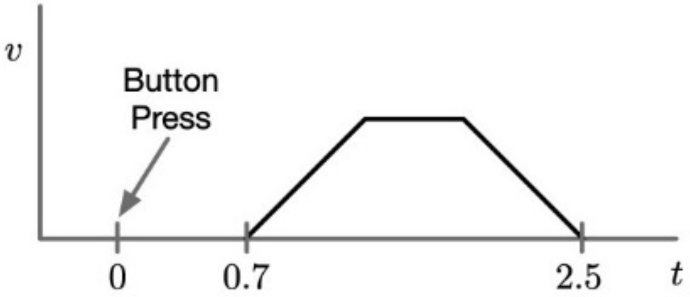
Discontinuous Functions - Motivation

1. **Fielding a dynamic system**. E.g. operator pushes a button, defining t=0. This triggers a pre-planned sequence of commands that are sent to the system.



2. **Designing a dynamic system**. You may be responsible for designing the input function to meet motion requirements.

   - Represent the input mathematically
   - Calculate or simulate the system response, iterate as needed

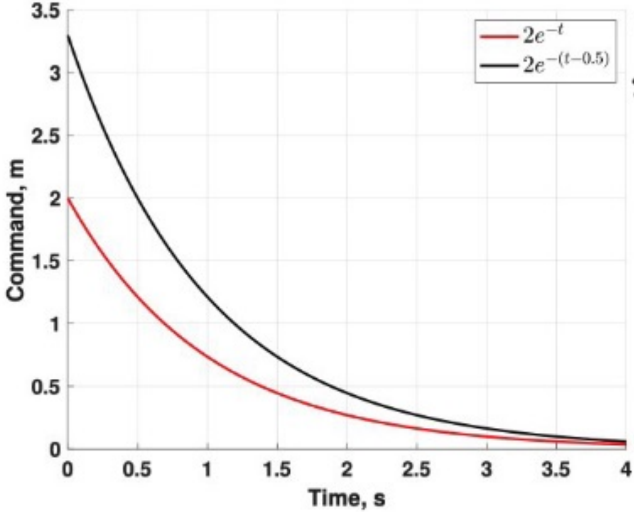
Skills

1. Identify time delayed functions
2. Given a discontinuous function **equation,** sketch it.
3. Given a discontinuous function **sketch**, create its equation
4. Write and use the definition of the **impulse**, **step** and **ramp** functions
5. Given a MATLAB dynamic system model create its **step and impulse responses**

Terminology

Unit, Unity: Another way of saying “one,” e.g. unit step, unity gain.

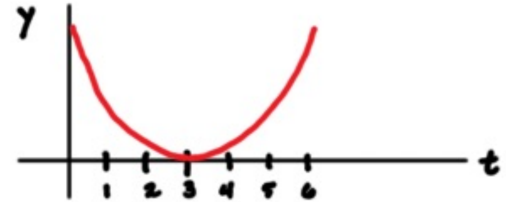
Time Delayed Functions



1. $2e^{-t}$ is the "undelayed" version of $2e^{-(t-.5)}$
2. The time delay is 0.5 sec.

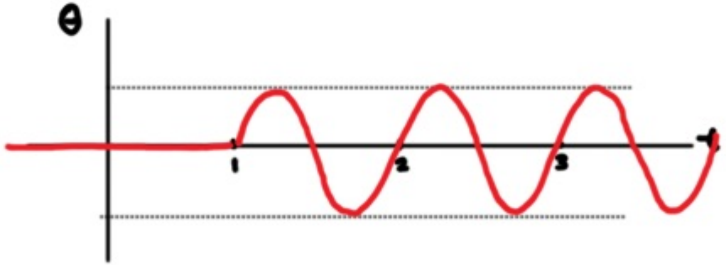
Identify Time-Delayed Functions - Examples

$y = (t-3)^2$



$$\theta = \begin{cases} 0 & t < 1 \\ \sin\left(2\pi(t-1)\right) & t \geq 1 \end{cases}$$

1 Hz



'starts at t=4'

$$r = \begin{cases} 0 & t < 4 \\ 3te^{-(t-4)} & t \geq 4 \end{cases}$$

$3te^{-t}e^{4} = 3e^{4}te^{-t}$

Note: e^{-t} ALWAYS dominates t^N.
E.g. $t^{100}e^{-.1t} \to 0$ as $t \to \infty$.

Impulse, Step and Ramp Functions

Math

$$\int_{-\infty}^{\infty} f(t)\,\delta(t-t_0)\,dt = f(t_0)$$

Not a fn. δ is <u>defined</u> by this property. Used to model dyn. sys: impulse response.

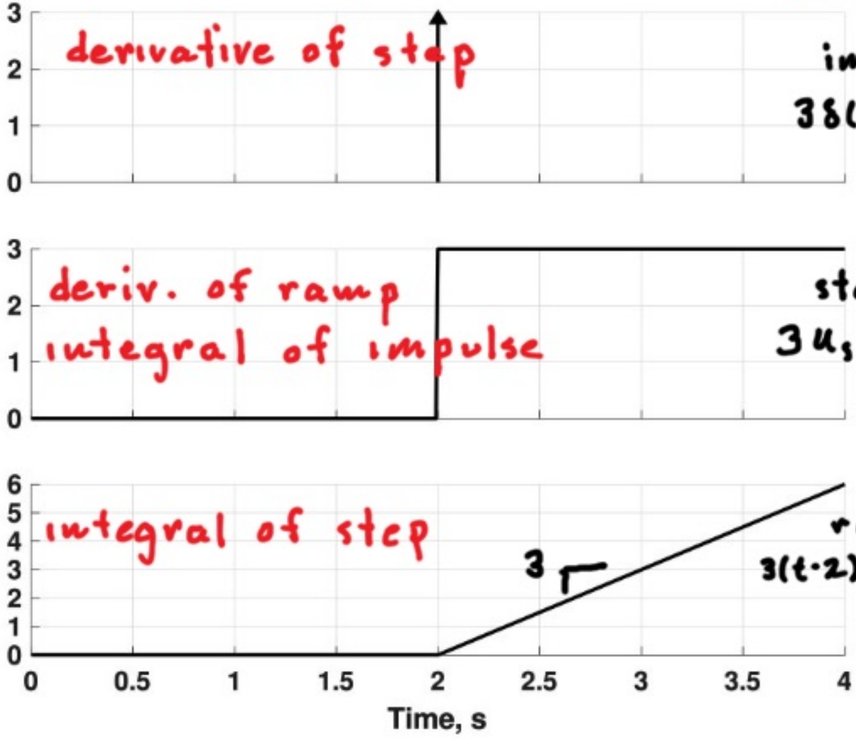
$$u_s(t-t_0) = \begin{cases} 0 & t < t_0 \\ 1 & t \geq t_0 \end{cases}$$

Great diagnostic input for a dynamic system: step response.

$$m(t-t_0)u_s(t-t_0) = \begin{cases} 0 & t < t_0 \\ m(t-t_0) & t \geq t_0 \end{cases}$$

1. Slope is the coefficient
2. Nice dyn. sys. input for 'gentle' transitions.

Relationships



Notation

impulse
$3\delta(t-2)$

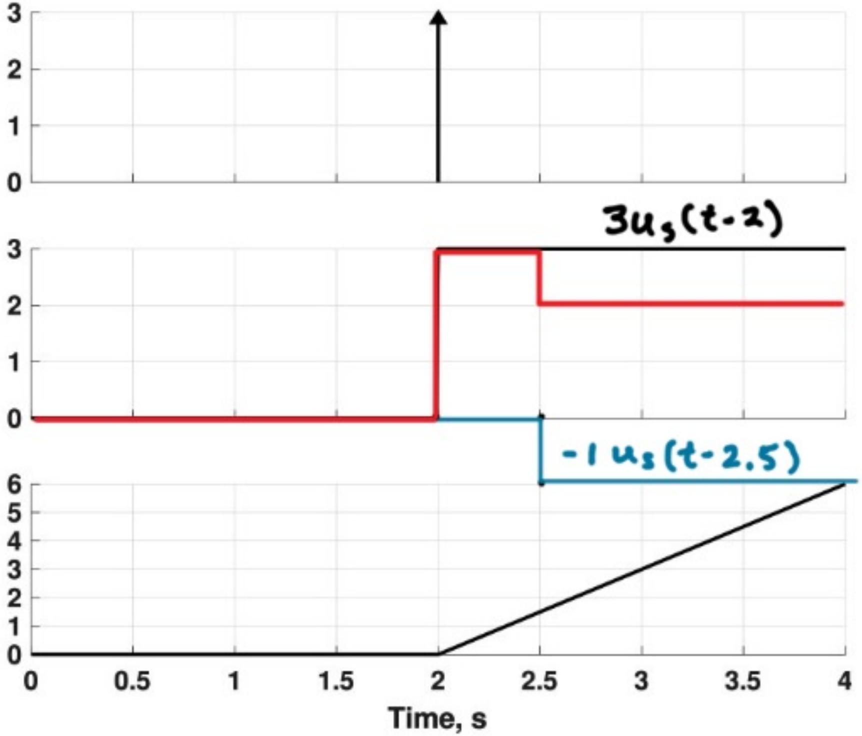
step
$3u_s(t-2)$

ramp
$3(t-2)u_s(t-2)$

Impulse, Step and Ramp Functions

create exotic fns by adding steps & ramps.

$r = 3u_s(t-2) - 1\,u_s(t-2.5)$



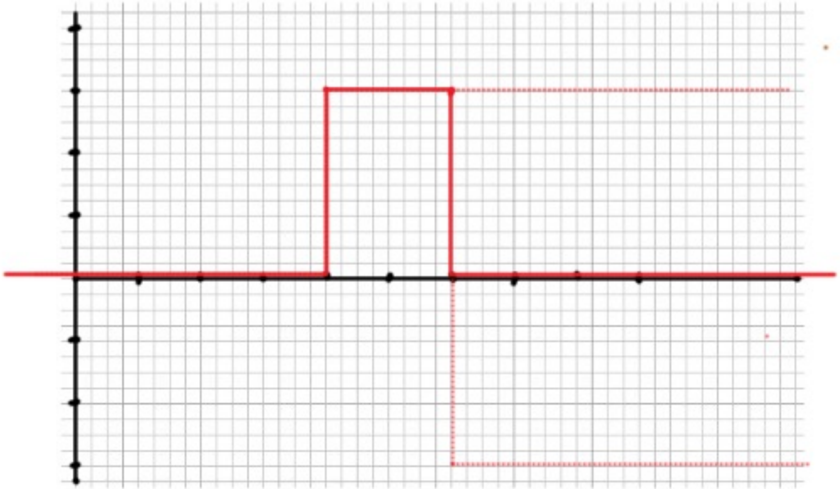
MATLAB and Simulink

1. The `step` and `impulse` commands create step and impulse responses of a dynamic system object
2. The `heaviside` function (named for Oliver Heaviside, 1850-1925, UK) can be used for symbolic operations, or to create an array of numeric step data. By default, `heaviside(0)` = 0.5. It really doesn't matter what value you assign it at t=0.
3. The `dirac` function (named for Paul Dirac, 1902-1984, UK) is mostly used for symbolic work.
4. The Simulink Library Browser, Sources folder has both step and ramp blocks. Clever addition and multiplication of these, with other blocks, can make exotic inputs.

Example 1

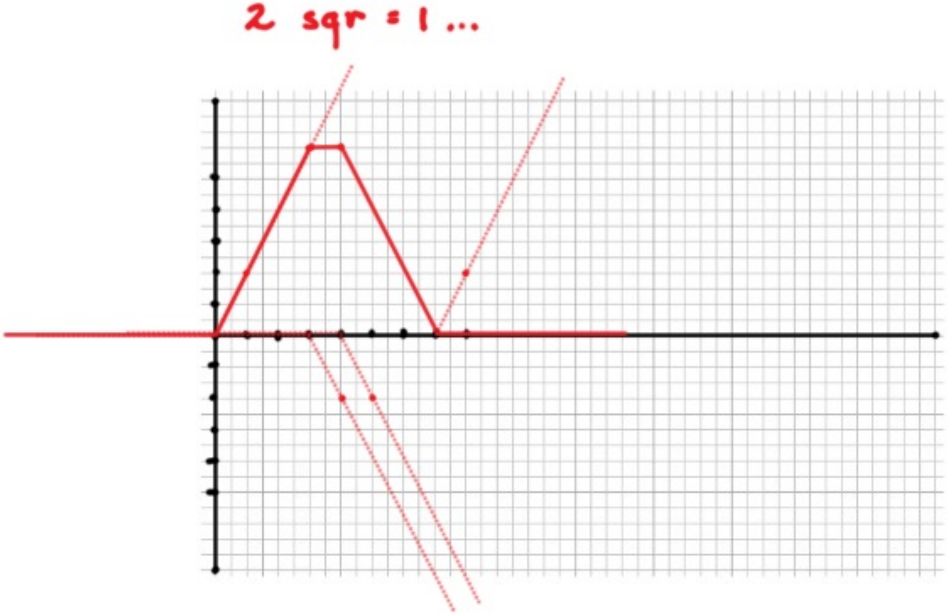
Sketch the function below and model it in Simulink.

$g(t) = 3u_s(t-4) - 3u_s(t-6)$ 4 sqr = 1 ...



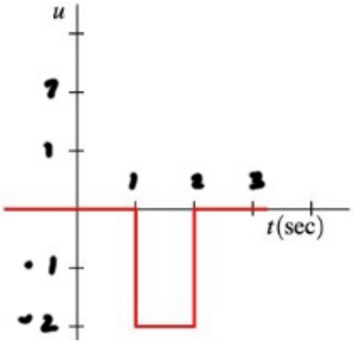
Example 2

Sketch the function: $y(t) = 2tu_s(t) - 2(t-3)u_s(t-3) - 2(t-4)u_s(t-4) + 2(t-7)u_s(t-7)$



Example 3

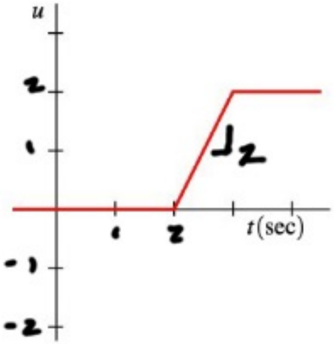
Write an expression for the function shown at right.



$$u = -2\,u_s(t-1) + 2\,u_s(t-2)$$

Example 4

Write an expression for the function shown at right and model it in Simulink.



$$u = 2(t-2)u_s(t-2) - 2(t-3)u_s(t-3)$$

Example 5

Given a MATLAB dynamic system, create its impulse and step responses. For now, you don't need to know how to create the MATLAB dynamic system - assume it's given.

```
>> mySys = tf(7,[1 1 5]);
>> figure(1); impulse(mySys); grid
>> figure(2); step(mySys); grid
```

Summary

- Recognize time-delayed functions, and when possible, extract out the "undelayed" function
- Use the definition of step and ramp functions to create and/or sketch complicated discontinuous functions
- Model discontinuous functions in Simulink
- Given a MATLAB dynamic system, create its step or impulse response