Exploiting Sparsity in M

We have $M = \alpha\left(P + \frac{1}{R} e d^T\right) + (1-\alpha)\frac{1}{R} e e^T$

Sparse. Not all web pages ~~are~~ are linked

dense in "dead-end" columns

fully dense!

Consider computing $M p^n = \underbrace{\alpha P p^n}_{①} + \underbrace{\frac{\alpha}{R} e d^T p^n}_{②} + \underbrace{\frac{(1-\alpha)}{R} e e^T p^n}_{③}$

Output p^{n+1} is a vector, and a sum of 3 vectors.

① is a sparse matrix-vector multiply. It can be done efficiently.

③ involves $e e^T p^n = e\underbrace{(e^T p^n)}_{\text{scalar!}}$ which requires the "dot-product" $e^T p^n$. This is just 1, since $\sum p_i^n = 1$ (it's a probability vector).
So we can simply add $\left(\frac{1-\alpha}{R}\right) e$.

② is similar: compute $\frac{\alpha}{R}\underbrace{(d^T p^n)}_{\text{scalar}} e$.

So, $p^{n+1} = M p^n = ① + ② + ③$, with no dense matrix-vector multiplication. No need to form M explicitly!