Energy - Motivation

When dynamic systems operate, energy is transferred between components and is dissipated. How does it fit into analysis and design?

1. Energy and power are the common currencies across all engineering fields. they provide understanding of how all system components interact

2. Power is the time derivative of energy. Knowing how much power a component needs aids in sizing it and the system overall

3. Conservative systems do not dissipate energy. A vast number of analysis and design methods can be applied to them

4. It's the cornerstone of advanced dynamic system analysis

Energy - Skills

1. Calculate the total energy of a mechanical system
2. For a conservative system
   a. Use its energy expression to determine natural frequency
   b. Use a tiny bit of information for its impulse response to determine its natural frequency

Background

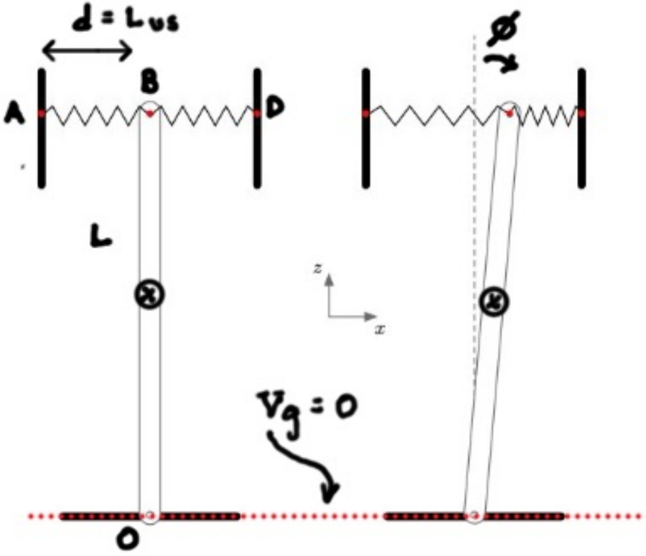
1. Total mechanical energy is the sum of PE and KE
2. A dynamic system whose external forces that do work are conservative (e.g. gravity and springs) is called conservative and its total energy, E, is constant for all time. If it starts moving, its moves forever.
3. If a system is conservative, then
    1. $\dot{E} = 0$
    2. If it's second order, then its motion is **undamped**, also called **harmonic**, and its natural frequency can be obtained from just two samples of data - the maximum value of its displacement and the maximum value of its velocity

$$\omega_n = \frac{\dot{x}_{max}}{x_{max}}$$

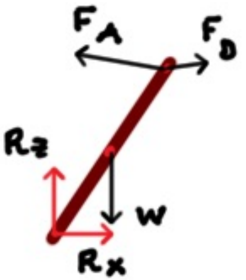
Example

Motion Var: ϕ, No Constraints, 1 DOF

Consider the homogenous rod that is free to rotate about O and is attached to side walls by two equal springs. Assuming small oscillation, calculate its total energy and then use it to: (1) estimate its natural frequency and (2) to create its DEQ model WITHOUT using $\sum M = I_o\ddot{\phi}$. Since the rods length is much larger than its radius, we can assume it to be "thin" and $I_c = mL^2/12$.



Conservative? FBD



The forces that do work, F_A, F_D, & W, are conservative. It is a cons. system.

$$\boxed{T = \frac{1}{2}mv_c^2 + \frac{1}{2}I_c\dot{\phi}^2 = \frac{1}{2}I_o\dot{\phi}^2}$$

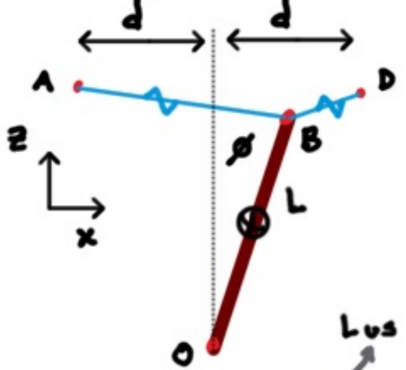
$$I_o = \frac{1}{12}mL^2 + m\left(\frac{L}{2}\right)^2 = \frac{1}{3}mL^2$$

$$V = V_s + V_g$$

$$\boxed{V_s = \underbrace{\frac{1}{2}k\Delta_1^2}_{\text{(left)}} + \underbrace{\frac{1}{2}k\Delta_2^2}_{\text{(right)}}}$$

$$V_g = mg\cdot\frac{L}{2}\cos\phi$$

Example - continued



Lus

$\Delta_1 = |\mathbf{r}_{B/A}| - d$, $\quad \mathbf{r}_{B/A} = \mathbf{r}_B - \mathbf{r}_A$

$\mathbf{r}_B = L\begin{Bmatrix} s\phi \\ 0 \\ c\phi \end{Bmatrix}$, $\mathbf{r}_A = \begin{Bmatrix} -d \\ 0 \\ L \end{Bmatrix}$

$\mathbf{r}_{B/A} = \begin{Bmatrix} Ls\phi + d \\ 0 \\ Lc\phi - L \end{Bmatrix} \approx \begin{Bmatrix} L\phi + d \\ 0 \\ 0 \end{Bmatrix}$

$$\boxed{|\mathbf{r}_{B/A}| \approx |L\phi + d|, \quad \Delta_1 \approx |L\phi|}$$

Build $E(\phi)$ Expression

$$\boxed{\begin{aligned} V_s &= \tfrac{1}{2}k\Delta_1^2 + \tfrac{1}{2}k\Delta_2^2 \\ V_g &= mg\cdot\tfrac{L}{2}\cos\phi \end{aligned}}$$

Same workflow for Δ_2, point D ...

$\Delta_2 \approx |L\phi|$

$$\boxed{\therefore V_s = kL^2\phi^2 \;\&\; V_g \approx \frac{1}{2}mgL}$$

$$T = \frac{1}{2}I_o\dot{\phi}^2 = \boxed{\frac{1}{2}\left(\frac{1}{3}mL^2\right)\dot{\phi}^2 = \frac{1}{6}mL^2\dot{\phi}^2}$$

$$E \approx \frac{1}{2}mgL + \frac{1}{6}mL^2\dot{\phi}^2 + kL^2\phi^2$$

Since mgL is constant & E is const., define E_o as

$$E_o = E - \frac{1}{2}mgL = \boxed{\frac{1}{6}mL^2\dot{\phi}^2 + kL^2\phi^2}$$

Example - continued

DEQ Model & Natural Frequency of Vibration

$$E_0 = \frac{1}{6} mL^2 \dot{\phi}^2 + KL^2 \phi^2$$

cons. sys: $\dot{E}_0 = 0$, so $\boxed{\frac{1}{3} mL^2 \dot{\phi}\ddot{\phi} + 2KL^2 \dot{\phi}\phi = 0}$

or, $\dot{\phi}\left(\frac{1}{3} mL^2 \ddot{\phi} + 2KL^2 \phi\right) = 0$ (1) $t = 0 \ldots \infty$

since $\dot{\phi}(t)$ will fluctuate, the only way to ensure (1) is: $\frac{1}{3} mL^2 \ddot{\phi} + 2KL^2 \phi = 0$. This is the DEQ model!!

Or

$$\boxed{\ddot{\phi} + \frac{6K}{m}\phi = 0}$$

Recall: $\ddot{\phi} + 2\zeta\omega_n \dot{\phi} + \omega_n^2 \phi = K_{dc}\,\omega_n^2 u$

So, the sys. is undamped & $\omega_n = \boxed{\sqrt{\frac{6K}{m}}}$

Example - continued

ϕ_{max}, $\dot{\phi}_{max}$, & ω_n

$$E_0 = \frac{1}{6} m L^2 \dot{\phi}^2 + K L^2 \phi^2$$

The rod vibrates forever. Let's guess:

$$\phi = \phi_0 \cos \omega_n t \longrightarrow \dot{\phi} = \boxed{-\phi_0 \omega_n \sin \omega_n t}$$

Sub. into E_0: $\boxed{E_0 = \frac{1}{6} m L^2 \phi_0^2 \omega_n^2 \sin^2 \omega_n t + K L^2 \phi_0^2 \cos^2 \omega_n t}$

We know E_0 = const. It doesn't look constant, unless... we can find ω_n so,

$$E_0 = E_0 \sin^2 \omega_n t + E_0 \cos^2 \omega_n t$$

$$E_0 = \frac{1}{6} m L^2 \phi_0^2 \omega_n^2 = K L^2 \phi_0^2 \longrightarrow \omega_n^2 = 6 \frac{K}{m}$$

Imagine an experiment: Rotate the rod by ϕ_0, release. The rod vibrates with $\phi_{max} = \phi_0$ & $\dot{\phi}_{max} = \phi_0 \omega_n$. You know $\phi_{max} = \phi_0$. Carefully measure $\dot{\phi}_{max}$.

$$\omega_n = \frac{\dot{\phi}_{max}}{\phi_{max}}$$

Example - continued

Summary

1. You've calculate the total energy of a conservative system. You'll soon get more experience doing this for nonconservative systems…
2. For a second oder conservative system you can…
    a. manipulate its energy expression to obtain its DEQ model and…
    b. calculate its natural frequency

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