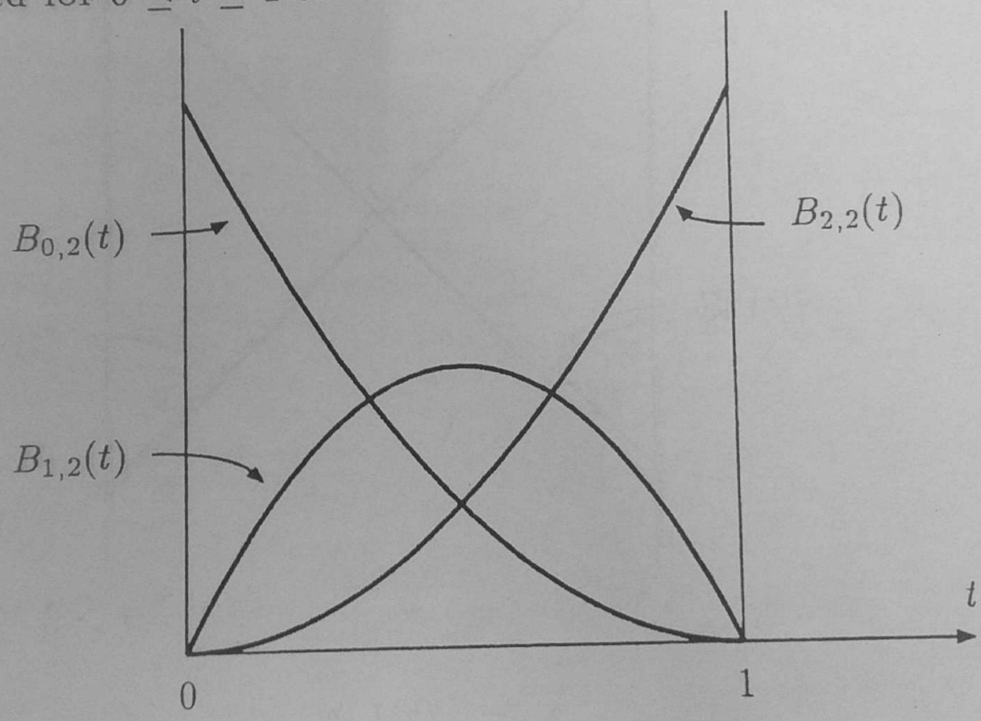
and can be plotted for $0 \leq t \leq 1$ as



- The Bernstein polynomials of degree 2 are

$$
\begin{aligned}
B_{0,2}(t) &= (1-t)^2 \\
B_{1,2}(t) &= 2t(1-t) \\
B_{2,2}(t) &= t^2
\end{aligned}
$$

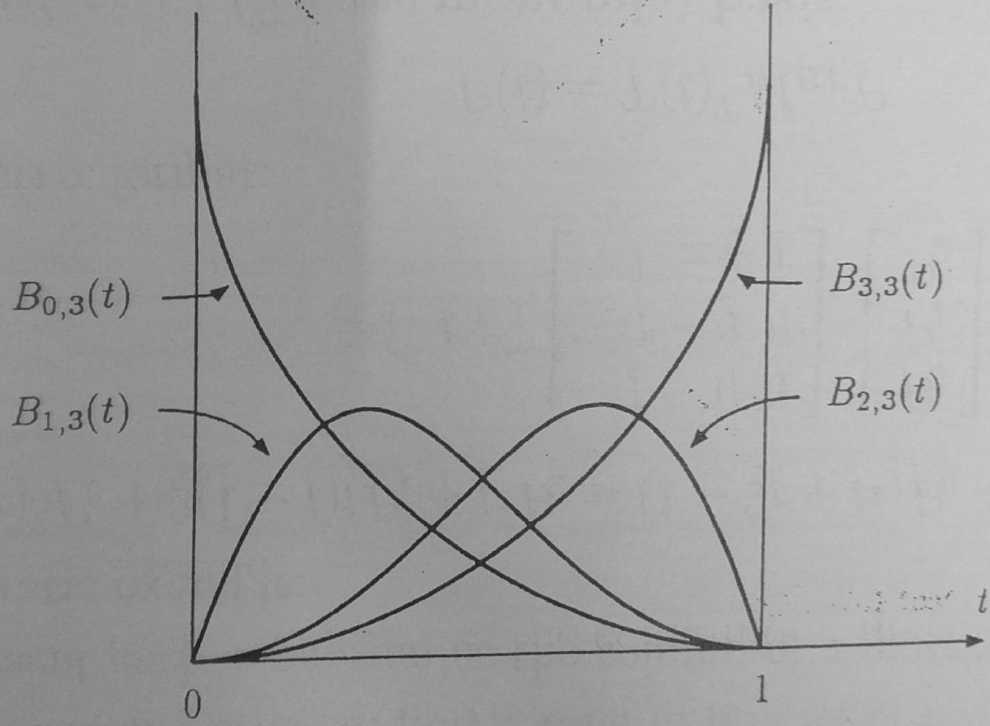
and can be plotted for $0 \leq t \leq 1$ as



- The Bernstein polynomials of degree 3 are

$$\begin{aligned}
B_{0,3}(t) &= (1-t)^3 \\
B_{1,3}(t) &= 3t(1-t)^2 \\
B_{2,3}(t) &= 3t^2(1-t) \\
B_{3,3}(t) &= t^3
\end{aligned}$$

and can be plotted for $0 \leq t \leq 1$ as



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Matrix view:

- Expand each Bernstein polynomial in powers of t
- Represent each expansion as the column of a matrix
- Quadratic example:

$$(1-t)^2P_0 + 2(1-t)tP_1 + t^2P_2 = (1-2t+t^2)P_0 + (2t-2t^2)P_1 + t^2P_2$$

$$= [1\ t\ t^2] \begin{bmatrix} 1 & 0 & 0 \\ -2 & 2 & 0 \\ 1 & -2 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

- In matrix format:

$$P(t) = T(t)^T M_{BT} P$$

  - $T(t)^T = [1\ t\ t^2]$ is the **monomial basis**
  - $P_T = M_{BT}P$ is a matrix containing the coefficients of the polynomials for each dimension of $p(t)$
  - M_{BT} is a **change of basis matrix** that converts a specification P of $P(t)$ relative to the Bernstein basis to one relative to the monomial basis