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Find the 1st two Lagrange basis functions for the four points:
$(-2,1)$, $(1,3)$, $(3,2)$, $(4,-1)$.

$$L_1(x) = \frac{(x-x_2)(x-x_3)(x-x_4)}{(x_1-x_2)(x_1-x_3)(x_1-x_4)} = \frac{(x-1)(x-3)(x-4)}{(-2-1)(-2-3)(-2-4)}$$
$$= \frac{(x-1)(x-3)(x-4)}{(-3)(-5)(-6)} = \frac{(x-1)(x-3)(x-4)}{-90}$$

$$L_2(x) = \frac{(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_1)(x_2-x_3)(x_2-x_4)} = \frac{(x--2)(x-3)(x-4)}{(1--2)(1-3)(1-4)}$$
$$= \frac{(x+2)(x-3)(x-4)}{18}$$

$$p(x) = y_1 L_1(x) + y_2 L_2(x) + y_3 L_3(x) + y_4 L(x)$$
$$= L_1(x) + 3L_2(x) + 2L_3(x) - L_4(x)$$

is the resulting Lagrange polynomial

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Hermite Interpolation

Fit a cubic function $p(x) = a + bx + cx^2 + dx^3$
given "Hermite data" at 2 points:

$$\left.\begin{aligned} p(x_1) &= y_1 \\ p(x_2) &= y_2 \end{aligned}\right\} \text{values}$$
$$\left.\begin{aligned} p'(x_1) &= s_1 \\ p'(x_2) &= s_2 \end{aligned}\right\} \text{slopes}$$

Find $p'(x)$ by differentiating:
$$p'(x) = b + 2cx + 3dx^2$$

Plug in values & slopes to get 4 eq'ns for the 4 unknown coefficients. Solve the system, that's it.

Example of Hermite interpolation.

$p(0) = 0 \quad p'(0) = 1$
$p(1) = 3 \quad p'(1) = 0$

What are the 4 equations?

$$p(0) = 0 = a \Rightarrow a = 0$$
$$p'(0) = 1 = b + 2c(0) + 3d(0)^2 \Rightarrow b = 1$$
$$p(1) = 3 = a + b + c + d$$
$$p'(1) = 0 = b + 2c + 3d$$

Solving gives $p(x) = x + 7x^2 - 5x^3$