Amath 250 Lecture 1

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Intro to Differential Equations (DEs)

A DE is an equation relating a function to its own derivative(s).
Solving it means identifying the function(s).
Eg. Solve:

$$\frac{dy}{dx} = y$$

We can guess the solution:

$$y = e^y$$

$$y = Ke^x | \exists K \epsilon R$$

(This gives the complete set)

Eg2:

$$y' = xy$$

Solution:

$$y = Ce^{\frac{1}{2}x^2}$$

Terminology:
We call the family of solutions the general solution. A single solution may be called a particular solution

To determine the value of C required to find a particular solution we need an initial condition. The value of f(x) at some value of x. A DE with an IC is called an initial value problem (IVP)

Eg1: Solve

$$y' = y^2, y(0) = 1$$

Possible Solution:

$$y = \frac{-1}{x}$$

The General solution is:

$$y = \frac{-1}{x+c}$$

If we set $y(0) = 1$ then $1 = \frac{-1}{c}$ so $c = -1$. The particular solution is:

$$y = \frac{1}{1-x}$$

The <u>order</u> of a DE is the order of the highest derivative.
Eg1 $(y''')^2 = y - 1$ is 3rd-order
Eg2 $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = x$ is a 2nd order equation
We will soon be able to solve this, the solution is:

$$y = C_1 e^{-2x} + C_2 x e^{-2x} + \frac{1}{4}(x-1)$$

We can <u>verify</u> this:

$$y' = -2C_1 e^{-2x} + C_2 e^{-2x} - 2C_2 x e^{-2x} + \frac{1}{4}$$

$$y'' = 4C_1 e^{-2x} - 2C_2 e^{-2x} - 2C_2 e^{-2x} + 4C_2 x e^{-2x}$$

$$y'' + 4y' + 4y = 4C_1 e^{-2x} - 4C_2 e^{-2x} + 4C_2 x e^{-2x} +$$

$$(-8C_1 e^{-2x} + 4C_2 e^{-2x} - 8C_2 x e^{-2x} + 1) + (4C_1 e^{-2x} + 4C_2 x e^{-2x} + x - 1) = x$$

Cancel out the terms and that is how we just get x