Introduction to Dimensional Analysis

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In applications we'll want to keep track of the units of measurement. For cimplicity we'll speak instead of the "dimensions" being measured.
New notation: The dimensions of mass are mass - square brackets define the dimensions of something
$[mass] = M$, $[length] = L$, $[time] = T$ etc. We will add more later

We start with two axioms:

1. (D1) Physical quantities may only be added ,subtracted, or equated if they have the same dimensions.

2. (D2) Quantities of different dimensions can only be combined by multiplication and division, in which case we have $[AB] = [A][B]$ and $[\frac{A}{B}] = \frac{[A]}{[B]}$

We can define dimensions for any quatntites which we believe shoudl obey these rules
In Physichs applications, we have 5 dimensions:
M, L, T and $[temperature] = U$ and $[charge] = Q$
(We can also define our own dimensions $[money]$, or $[applies]$ and $[oranges]$ etc.)

We can use D2 to calculate dimensions of secondary quantities:
eg) $[speed] = \frac{[length]}{[time]} = LT^{-1}$
from: $v = \frac{ds}{dt} = \lim_{\Delta t} \frac{\Delta s}{\Delta t}$

$[acceleration] = [\frac{dv}{dt}] = \frac{[speed]}{[time]} = LT^{-2}$
$[force] = [mass][acceleration] = MLT^{-2}$
$[work] = [force][distance] = ML^2T^{-2}$
$[voltage] = \frac{[work]}{[charge]} = ML^2T^{-2}Q^{-1}$

Comment: Angles are.... <u>dimensionless</u>! (in radians, $\Theta = \frac{s}{r}$) with radius r and length of the side is s. so $[\Theta] = \frac{[s]}{[r]} = \frac{L}{L} = 1$