Amath 250 Lecture 3

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The Existence/Uniqueness theorem (for 1st order IVPs)

The IVP:

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

Has a unique solutino provided that both f(x,y) and $f_y(x, y)$ are continuous within a neighbourhood of (x_0, y_0)

Corollary: If f and f_y are cts in a region of the xy-plane, then the various solution curves of $y' = f(x, y)$ will not intersect in that region.

Sketching Families of Solutions

If you have problems that are too difficult - you can look at the derivitive. Using the DE we already have info about the derivitive and we can use that info directly.

We can often tell a lot about he graphs of the solutions from the DE itself.

Example 1: Consider the equation:
$\frac{dy}{dx} = y^2 - 4$

If we set the $\frac{dy}{dx} = 0$ we get $y = +/-2$
This means that:

- the constant functions $y = +/-2$ are solutions (They are called "equilibrium solutions)
- none of the other solutions have any critical points
- $\frac{dy}{dx} > 0$ iff $|y| > 2$ and $\frac{dy}{dx} < 0$ iff $|y| < 2$
- in fact, $\frac{dy}{dx} \neq 0$ when $y \approx +/-2$, and $|\frac{dy}{dx}|$ increases with distance from $y = +/-2$