Given an equation $Ax=b$, perturb b by Δb.

This gives

$$A(x+\Delta x) = b + \Delta b.$$

Subtracting $Ax=b$ gives

$$A\Delta x = \Delta b, \text{ or } \Delta x = A^{-1}\Delta b.$$

We want to know the relative change in x, $\frac{\|\Delta x\|}{\|x\|}$, given the relative change in b, $\frac{\|\Delta b\|}{\|b\|}$.

Applying norm rules on $Ax=b$...

$$\|b\| = \|Ax\| \leq \|A\|\|x\| \text{ or } \frac{\|A\|}{\|b\|} \geq \frac{1}{\|x\|} \quad (1)$$

and on $\Delta x = A^{-1}\Delta b$...

$$\|\Delta x\| \leq \|A^{-1}\|\|\Delta b\| \quad (2).$$

Combining (1) and (2) we find

$$\|\Delta x\| \cdot \frac{1}{\|x\|} \leq \left(\|A^{-1}\|\|\Delta b\|\right)\left(\frac{\|A\|}{\|b\|}\right)$$

or

$$\frac{\|\Delta x\|}{\|x\|} \leq \underbrace{\left(\|A^{-1}\|\|A\|\right)}_{K} \frac{\|\Delta b\|}{\|b\|}$$

The "condition number" K bounds the relative change in x due to relative change in b.

$$K = \|A\|\|A^{-1}\|$$

Now consider perturbing A.

$$(A+\Delta A)(x+\Delta x)=b$$

$$Ax+A\Delta x+\Delta A x+\Delta A\Delta x=b.$$

Subtract $Ax=b$ and rearrange:

$$A\Delta x=-\Delta A(x+\Delta x)$$

or

$$\Delta x=-A^{-1}\Delta A(x+\Delta x).$$

Taking norms gives

$$\|\Delta x\|\leq\|A^{-1}\|\|\Delta A\|\|x+\Delta x\|.$$

Multiply by $1=\frac{\|A\|}{\|A\|}$ and take x terms to LHS gives

$$\frac{\|\Delta x\|}{\|x+\Delta x\|}\leq\underbrace{\left(\|A\|\|A^{-1}\|\right)}_{K(A)}\frac{\|\Delta A\|}{\|A\|}.$$

Again, condition number dictates a bound on change in x.