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Fit a Hermite interpolant to...

$x_1 = 0 \quad y_1 = 0 \quad s_1 = 1$

$x_2 = 1 \quad y_2 = 3 \quad s_2 = 0$

$$\Delta x_1 = x_2 - x_1 = 1 - 0 = 1$$

$$y_1' = \frac{y_2 - y_1}{\Delta x_1} = \frac{3-0}{1} = 3$$

$$a_1 = y_1 = 0$$
$$b_1 = s_1 = 1$$

$$c_1 = \frac{3y_1' - 2s_1 - s_2}{\Delta x_1} = \frac{3(3) - 2(1) - 0}{1} = 7$$

$$d_1 = \frac{s_2 + s_1 - 2y_1'}{\Delta x_1^2} = \frac{0 + 1 - 2(3)}{1^2} = -5$$

$$p_1(x) = a_1 + b_1(x - x_1) + c_1(x - x_1)^2 + d_1(x - x_1)^3$$
$$= 0 + x + 7x^2 - 5x^3$$

Same solution as before!

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Spline Problem Example

Give the conditions/equations that should be satisfied for $S(x)$ to be a valid cubic spline, where

$$S(x) = \begin{cases} 5/3 + \frac{16}{3}x + ax^2 + x^3 & \text{on } [-1, 1] \\ -7/3 + bx + \frac{22}{3}x^2 + \frac{2}{3}x^3 & \text{on } [1, 2] \end{cases}$$

Value: $S_1(1) = S_2(1)$

$$\frac{5}{3} + \frac{16}{3} + a + 1 = -\frac{7}{3} + b + \frac{22}{3} + \frac{2}{3} \quad (1)$$

1st deriv: $S_1'(x) = \frac{16}{3} + 2ax + 3x^2$

$$S_2'(x) = b + \frac{44}{3}x + 2x^2$$

Evaluate at $x=1$, and equate:

$$\frac{16}{3} + 2a + 3 = b + \frac{44}{3} + 2 \quad (2)$$

2nd deriv:

$$S_1''(x) = 2a + 6x$$

$$S_2''(x) = 44/3 + 4x$$

$$\therefore 2a + 6 = \frac{44}{3} + 4 \quad (3)$$

If we try to solve, there is no solution satisfying all 3 conditions. This cannot be a cubic spline for any choice of a and b.