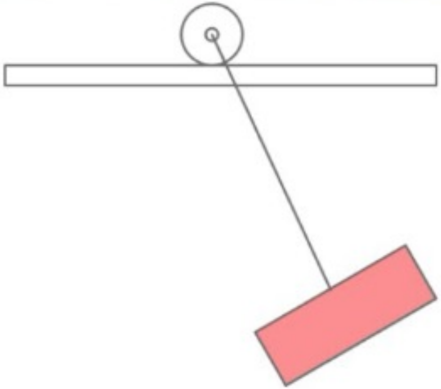
Dynamics - Motivation

1. System design and analysis often begin with a simple model

2. Use a model to probe the system's sensitivities and response to inputs to inform design modifications or the control system

3. Determine power requirements and size components

4. Knowing what a simple model should (and should not) include requires experience - it all starts with being able to perform dynamic analysis



Example - Ship Drivetrain Vibration

Possible Analysis Objectives

1. Specify the properties of the Torsional Elastic Coupling such that propeller vibration, due to engine firing, is below some threshold

2. Specify the Misalignment Coupling angular range due to the vertical vibration of the engine and/or specify the elastic engine mount properties.

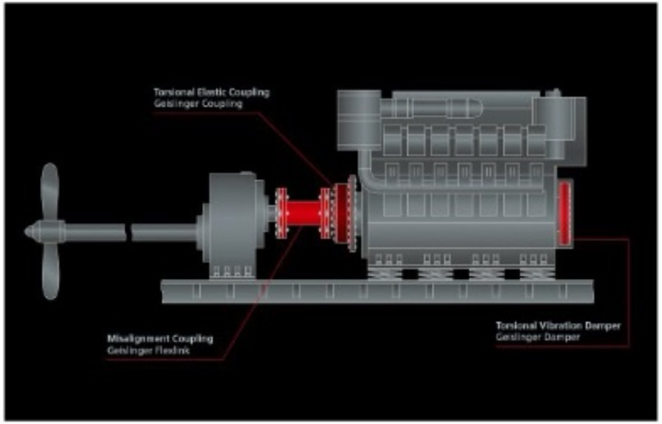
Possible Analysis Approach

1. Declare assumptions (different for each analysis objective)

2. Postulate a **lumped parameter** model and generate differential equations

3. Develop an understanding of the sensitivities of motion outputs to force/torque inputs

4. Evolve a simulation - start simple, add complexity as needed

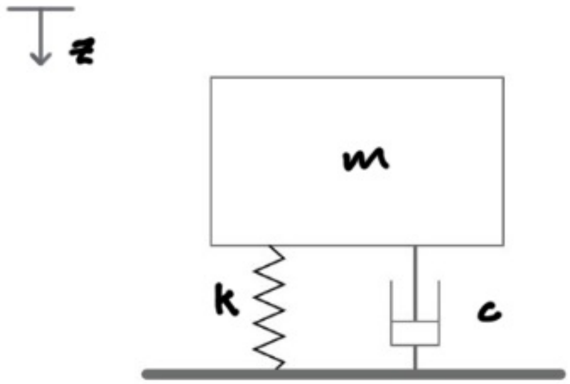


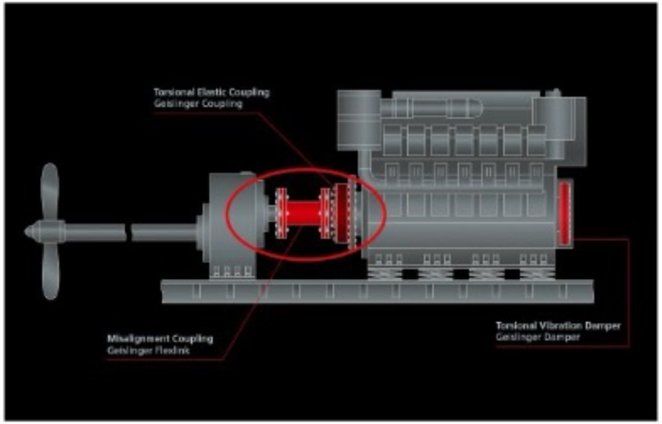
https://www.geislinger.com/en/products/product/coupling

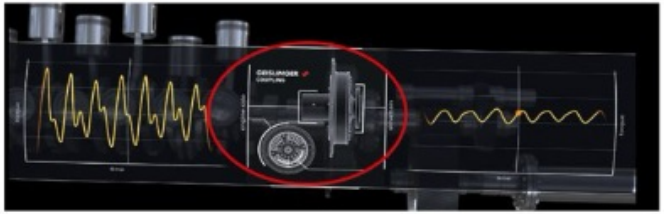
Example - Ship Drivetrain Vibration

Lumped Parameter Model - An approximate model of a physical system/device where its numerous parts are replaced with a few, idealized ones based on stated assumptions.

constrained to vertical only







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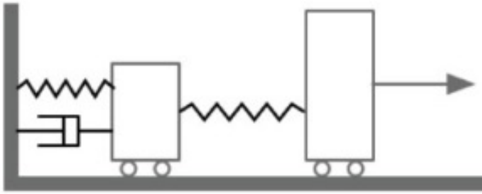
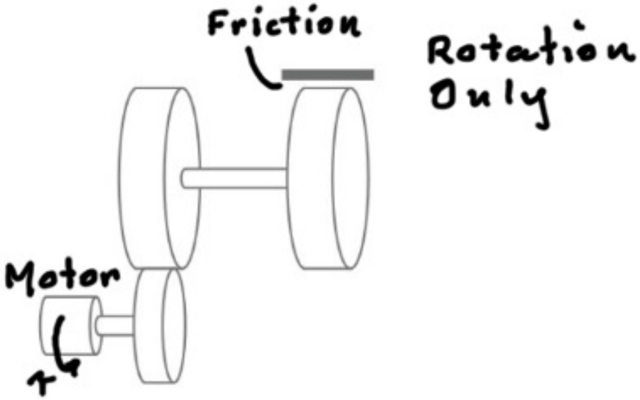
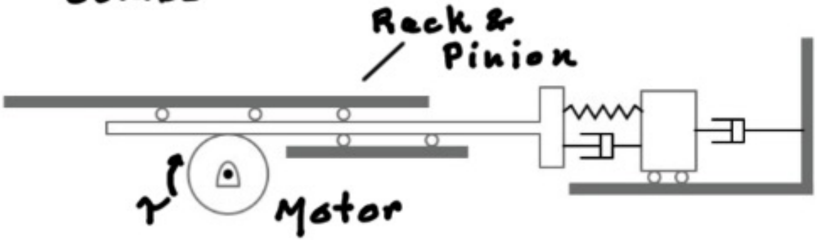
Lumped Model Samples

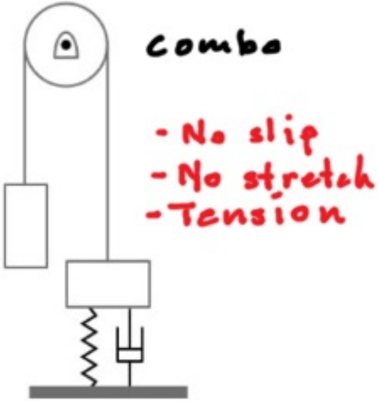
Translation Only





combo



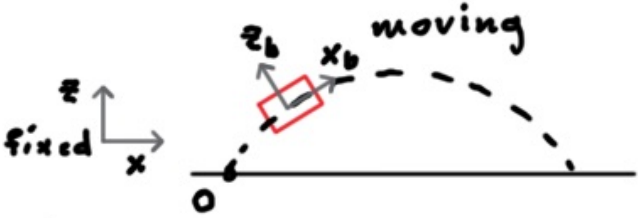


Objectives and Skills

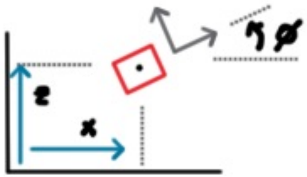
1. Recognize a lumped parameter model
2. Review concepts and terminology from MEEM 2700, dynamics
3. Develop a slightly different method for FBDs
4. Create a FBD of a hanging rod, maybe generate its DEQ model

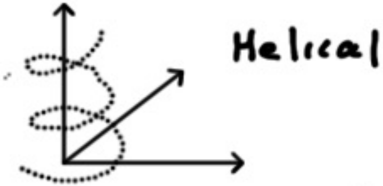
Concepts and Terminology

1. Reference frame - A right-hand set of 3-axes that is either fixed (inertial) or moving/rotating



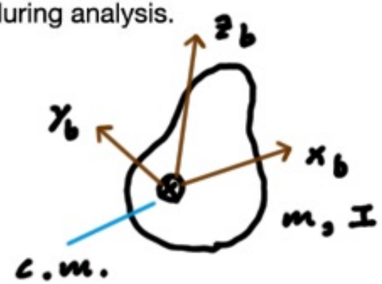
2. Plane Motion - All motion occurs in a single, fixed plane





3. Particle - A conceptual “body” with mass but no dimensions, thus no inertia

4. Rigid body - A real body with mass properties - mass, inertia, and center of mass. Sometimes we can represent rigid bodies as particles during analysis.

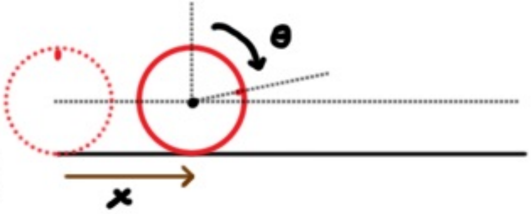


Concepts and Terminology

5. Kinematics - Equations that relate motions within or between bodies

$$x = r\theta$$
$$\dot{x} = r\dot{\theta}$$
$$\ddot{x} = r\ddot{\theta}$$

x & θ are NOT independent. They are related by kinematics.



6. Statics $\Sigma \underline{F} = \underline{0}$

$$\Sigma \underline{M} = \underline{0}$$

7. Dynamics

$$\Sigma \underline{F} = m\underline{a}$$

External Forces

Absolute acc. of c.m.

$$\Sigma \underline{M} = \underline{\underline{I}}\,\dot{\underline{\omega}} + \underline{\omega} \times \underline{\underline{I}}\,\underline{\omega}$$

$$\Sigma M = I\ddot{\phi} \quad \text{(planar motion)}$$

External moments about c.m

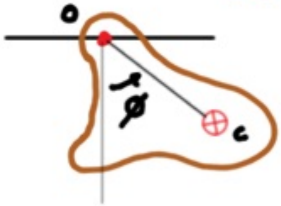
Mass moment of inertia, c.m.

Absolute angular acc.

Special Rotation Case:

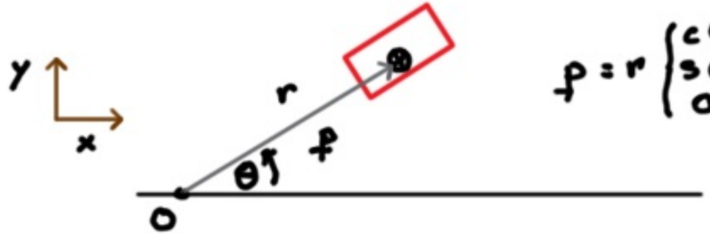
Fixed Axis

$$\Sigma M_0 = I_0\ddot{\phi}$$



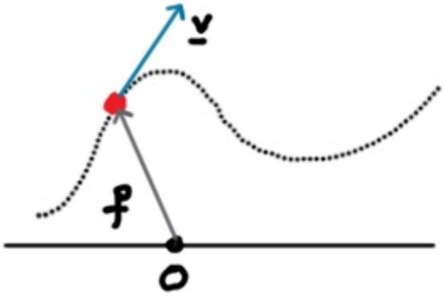
Concepts and Terminology

8. Absolute position vector (m) - A vector from a fixed point to some point of interest, usually a center of mass



$$\underline{\rho} = r\begin{Bmatrix} c\theta \\ s\theta \\ 0 \end{Bmatrix}$$

$\underline{v}$ is tangent to the path.



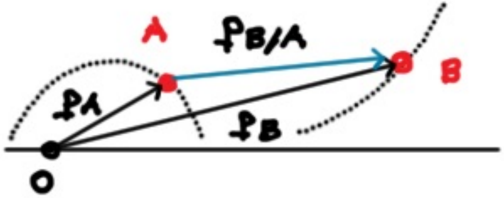
9. Absolute velocity and acceleration $(m/s, m/s^2)$- Time rate of change of the position and velocity vectors respectively

Fixed Frame: $\underline{v} = \dot{r}\begin{Bmatrix} c\theta \\ s\theta \\ 0 \end{Bmatrix} + r\dot{\theta}\begin{Bmatrix} -s\theta \\ c\theta \\ 0 \end{Bmatrix}$

$$\underline{a} = \ddot{r}\begin{Bmatrix} c\theta \\ s\theta \\ 0 \end{Bmatrix} + \dot{r}\dot{\theta}\begin{Bmatrix} -s\theta \\ c\theta \\ 0 \end{Bmatrix} + r\ddot{\theta}\begin{Bmatrix} -s\theta \\ c\theta \\ 0 \end{Bmatrix} + \dot{r}\dot{\theta}\begin{Bmatrix} -s\theta \\ c\theta \\ 0 \end{Bmatrix} + r\dot{\theta}^2\begin{Bmatrix} c\theta \\ -s\theta \\ 0 \end{Bmatrix}$$

Concepts and Terminology

10. Relative position (m) - A vector from one moving point to another



$$\rho_A + \rho_{B/A} = \rho_B$$

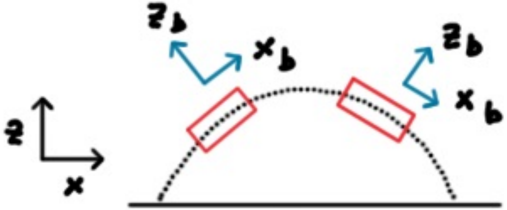
or

$$\rho_{B/A} = \rho_B - \rho_A$$

11. Relative velocity and acceleration $(m/s, m/s^2)$- The velocity and acceleration of some point as viewed by an observer fixed at another point

$$\underline{v}_{B/A} = \underline{v}_B - \underline{v}_A \;\&\; \underline{a}_{B/A} = \underline{a}_B - \underline{a}_A$$

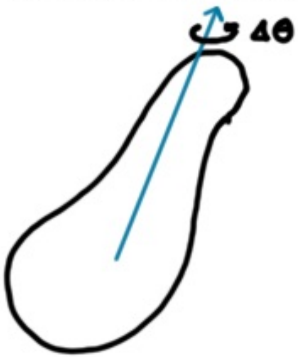
12. Rotation - A situation where two reference frames are not aligned



Concepts and Terminology

13. Angular Velocity (rad/s) - A vector that describes the rate of change of a body's rotation. NOTE: It is defined and NOT the derivative of a "rotation vector," and thus is unlike velocity which is the derivative of a defined absolute position vector. Rigid bodies can have angular velocity. We never say a point on the body has an angular velocity

$$\underset{\sim}{\omega} = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}$$



14. Angular Acceleration (rad/s^2) - The time rate of change of the angular velocity vector

$$\underset{\sim}{\alpha} = \frac{d}{dt}\underset{\sim}{\omega}$$

$\ddot{\theta}, \ddot{\phi}$, etc.

Concepts and Terminology

15. ~~Linear~~ Momentum $(kg \cdot m/s)$ Product of mass and velocity

$a_1 > a_2$

$$\underline{p} = m\underline{v} \qquad \dot{\underline{p}} = \underline{F} \qquad \dot{\underline{p}} = m\underline{a} = \underline{F}$$

mass is a measure of resistance to accelerate.

16. Angular Momentum $(kg \cdot m^2/s)$, Product of mass moment of inertia and angular velocity

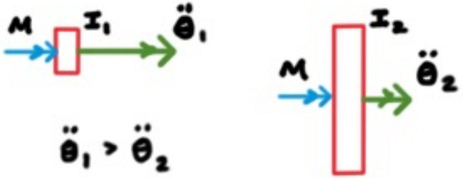
$$\underline{H} = \underline{\underline{I}}\,\underline{\omega}, \quad \dot{\underline{H}} = \underline{M},$$

General

$$\dot{\underline{H}} = \underline{\underline{I}}\,\dot{\underline{\omega}} + \underline{\omega} \times \underline{\underline{I}}\,\underline{\omega} = \underline{M}$$

$$\dot{H}_z = I_z \ddot{\theta} = M_z$$

Planar Motion

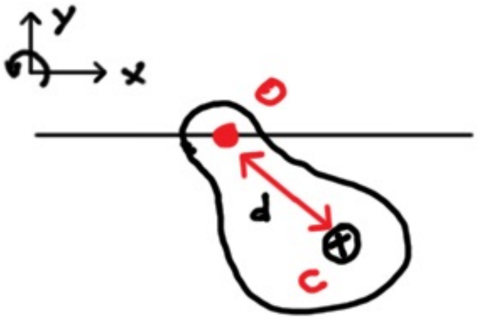


$\ddot{\theta}_1 > \ddot{\theta}_2$

Inertia is a measure of resistance to angular acc.

Concepts and Terminology

17. Mass Moment of Inertia ($kg \cdot m^2$) - A property of a rigid body



Given:
$m = 40\,kg$

$I_{c_z} = .2\,kg \cdot m^2$

Find: I_{o_z}

Tables, CAD

Move the reference point using the parallel axis theorem.

$I_{o_z} = I_{c_z} + md^2$

we need d ...

$I_{o_z} = .2 + 40d^2$

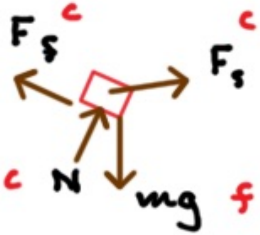
why do it ?? Makes KE easier to calculate. Makes rotational DEQs easier too when there is rotation about a fixed pt.

$\Sigma M_o = I_o \ddot{\theta}$

Note: Whenever you move the ref. pt. for I, its value increases...

Concepts and Terminology

18. Force $(N, kg \cdot m/s^2)$. For us, divided into two categories: contact and field. Several ways to describe it: (1) if there's a force, there's the possibility of motion, (2) if there's motion, there is or was a force, (3) product of mass and acceleration



19. Moment $(N \cdot m, kg \cdot m^2/s^2)$. Created by a force, so the comments above apply to moments too. Regardless of where a moment is applied to a rigid body, its effect on its motion is the same.

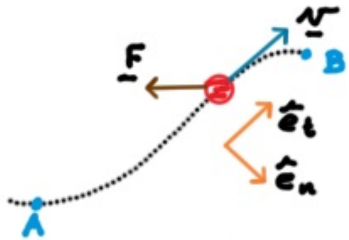
$\underset{\sim}{M} = \underset{\sim}{r}_{A/c} \times \underset{\sim}{F}_T$

Pos. vec from c.m. to the point of application of the force.

Force Vector

Concepts and Terminology

20. Work ($N \cdot m, kg \cdot m^2/s^2$). If a body is moving, and there is an external force present, then work is being done. If the force is in the direction of motion then the work is positive. If the force is opposing the motion, then the work is negative. We never say a body "has work." Moments do work too...



$W = \int_A^B \underline{F} \cdot d\underline{r}$ Note: $d\underline{r} = ds\,\hat{e}_t \longrightarrow$ tangent to the path. Just like $\underline{v}$!!

$\therefore$ A force does work when a component is in the drxn of $\underline{v}$. Also Note: work is a scalar.

21. Kinetic Energy ($kg \cdot m^2/s^2$). Defined at any instant in time as: $T = \dfrac{1}{2} m \mathbf{v} \cdot \mathbf{v}$

with the def'n of W & $\underline{F} = m\underline{a}$ & some calculus...

$W = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2 = T_B - T_A$ (Princ. of W & KE)

Note: Dot product of 2 vecs

$\begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} \cdot \begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = v_x^2 + v_y^2 + v_z^2$, or $\underline{v} \cdot \underline{v} = |\underline{v}|^2$ which we write as v^2.

Concepts and Terminology

$G = 6.6743\times10^{-11}\ \frac{Nm^2}{kg^2}$, $M = 5.9722\times10^{24}$ kg, $R = 6.3750\times10^{3}$ m (Houghton, effective)

22. Potential Energy ($kg \cdot m^2/s^2$). Energy associated with some very special forces, called **conservative forces**, that have the properties: (1) they are functions of position only, not time, not speed and (2) the work done on a body by a conservative force as it moves from configuration A to configuration B only depends on the description of these two end points and has nothing to do with how it moved between them. An interesting result that relates a conservative force and PE is:

$$\mathbf{F} = -\left(\frac{\partial V}{\partial x}\mathbf{i} + \frac{\partial V}{\partial y}\mathbf{j} + \frac{\partial V}{\partial z}\mathbf{k}\right)$$

Gravity

$$F = \frac{-MmG}{r^2} = \frac{-MmG}{(R+h)^2}$$
$$\approx \frac{-MmG}{R^2}(1 - 2^h/R)$$
$$\approx -mg \quad (h \ll R)$$
$$g = MG/R^2 \approx 9.808$$

$$V_0 = \frac{-MmG}{r} = \frac{-MmG}{R+h} \approx \frac{-MmG}{R}\left(1 - \frac{h}{R}\right)$$
$$\approx -mgR(1 - h/R) \approx -mgR + mgh$$
$$V = V_0 + mgR \approx mgh \quad (h \ll R)$$

Spring

Δ = distance − Lus

$$F_s = -k\Delta$$
$$= -k(x - x_{us})$$
$$V = \frac{1}{2}k\Delta^2$$
$$V = \frac{1}{2}k(x - x_{us})^2$$

Nonconservative Forces

- Thruster, $F(t)$
- Motor, $\tau(t)$

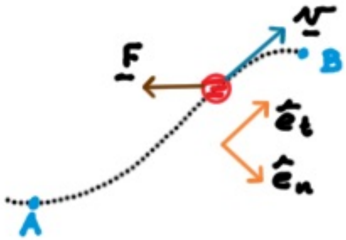
Friction, $F_f = -\mu N$ ×

$F_f = -\mu N\,\mathrm{sign}(\dot{x})$

Concepts and Terminology

24. Power $(W, N \cdot m/s)$. Time rate of change of energy. Used for sizing mixed physics components.

Related to work...



$P = \dot{W} = \underline{F} \cdot \underline{v}$

If $\underline{F}$ & $\underline{v}$ are in the same drxn, then nrg is going IN. If $\underline{F}$ & $\underline{v}$ are opposing then nrg is being extracted. Key to nrg generation, force & velocity.

Same concept for $\underline{M}$ & $\underline{\omega}$ (wind turbine).

$P = \underline{M} \cdot \underline{\omega} = \underline{F} \cdot \underline{v} = (\text{Volt})(\text{current}) = iR^2 = \text{etc.}$

($\approx .3$ ft·lb)

Example: A battery operated motor must generate .4 N·m @ 1000 rpm. The battery is 12 V. How much current will it draw?

$P = .4 \cdot 1000 \cdot \dfrac{2\pi \text{ rad}}{\text{rev}} \approx 2.5 \text{ kW} = (12)(i), \quad i \approx 200 \text{ amps}$

Note: A 48 A-hr battery would last < 14 min...

Key Principles

Starting from Newton's Law of Motion, we can derive and exploit the principles of …

1. Work and Kinetic Energy
2. Conservation of Mechanical Energy
3. Linear Impulse and Momentum
4. Conservation of Linear Impulse
5. Angular Momentum and Impulse
6. Conservation of Angular Momentum

Free Body Diagrams - A Checklist

1. Sketch out one or more coordinate frames and define the motion variables, x, y, θ, ϕ, etc. Take care to define what positive means for each

2. Separate each body from the system and sketch it in roughly the same location as it was in the system, but with room to annotate it with vectors

3. Immediately label each with appropriate translation and rotation inertial forces (i.e. $m\ddot{x}, m\,\ddot{y}, I\ddot{\theta}, I\ddot{\phi}$, etc.) with the vector point in the OPPOSITE direction of what you declared as positive in Step #1.

4. If the motion is in a vertical plane (gravity) then immediately annotate each body with its weight vector, $\mathbf{w} = m\mathbf{g}$

5. Identify all forces and moments that are shared between bodies, e.g. springs, cords, dampers, normal forces, etc. Apply them as vectors and include equal and opposite on affected bodies

6. Assign normal forces between bodies and/or fixed surfaces - ALWAYS normal to their interface surface

7. Assign externally applied forces and moments, e.g. thrusters, motor torques, etc. Classify: Conservative, nonconservative

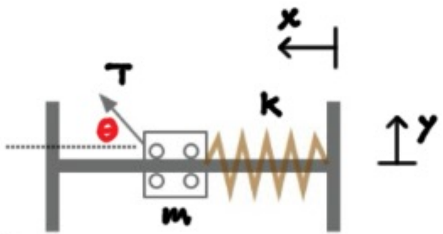
8. If any body can slide or roll relative to another body or surface, then assign a friction force. If between bodies, then make sure you apply equal and opposite vectors. NOTE: you can always set any of these to zero if there is no friction

9. If there are joints with reaction forces and moments, then assign them, e.g. pins, cantilever, etc.

Free Body Diagram - One Figure, Not Two - Translation

Newton's Law of Motion

$$\sum \mathbf{F} = m\,\mathbf{a}$$



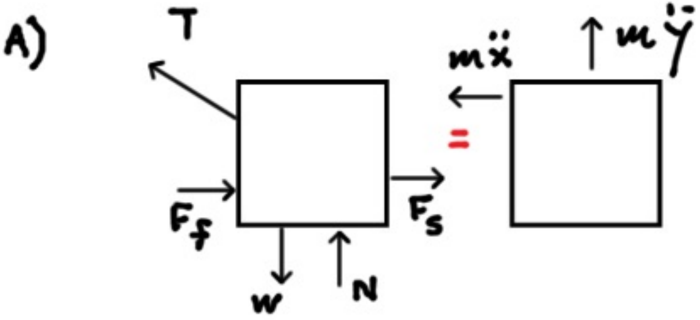
Block on a guide

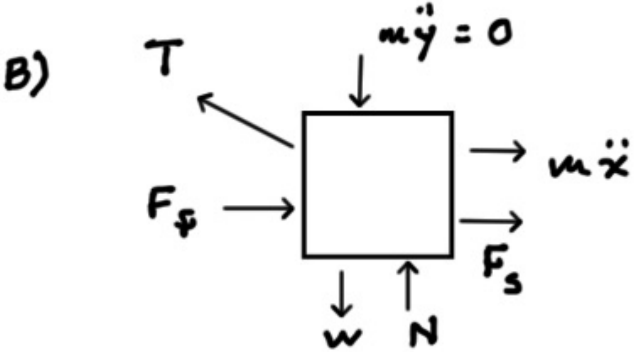
A) $T c\theta - F_f - F_s = m\ddot{x}$

$T s\theta + N = m\ddot{y} = 0$

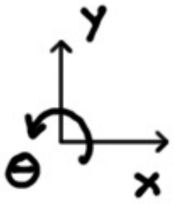
B) $T c\theta - F_f - F_s - m\ddot{x} = 0$

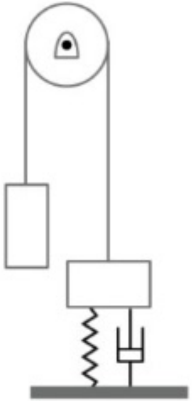
$T s\theta + N - \cancel{m\ddot{y}} = 0$





Free Body Diagram - One Figure, Not Two - Rotation

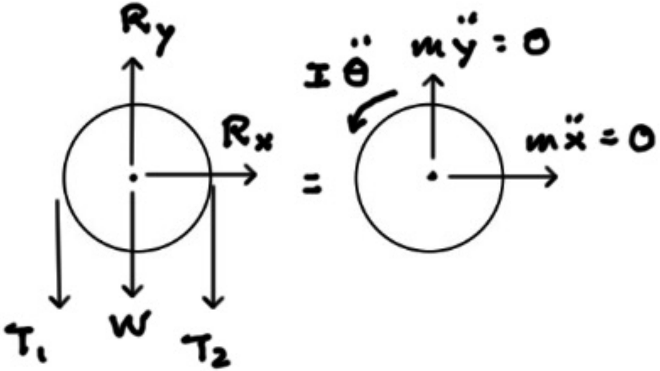


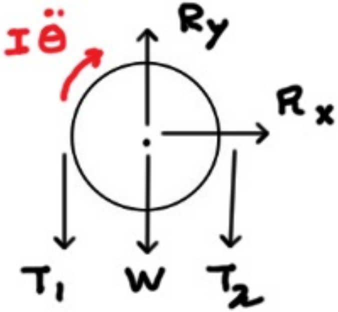


Euler's Equation

$$\sum \mathbf{M} = I\dot{\omega} + \omega \times I\omega$$

$$\sum M = I\ddot{\theta}$$

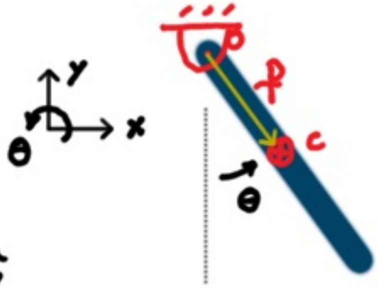




Pulley Only

Example 1

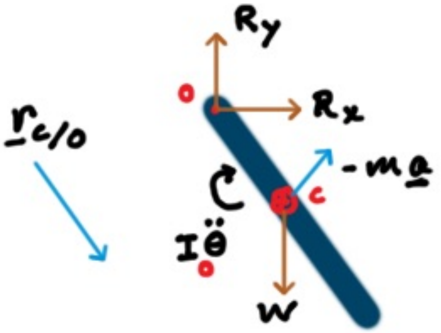
Create the DEQ model, using an FBD, of a hanging rod.



L, m, I_c, r

Uniform ρ

$I_c = \frac{m}{12}(3r^2 + L^2)$



$\underline{a} = ?$

Kinematics

$\underline{\rho} = \frac{L}{2} s\theta \,\hat{x} - \frac{L}{2} c\theta \,\hat{y}$

Take 2 derivs ... lots of terms.

$\underline{a}(\theta, \dot{\theta}, \ddot{\theta})$

$$\boxed{\begin{gathered} \Sigma \underline{F} = m\underline{a} \\ \& \\ \Sigma M_c = I_c \ddot{\theta} \\ \text{or} \\ \Sigma M_o = I_o \ddot{\theta} \end{gathered}}$$

$$\left.\begin{aligned} x)\;& R_x - m a_x = 0 \\ y)\;& R_y - W - m a_y = 0 \end{aligned}\right\} \text{Important if we want } R_x \;\&\; R_y.$$

θ) Use o, not c, easier

$$\Sigma M_o = \underline{r}_{c/o} \times \underline{W} = \frac{L}{2}\begin{Bmatrix} s\theta \\ -c\theta \\ 0 \end{Bmatrix} \times \begin{Bmatrix} 0 \\ -W \\ 0 \end{Bmatrix} = \frac{LW}{2}\begin{Bmatrix} 0 \\ 0 \\ -s\theta \end{Bmatrix}$$

$$\Sigma M_o = I_o \ddot{\theta}$$

$$\boxed{-\frac{LW s\theta}{2} = I_o \ddot{\theta}}$$

Example 1

Create the DEQ model, using an FBD, of a hanging rod.

$$-\frac{LW\,s\theta}{2} = I_o\ddot{\theta}$$

or, $\boxed{I_o\ddot{\theta} + \frac{mgL}{2}s\theta = 0}$ Nonlinear DEQ...

small θ, linearize about $\theta_o = 0$

$$I_o\ddot{\theta} + \frac{mgL}{2}\theta = 0, \quad I_o = I_c + md^2 = I_c + m\left(\frac{L}{2}\right)^2$$

$$I_c = \frac{m}{12}(3r^2 + L^2) \quad \text{If } r \ll L,\ I_c \approx \frac{1}{12}mL^2,\ I_o \approx \frac{1}{12}mL^2 + \frac{1}{4}mL^2 \approx \frac{1}{3}mL^2$$

so...

$$\frac{1}{3}mL^2\ddot{\theta} + \frac{1}{2}mgL\theta = 0$$

$\boxed{\ddot{\theta} + \frac{3}{2}\frac{g}{L}\theta = 0}$ undamped, $\omega_n = \sqrt{\frac{3g}{2L}}$

If $L = .37\text{ m}$ ($\approx 14''$) then

$$T = \frac{2\pi}{\omega_n} = 2\pi\sqrt{\frac{2L}{3g}} \approx 1\text{ sec}$$

Example 1

Create the DEQ model, using an FBD, of a hanging rod.

Summary

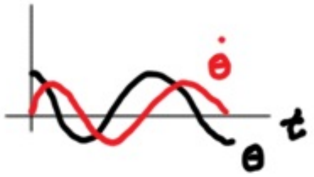
1. $R_x - m\cdot\frac{L}{2}(\ddot{\theta} c\theta - \dot{\theta}^2 s\theta) = 0$

2. $R_y - W - m\cdot\frac{L}{2}(\ddot{\theta} s\theta + \dot{\theta}^2 c\theta) = 0$

3. $\ddot{\theta} + \frac{3}{2}\frac{g}{L}\theta = 0$

What to do with these? Solve for rxn forces during simulation.

1. Simulate Eq. 3 $\longrightarrow$



2. Sub 3 into 1 & 2

3. Calc R_x & R_y from $\theta(t)$, $\dot{\theta}(t)$

Summary

You can (with a little more practice)

1. Distinguish the difference between a real dynamic system and a lumped parameter representation/model

2. Create a FBD of a plane motion, rigid body including declaring variables of motion and coordinate frames

3. Use 1-sketch FBDs

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