# Inverse problems

* <http://www-eng.lbl.gov/~shuman/NEXT/MATERIALS&COMPONENTS/Xe_damage/INVERSE-HEAT-TRANSFER-OZISIK.pdf> - the main source of information below
* <https://en.wikipedia.org/wiki/Inverse_problem> - also a very nice source of info, mainly about the various applications

In the majority of scientific fields, the classical (direct) problems mean the cause of some action (boundary condition) is given, and our goal is to determine the effect that this action will have on the object.

Inverse problems are being solved when we know the final results of an action, but we are searching for the (boundary) conditions that led to this result. In the context of heat transfer, it could be translated into knowing the temperature distribution in the object after heating it up but being interested in the heat flux applied on the object. Temperature distribution inside the object is usually determined by measuring the temperature below the object’s surface with an appropriate method.

## Classical vs inverse problems

*“In the direct problem the causes are given, the effect is determined; whereas in the inverse problem the effect is given, the cause (or causes) is estimated.”*

The difference between classical and inverse problems is also in the precision and correctness of the result. It can be said that the result of the classical problem can be unambiguously determined. The solution to the inverse problem, on the other hand, can be only estimated, and there will always be some uncertainty.

From a mathematical perspective, they differ in the aspect called posedness. Classical problems are described as “well-posed”, in contrast to inverse problems, which are “ill-possed”.

Well-posedness is defined as a mathematical model of a physical process that has three main characteristics – a solution exists, it is unique, and its behaviour changes continuously with the initial conditions (is stable). Examples of this include the heat equation with clearly defined initial and boundary conditions.

Ill-possed problems’ main difference lies in the stability of the result – the third point. The solution of an ill-posed problem can be highly unstable and sensitive to small changes in the input data (random errors). It means that even small errors encountered in measurements can have a big impact on the result.

It is also almost impossible to prove the uniqueness of the solution resulting from the inverse problem.

* <https://en.wikipedia.org/wiki/Well-posed_problem>

There is also a terminology difference in describing the way the results are created between classical and inverse problems. In the case of classical problems, the results are *determined*, and in the case of inverse problems, the results are *estimated*. This is because while solving the inverse problem, errors in the measurements can influence the final result much more than while solving the classical problem.

## Application of inverse problems

This discipline has numerous practical usages, as usually, the measurement of boundary conditions applied on an object is harder than measuring the temperature distribution inside the body.

* For example, placing sensors on the desired place would interfere with the measurement itself, as the sensors would influence the experiment in an undesirable way.

What is always necessary, however, is the known solution for a forward problem.

One of the most widespread usages comes from the manufacturing system. The temperature cycle for a component to gain the desired characteristics is usually well described. The boundary conditions like heat flux or pressure are to be determined, either by a trial and error or by solving an inverse problem.

* <https://www.researchgate.net/publication/325221371_SOLUTION_OF_AN_INVERSE_PROBLEM_TO_DETERMINE_HEAT_SOURCE_STRENGTH_AND_LOCATION>

The solution of heat transfer problems contributed to the space exploration programs in the 1950s and 1960s – as engineers were unable to measure the temperature at the outside of the spaceships during their re-entry from the space in the atmosphere because the heat flux there was enormously high. Therefore, temperature measurements were taken not on the very outside, but below the surface, where the temperature could be measured more easily. With this knowledge, the temperature at the surface of the spaceship could be estimated.

Modern materials have their thermophysical properties dependent on the temperature and the position in the object, and therefore it is harder to determine their actual properties at the exact moment. With the use of inverse problems, they can be estimated at almost any given point.

With the use of inverse problems, industrial devices and their properties can be more easily evaluated during real operating conditions, not having to depend only on simulation or other estimates.

“*The principal advantage of the IHTP is that it enables to conduct experiments as close to the real conditions as possible”*

*“An inverse solution is developed to determine source strength and location from downstream data.”*

* <https://www.sciencedirect.com/science/article/abs/pii/S0017931017344617?via%3Dihub#ab010>

Research and advancements in inverse problems are tightly connected with the advancements in computer science, as more powerful computers allowed for better usage of numerical methods and computing, which is broadly used in this area.

Inverse Heat Transfer Problems (IHTP) does not limit themselves to estimating heat flux on the boundary from the temperature distribution, they can also be used for estimating object material properties like specific heat capacity (cp) or thermal conductivity (lambda).

IHTP are usually connected with conduction, however, they are also applicable in other heat transfer mechanisms – convection and radiation, and even in mixes problems. One-dimensional problems are also the most usual ones, in recent years, however, even multi-dimensional problems are being solved in an inverse manner.

The temperature response for applied heat flux is delayed, and this delay is higher in points further from the source. Therefore, if we want to estimate current heat flux, we need to measure the temperatures not only in the current moment but also in the near future.

The measurement errors are also magnified more with the increasing distance from the source of heat flux.

# Heat Transfer

* **BERGMAN, T. a Frank P. INCROPERA. Fundamentals of heat and mass transfer.** – The main source of further information

In our physical world energy can be transferred from one system to another either by work or by heat.

In contrast to thermodynamics, which is only concerned with the initial and final state of these systems, the field of heat transfer also determines detailed information about the way this transfer of energy is occurring, like time or rate.

*“Heat transfer (or heat) is thermal energy in transit due to a spatial temperature difference.”*

*“Whenever a temperature difference exists in a medium or between media, heat transfer must occur.”*

## Modes of heat transfer

There are three main types of heat transfer, so-called modes. They include conduction, convection, and radiation.

### Conduction

Pure conduction happens inside stationary solid or fluid objects when there is a temperature gradient inside that object.

*“Conduction may be viewed as the transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles.”*

Particles with higher temperature (higher energy) are randomly colliding with less energetic particles. During the collisions, energy in the form of heat is transferred from the more energetic particles to the less ones. Therefore, the heat goes from the warmer region of the object to the colder region, and this is also the direction of the heat transfer.

Random molecular motion, which is contributing to the heat transfer by conduction, is also called diffusion.

When comparing the conduction in gas, liquid and solid objects, the mechanism is almost exactly the same. However, there is a difference in the particle density (distance between particles in the object) between these three states – in case of gas, particles are much more far away from each other than in case of solid. This contributes to the fact that conduction in solid materials (metals) is much more intensive and quicker compared to gases.

The rate equation for conduction, which is used to calculate the transferred energy, is called Fourier’s law.

### Convection

Convection is described as a heat transfer between a surface and moving fluid when these have different temperatures.

One mechanism of convection is already known from conduction – the heat transfer due to random movements and collisions between particles which are of different temperatures (diffusion).

The other mechanism causing convection is the bulk motion of the fluid, which is represented by a large number of molecules moving collectively. This bulk movement is also known as advection.

The final heat transfer by convection is then determined as an aggregation of these two mechanisms – because even in the large bulk of fluid, which is flowing “as one piece”, there exists a random movement of molecules in the bulk.

The rate equation for convection is known as Newton’s law of cooling.

### Radiation

Radiation means the emission of heat by all objects that have a non-zero temperature (in Kelvins).

The emission of energy by radiation happens thanks to electromagnetic waves.

Opposed to conduction and convection, where there needs to be a material medium for the heat transfer to happen, radiation does not require that. On the contrary, heat transfer by radiation happens most efficiently in a vacuum, when there is no medium at all.

The rate equation for radiation is called Stefan-Boltzmann law.

# Possible solutions for heat transfer problems

## Analytical vs numerical solution

Analytical solutions are more convenient, but they exist only for a small subset of problems and are applicable mostly for the simplest geometries and boundary conditions.

The numerical solution usually takes more effort, but when already created, it is much more versatile, as we can freely change parameters or geometries of the simulation without redoing the numerical model itself.

The most common numerical methods include the finite elements and finite volumes methods.

Finite elements have the ability of working with arbitrary shapes.

Coming up with a simple solution for the small element, and then just replicate it on the whole domain

* <https://www.youtube.com/watch?v=aLJMDn_2-d8> - Finite Element Method (FEM) - Finite Element Analysis (FEA): Easy Explanation
* <https://www.youtube.com/watch?v=sSaUHDQf204> – FEA 01: What is FEA?
* <https://www.youtube.com/watch?v=WCpJYskd0_Y> - FEA 02: What could go wrong?
  + It is easy to make mistakes, as there are many inputs
    - Always sanity-check the results
  + Element size matters
    - Check the elements size does not have a significant impact on the results (convergence study)
  + Element shape matters (it should be consistent and same in the whole object)
  + Numerical errors that build up as the calculation progresses (computers have a finite precision, and the errors can propagate further)
* <https://www.youtube.com/watch?v=YnIOyzUQ4H8> - FEA 03: Strength of Materials Review
* <https://www.youtube.com/watch?v=x2HnV3mU1Cg> - FEA 04: Matrix Algebra Review
  + We need matrixes in FEA to both formulate and solve the problem
  + Taking advantage of linear (matrix) algebra we can solve multiple variables at the same time in an efficient manner
  + Matrixes are representing systems of linear equations

## The partial differential equation (PDE) for heat transfer

* [*https://en.wikipedia.org/wiki/Heat\_equation*](https://en.wikipedia.org/wiki/Heat_equation)
* [*https://en.wikipedia.org/wiki/Partial\_differential\_equation*](https://en.wikipedia.org/wiki/Partial_differential_equation)
* *“heat equation is a partial differential equation that describes how the distribution of some quantity (such as heat) evolves over time in a solid medium, as it spontaneously flows from places where it is higher towards places where it is lower”*
* It was first described by Joseph Fourier, after whom the rate of conduction is being calculated by a Fourier’s law.
* **TODO: Fundamentals of Heat and Mass Transfer** – pages 82-85
* Terms to be explained
  + PDE
  + Implicitness/explicitness
  + Boundary conditions and their 3 kinds – Dirichlet, Neumann, and Newton
    - Dirichlet boundary condition is describing a situation when we know the exact temperature at the surface of an object.
    - Neumann boundary condition means knowing the heat flux at the surface of an object. When the object is insulated, this heat flux is equal to zero.
    - When a Newton boundary condition is present, we know there is a certain convection heat transfer (cooling or heating) happening on the object surface.
  + Linear vs non-linear problems (when material properties are highly dependent on temperature)
  + Regularisation of the ill-posed problem (is being done by a higher number of window span)
    - With a higher window span, the simulation time grows, and the solution gets smoother, which can be bad when there are some spikes in heat flux that need to be caught.
    - The optimal value of the window span is highly individual for each simulation, there is no single value that would be a good fit for all simulation.
      * Parameters that can change the optimal window span are mainly material properties, the position of the interest and sample frequency.
    - Its value should be high enough to eliminate oscillations and allow for the delay in the heat flux change having the effect of changing temperature inside the object
* Our inverse solution
  + ***Vývoj inverzní sub-doménové metody pro výpočet okrajových podmínek vedení tepla***– source for some info, citing mainly the below-mentioned
    - *BECK, J. V., BLACKWELL, B., CHARLES, R. C. Inverse heat conduction: ill-posed problems. New York: Wiley, 1985, 308 s. ISBN 04-710-8319-4.*
  + Seems to be using the sequence method (page 35)
  + When window span is 1, it is a one-step method (page 29)
  + Having a window span bigger than 1 is increasing the stability by decreasing the influence of random errors from measurements. However, it can also have softening effects, so we could get lower heat fluxes than in reality.
* **PDE of heat transfer**
* M – tridiagonal sparse mass matrix … … is multiplied by rho, cp, and dx
* K – tridiagonal sparse mass stiffness … … is multiplied by lambda/dx
  + Multiply by a test function v(x)
  + Integrate through the whole body domain
  + Apply Gauss-Ostrogradsky theorem (TODO: describe more and source it)
  + Weaken the form in integrals by dS (TODO: describe and source it)
  + We have two boundary conditions
  + On the left side () we have Neumann – second order …
  + On the right side () we have Robin – third order …
    - = surrounding temperature (ambient temperature)
  + Separate entries with on the left and rest on the right
  + Replacing functions (v and T) with finite elements

# TODOS FOR THEORY

* Describe the real experiment (with some photos in the best case)
* Deepen the description and derivation of the numerical solution (with some sources)
* Add some pictures where possible