

Line equation

$$y = ax + b \quad \text{Points: } (1, 2), (-1, 4)$$

Plug in to find coefficients a and b :

$$\begin{aligned} 1) \quad 2 &= a + b \rightarrow a = 2 - b \\ 2) \quad 4 &= -a + b \rightarrow a = b - 4 \end{aligned}$$

$$\begin{aligned} 2 - b &= b - 4 & a &= 2 - (3) = -1 \\ 6 &= 2b \\ b &= 3 \end{aligned}$$

∴ Line is $y = -x + 3$.

Fitting a cubic!

Data: $(0,1), (1,2), (2,0), (3,3)$

Find system of eq's to fit a cubic!

$$y = C_1 + C_2x + C_3x^2 + C_4x^3$$

$$1) \quad y_1 = C_1 + C_2x_1 + C_3x_1^2 + C_4x_1^3 \rightarrow 1 = C_1 + C_2(0) + C_3(0)^2 + C_4(0)^3 \\ \rightarrow 1 = C_1$$

$$2) \quad 2 = C_1 + C_2 + C_3 + C_4$$

$$3) \quad 0 = C_1 + 2C_2 + 4C_3 + 8C_4$$

$$4) \quad 3 = C_1 + 3C_2 + 9C_3 + 27C_4$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 2 & 4 & 8 \\ 1 & 3 & 9 & 27 \end{bmatrix} \vec{C} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 3 \end{bmatrix}$$

The linear system
for the coefficients.

Solution is:

$$C_1 = 1$$

$$C_2 = 31/6$$

$$C_3 = -11/2$$

$$C_4 = 4/3$$

Lagrange polynomial for 2 pts

Data: $(1, 2), (-1, 4)$

$$L_1(x) = \frac{(x - x_2)}{(x_1 - x_2)} = \frac{x - -1}{1 - -1} = \frac{x + 1}{2}$$

$$L_2(x) = \frac{x - x_1}{x_2 - x_1} = \frac{x - 1}{-1 - 1} = \frac{1 - x}{2}$$

The polynomial is

$$\begin{aligned} p(x) &= y_1 L_1(x) + y_2 L_2(x) \\ &= 2 \left(\frac{x+1}{2} \right) + 4 \left(\frac{1-x}{2} \right) \\ &= x+1 - 2x+2 \\ &= -x+3 \end{aligned}$$

Same line, found differently.