

Google Matrix example

$$P = \begin{bmatrix} 0 & 0 & 1/4 & 0 & 1/4 \\ 0 & 0 & 1/4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/4 \\ 0 & 1/2 & 1/4 & 0 & 1/4 \\ 0 & 1/2 & 0 & 0 & 1/4 \\ 0 & 0 & 1/4 & 0 & 0 \end{bmatrix}$$

$$d = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$e = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$R=6 \quad \alpha = \frac{1}{2}$$

$$M = \alpha(P + \frac{1}{R}ed^T) + (1-\alpha)\frac{1}{R}ee^T$$

$$\alpha(P + \frac{1}{R}ed^T) = \frac{1}{2} \begin{bmatrix} 1/6 & 0 & 1/4 & 1/6 & 0 & 1/4 \\ 1/6 & 0 & 1/4 & 1/6 & 0 & 0 \\ 1/6 & 0 & 0 & 1/6 & 0 & 1/4 \\ 1/6 & 1/2 & 1/4 & 1/6 & 1/2 & 1/4 \\ 1/6 & 1/2 & 0 & 1/6 & 0 & 1/4 \\ 1/6 & 0 & 1/4 & 1/6 & 1/2 & 0 \end{bmatrix}$$

$$\frac{(1-\alpha)}{R}ee^T = \left(\frac{1}{2}\right)\left(\frac{1}{6}\right) \begin{bmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \dots & \dots & 1 \end{bmatrix} \quad \text{i.e. } \frac{1}{12} \text{ everywhere}$$

Result:

$$M = \begin{bmatrix} 1/6 & 1/12 & 5/24 & 1/6 & 1/12 & 5/24 \\ 1/6 & 1/12 & 5/24 & 1/6 & 1/12 & 1/12 \\ 1/6 & 1/12 & 1/12 & 1/6 & 1/12 & 5/24 \\ 1/6 & 1/3 & 5/24 & 1/6 & 1/3 & 5/24 \\ 1/6 & 1/3 & 1/12 & 1/6 & 1/12 & 5/24 \\ 1/6 & 1/12 & 5/24 & 1/6 & 1/3 & 1/12 \end{bmatrix}$$

Observe that columns each sum to 1.