Stability Analysis

May 6, 2016 9:26 AM

Consider the recursive expression

$$I_0 = \log\left(\frac{1+\alpha}{\alpha}\right) \quad \text{and} \quad I_n = \frac{1}{n} - \alpha I_{n-1}$$

Assume some initial error ϵ_0 in I_0

ie.

$$\epsilon_0 = \underbrace{(I_0)_A}_{\text{numerical sol'n}} - \underbrace{(I_0)_E}_{\text{true sol'n}}$$

What is ϵ_n after n steps?

Exact and approx. sol'ns both follow the recurrence. So...

$$\begin{aligned}
\epsilon_n &= (I_n)_A - (I_n)_E \\
&= \left[\cancel{\frac{1}{n}} - \alpha (I_{n-1})_A\right] - \left[\cancel{\frac{1}{n}} - \alpha (I_{n-1})_E\right] \\
&= -\alpha \left((I_{n-1})_A - (I_{n-1})_E\right) \\
&= -\alpha\, \epsilon_{n-1}
\end{aligned}$$

This is a simple recurrence, with sol'n

$$\epsilon_n = (-\alpha)^n \epsilon_0$$

Initial error ϵ_0 is scaled by $(-\alpha)^n$.

Is this stable or unstable?

2 cases

1) $|-\alpha| < 1$: Error is scaled down. Stable!

2) $|-\alpha| > 1$: Error is magnified. <u>Unstable</u>!

FP Example

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What is 14.375 in $F = \{\underset{\beta}{2}, \underset{t}{6}, \underset{L}{-5}, \underset{U}{5}\}$ with rounding?

In binary:

$$\begin{array}{cccccccc} 8+ & 4+ & 2+ & 0+ & 0+ & \frac{1}{4}+ & \frac{1}{8} \\ 1 & 1 & 1 & 0. & 0 & 1 & 1 \end{array}$$

Rounded (up) to 6 digits

1110,10

Normalize to get p:

$$0.111010 \times 2^4$$

In decimal this is 14.5

(We will always round ties (i.e. ½) up, for simplicity.)

With truncation, we would get

$$0.111001 \times 2^4 = 14.25 \text{ instead.}$$