Machine Epsilon

May 4, 2016 3:04 PM

What is E for $F = \{\beta = 10, t = 3, L = -5, U = 5\}$ with rounding to nearest?

$$E = \frac{1}{2}\beta^{1-t} = \frac{1}{2}(10)^{1-3} = 0.5 \times 10^{-2} = 5 \times 10^{-3}.$$

Why? What is smallest # in F above 1?

$1 = 0.100 \times 10^1$

Next # is 0.101×10^1.

Would need to add 0.0005×10^1 for rounding up to occur. i.e. 5×10^{-3}.

Under truncation, $E = \beta^{1-t} = 10^{-2}$.

Error Analysis

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$$\text{Rel. err} = \frac{|(a \oplus b) \oplus c - (a+b+c)|}{|a+b+c|}$$

$$= \frac{|(a+b)(1+\delta_1) \oplus c - (a+b+c)|}{|a+b+c|}$$

$$= \frac{|((a+b)(1+\delta_1)+c)(1+\delta_2) - (a+b+c)|}{|a+b+c|}$$

by Δ ineq. $|x+y| \le |x|+|y|$

$$= \frac{|\cancel{a+b+c} + a\delta_1 + b\delta_1 + (a+b+c+a\delta_1+b\delta_1)\delta_2 - \cancel{(a+b+c)}|}{|a+b+c|}$$

$$= \frac{|a\delta_1 + b\delta_1 + a\delta_1\delta_2 + b\delta_1\delta_2 + (a+b+c)\delta_2|}{|a+b+c|}$$

$$\le \frac{|(a+b)\delta_1| + |(a+b)\delta_1\delta_2| + |(a+b+c)\delta_2|}{|a+b+c|}$$

use $|xy| = |x||y|$

$$\le \frac{|a+b||\delta_1| + |(a+b)||\delta_1||\delta_2| + |a+b+c||\delta_2|}{|a+b+c|}$$

use $|\delta| \le E$

$$\le \frac{|a+b|E + |a+b|E^2 + |a+b+c|E}{|a+b+c|}$$

$$\le \frac{|a+b|}{|a+b+c|}(E+E^2) + E$$