# Inverse problems

* <http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Xe_damage/INVERSE-HEAT-TRANSFER-OZISIK.pdf> - the main source of information below
* <https://en.wikipedia.org/wiki/Inverse_problem> - also a very nice source of info, mainly about the various applications
* In the majority of science fields, the classical (direct) problems mean the cause of some action (boundary condition) is given, and our goal is to determine the effect that this action will have on the object.
* Inverse problems are being solved when we know the final results of an action, but we are searching for the (boundary) conditions that led to this result. In the context of heat transfer it could be translated into knowing the temperature distribution in the object after heating it up but being interested in the heat flux applied on the object. Temperature distribution inside the object is usually determined by measuring the temperature below the object’s surface with an appropriate method.
* *“In the direct problem the causes are given, the effect is determined; whereas in the inverse problem the effect is given, the cause (or causes) is estimated.”*
* The difference between classical and inverse problems is also in the precision and correctness of the result. It can be said that the result of the classical problem can be unambiguously determined. The solution of the inverse problem, on the other hand, can be only estimated, and there will always be some uncertainty.
  + From a mathematical perspective they differ in the aspect called posedness. Classical problems are described as “well-posed”, in contrast to inverse problems, which are “ill-possed”.
  + Well-possedness is defined as a mathematical model of a physical process that has three main characteristics – solution exists, it is unique, and its behaviour changes continuously with the initial conditions (is stable). Examples of this include heat equation with clearly defined initial and boundary conditions.
  + Ill-possed problems’ main difference lies in the stability of the result – the third point. Solution of an ill-posed problem can be highly unstable and sensitive to small changes in the input data (random errors). It means that even small errors encountered in measurements can have big impact on the result.
    - It is also almost impossible to prove the uniqueness of the solution resulting from the inverse problem.
    - <https://en.wikipedia.org/wiki/Well-posed_problem>
  + There is also a terminology difference in describing the way the results are created between classical and inverse problems. In the case of classical problem, the results are *determined*, and in case of inverse problem the results are *estimated*. This is because while solving the inverse problem, errors in the measurements can influence the final result much more than while solving the classical problem.
* This discipline has numerous practical usages, as usually the measurement of boundary conditions applied on an object is harder than measuring the temperature distribution inside the body.
* What is always necessary, however, is the known solution for a forward problem.
* Applications of inverse problems
  + One of the most widespread usages comes from manufacturing system. Temperature cycle for a component to gain desired characteristics is usually well described. The boundary conditions like heat flux or pressure are to be determined, either by a trial and error, or by solving an inverse problem.
    - <https://www.researchgate.net/publication/325221371_SOLUTION_OF_AN_INVERSE_PROBLEM_TO_DETERMINE_HEAT_SOURCE_STRENGTH_AND_LOCATION>
  + Solution of a heat transfer problems contributed to the space exploration programs in 1950’s and 1960’s – as engineers were unable to measure the temperature at the outside of the spaceships during their re-entry from the space in the atmosphere, because the heat flux there was enormously high. Therefore, temperature measurements were taken not on the very outside, but below the surface, where the temperature could be measured more easily. With this knowledge the temperature at the surface of the spaceship could be estimated.
  + Modern materials have their thermophysical properties dependent on the temperature and the position in the object, and therefore it is harder to determine their actual properties at the exact moment. With the use of inverse problems, they can be estimated at almost any given point.
  + With the use of inverse problems, industrial device and its properties can be more easily evaluated during real operating conditions, not having to depend only on simulation or other estimates.
  + “*The principal advantage of the IHTP is that it enables to conduct experiments as close to the real conditions as possible”*
* *“An inverse solution is developed to determine source strength and location from downstream data.”*
  + <https://www.sciencedirect.com/science/article/abs/pii/S0017931017344617?via%3Dihub#ab010>
* Research and advancements in inverse problems are tightly connected with the advancements in computer science, as more powerful computers allowed for better usage of numerical methods and computing, which is broadly used in this area.
* Inverse Heat Transfer Problems (IHTP) does not limit themselves to estimating heat flux on the boundary from the temperature distribution, they can also be used for estimating object material properties like specific heat capacity (cp) or thermal conductivity (lambda).
* IHTP are usually connected with conduction, however, they are also applicable in other heat transfer mechanisms – convection and radiation, and even in mixes problems. One-dimensional problems are also the most usual ones, in recent years however, even multi-dimensional problems are being solved in inverse manner.
* Temperature response for applied heat flux is delayed, and this delay is higher in points further from the source. Therefore, if we want to estimate current heat flux, we need to measure the temperatures not only in the current moment, but also in the near future.
* The measurements errors are also magnified more with the increasing distance from the source of heat flux.

# Heat Transfer