Dynamics Practice - Skills

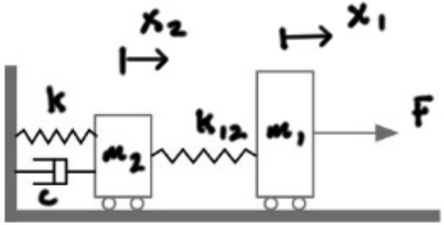
Practice the concepts developed since the beginning of class to analyze dynamic systems including

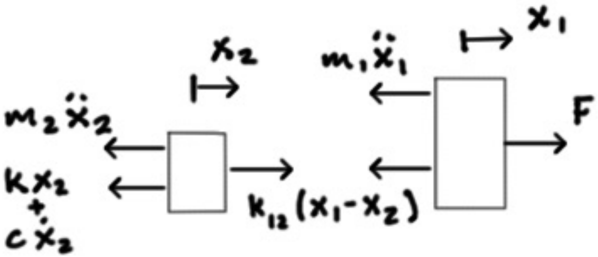
1. Kinematics
2. Principles of Dynamics
3. Constraint Equations
4. Equilibrium
5. Energy analysis
6. FBDs
7. Springs and Dampers
8. Simulation
9. Response Characteristics

Example 1

2 DOFs

Create a state space model, implement it into MATLAB/ Simulink and generate its unit step response.





$$m_1\ddot{x}_1 + K_{12}(x_1-x_2) = F$$

$$m_2\ddot{x}_2 + c\dot{x}_2 + Kx_2 - K_{12}(x_1-X_2) = 0$$

$$\ddot{x}_1 + \frac{K_{12}}{m_1}(x_1-x_2) = \frac{1}{m_1}F$$

$$\ddot{x}_2 + \frac{c}{m_2}\dot{x}_2 + \frac{K}{m_2}x_2 - \frac{K_{12}}{m_2}(x_1-X_2) = 0$$

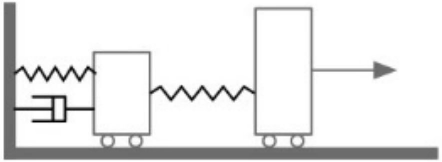
$$\underline{\dot{z}} = \underline{\underline{A}}\,\underline{z} + \underline{\underline{B}}\,\underline{u}, \quad \underline{y} = \underline{\underline{C}}\,\underline{z} + \underline{\underline{D}}\,\underline{u}$$

$$\begin{aligned}
z_1 &= x_1 & \dot{z}_1 &= z_3 \\
z_2 &= x_2 & \dot{z}_2 &= z_4 \\
z_3 &= \dot{x}_1 & \dot{z}_3 &= \ddot{x}_1 = -\frac{K_{12}}{m_1}z_1 + \frac{K_{12}}{m_1}z_2 + \frac{1}{m_1}F \\
z_4 &= \dot{x}_2 & \dot{z}_4 &= \ddot{x}_2 = -\frac{c}{m_2}z_4 - \frac{K}{m_2}z_2 + \frac{K_{12}}{m_2}(z_1-z_2)
\end{aligned}$$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{K_{12}}{m_1} & \frac{K_{12}}{m_1} & 0 & 0 \\ \frac{K_{12}}{m_2} & -\frac{(K_{12}+K)}{m_2} & 0 & -\frac{c}{m_2} \end{bmatrix} \quad \underline{\underline{B}} = \begin{Bmatrix} 0 \\ 0 \\ 1/m_1 \\ 0 \end{Bmatrix}$$

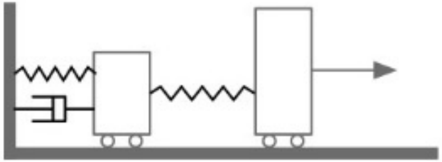
Example 1

Create a state space model, implement it into MATLAB/
Simulink and generate its unit step response.



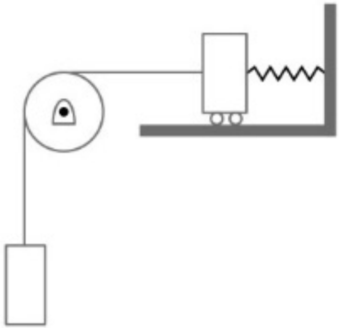
Example 1

Create a state space model, implement it into MATLAB/ Simulink and generate its unit step response.



Example 2

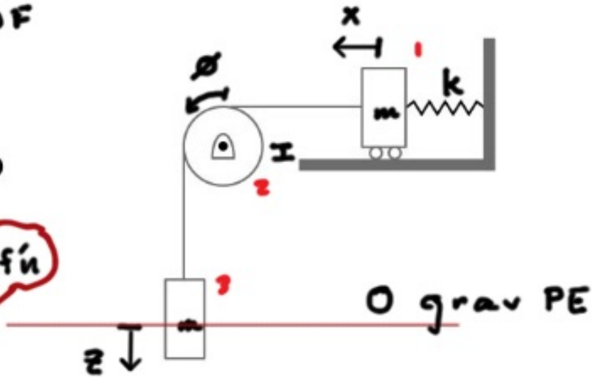
Create an energy expression and determine is natural frequency. Then, create a state space model, implement it into MATLAB/Simulink and generate its initial condition response.



Example 2

3 motion vars: x, ϕ, z ; 2 constraints: $r\phi = x$, $x = z$ } 1 DOF

Create an energy expression and determine is natural frequency. Then, create a state space model, implement it into MATLAB/Simulink and generate its initial condition response.



$$E = T + V = T_1 + T_2 + T_3 + V_{spr} + V_{grav}$$

included via S.E. def'n of x y z.

$$E = \tfrac{1}{2} m \dot{x}^2 + \tfrac{1}{2} I \dot{\phi}^2 + \tfrac{1}{2} m \dot{z}^2 + \tfrac{1}{2} k x^2$$

$$= m \dot{z}^2 + \tfrac{1}{2} \cdot \frac{I}{r^2} \dot{z}^2 + \tfrac{1}{2} k z^2 = \left(m + \frac{I}{2r^2}\right) \dot{z}^2 + \tfrac{1}{2} k z^2$$

$$\dot{E} = 2\left(m + \frac{I}{2r^2}\right) \dot{z} \ddot{z} + k z \dot{z} = 0$$

$$2\left(m + \frac{I}{2r^2}\right) \ddot{z} + k z = 0$$

$\zeta = 0$, undamped

$$\omega_n = \sqrt{k \Big/ 2\left(m + \frac{I}{2r^2}\right)}$$

$$\ddot{z} + \omega_n^2 z = 0$$

$x_1 = z \qquad \dot{x}_1 = x_2$

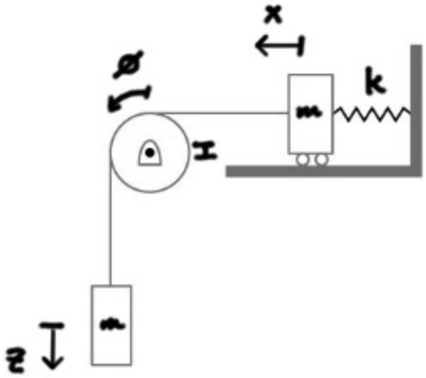
$x_2 = \dot{z} \qquad \dot{x}_2 = \ddot{z} = -\omega_n^2 x_1$

$$\dot{\underline{x}} = \underline{\underline{A}}\,\underline{x} + \underline{\underline{B}}\,\underline{u}, \quad \underline{y} = \underline{\underline{C}}\,\underline{x} + \underline{\underline{D}}\,\underline{u}$$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & 0 \end{bmatrix} \quad \underline{\underline{B}} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

Example 2

Create an energy expression and determine is natural frequency. Then, create a state space model, implement it into MATLAB/Simulink and generate its initial condition response.



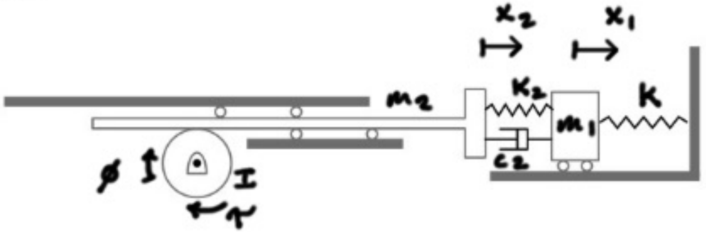
Example 3

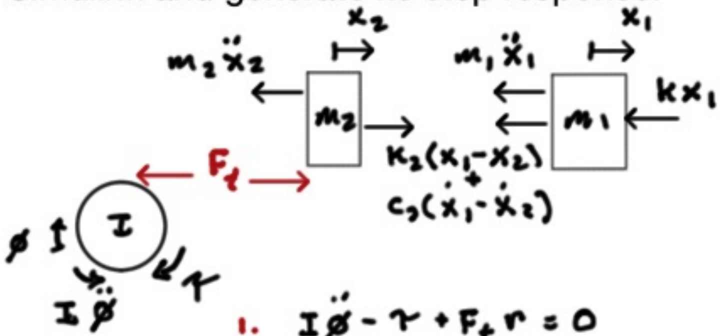
3 vars

1 constraint: $r\phi = x_2$

2 DOFs

Create a state space model, implement it into MATLAB/Simulink and generate its step response.





1. $I\ddot{\phi} - \tau + F_t r = 0$
2. $m_2\ddot{x}_2 - F_t - c_2(\dot{x}_1-\dot{x}_2) - k_2(x_1-x_2) = 0$
3. $m_1\ddot{x}_1 + c_2(\dot{x}_1-\dot{x}_2) + k_2(x_1-x_2) + kx_1 = 0$

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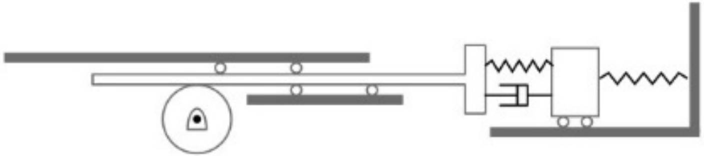
2. $m_2 r\ddot{\phi} - c_2(\dot{x}_1 - r\dot{\phi}) - k_2(x_1 - r\phi) = F_t$
1. $I\ddot{\phi} + m_2 r^2\ddot{\phi} - rc_2(\dot{x}_1 - r\dot{\phi}) - rk_2(x_1 - r\phi) = \tau$
3. $m_1\ddot{x}_1 + c_2(\dot{x}_1 - r\dot{\phi}) + k_2(x_1 - r\phi) + kx_1 = 0$

Example 3

Create a state space model, implement it into MATLAB/ Simulink and generate its step response.

$$\dot{\underline{z}} = \underline{\underline{A}}\,\underline{z} + \underline{\underline{B}}\,\underline{u}, \quad \underline{y} = \underline{\underline{C}}\,\underline{z} + \underline{\underline{D}}\,\underline{u}$$



$$1.\quad \underbrace{I\ddot{\phi} + m_2 r^2 \ddot{\phi}}_{I_{eq}\ddot{\phi}} - r c_2(\dot{x}_1 - r\dot{\phi}) - r k_2 (x_1 - r\phi) = \tau$$

$$3.\quad m_1 \ddot{x}_1 + c_2(\dot{x}_1 - r\dot{\phi}) + k_2(x_1 - r\phi) + k x_1 = 0$$

$$z_1 = \phi \qquad \dot{z}_1 = z_3$$

$$z_2 = x_1 \qquad \dot{z}_2 = z_4$$

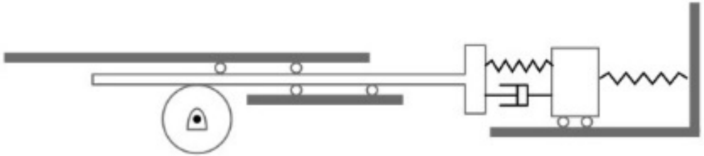
$$z_3 = \dot{\phi} \qquad \dot{z}_3 = \ddot{\phi} = \frac{r c_2}{I_{eq}}(z_4 - r z_3) + \frac{r k_2}{I_{eq}}(z_2 - r z_1) + \frac{1}{I_{eq}}\tau$$

$$z_4 = \dot{x}_1 \qquad \dot{z}_4 = \ddot{x}_1 = -\frac{c_2}{m_1}(z_4 - r z_3) - \frac{k_2}{m_1}(z_2 - r z_1) - \frac{k}{m_1} z_2$$

$$\underline{\underline{A}} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{-k_2 r^2}{I_{eq}} & \frac{k_2 r}{I_{eq}} & \frac{-c_2 r^2}{I_{eq}} & \frac{c_2 r}{I_{eq}} \\ \frac{k_2 r}{m_1} & \frac{-(k+k_2)}{m_1} & \frac{c_2 r}{m_1} & \frac{-c_2}{m_1} \end{bmatrix} \quad \underline{\underline{B}} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{I_{eq}} \\ 0 \end{bmatrix}$$

Example 3

Create a state space model, implement it into MATLAB/
Simulink and generate its step response.

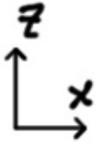


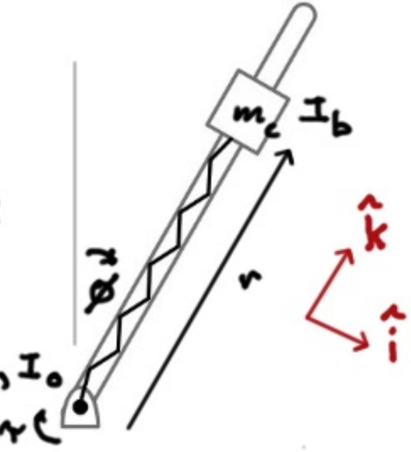
Horizontal Plane

Example 4

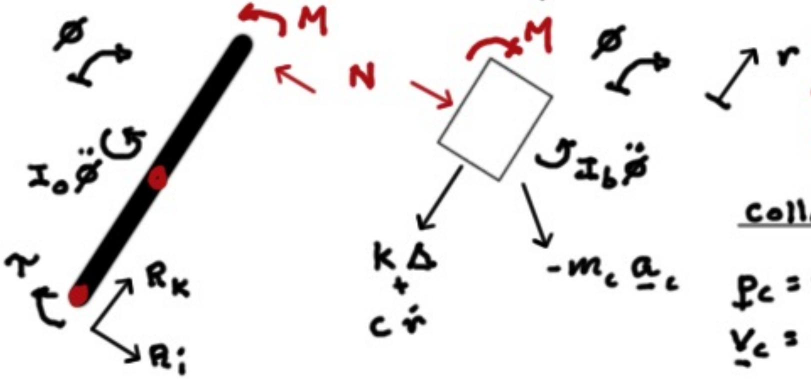
2 mo vars, 0 constraint, 2 DOFs

The internal spring also has some damping. Create a state space model, implement it into MATLAB/Simulink and generate its step response.





Discussion: Is the collar a point mass, or a body?



What causes collar rotation?

Rod applies a moment.

Collar Kinematics

$$\underline{p}_c = r\hat{k}$$

$$\underline{v}_c = \dot{r}\hat{k} + \underline{\omega}\times\underline{p}_c, \quad \underline{\omega} = \dot{\phi}\hat{j}$$

$$\underline{v}_c = \dot{r}\hat{k} + r\dot{\phi}\hat{i}$$

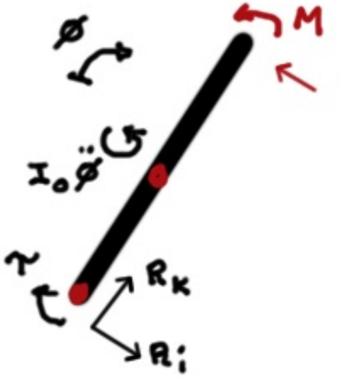
$$\underline{a}_c = \ddot{r}\hat{k} + \dot{r}\dot{\phi}\hat{i} + r\ddot{\phi}\hat{i} + \underline{\omega}\times\underline{v}_c$$

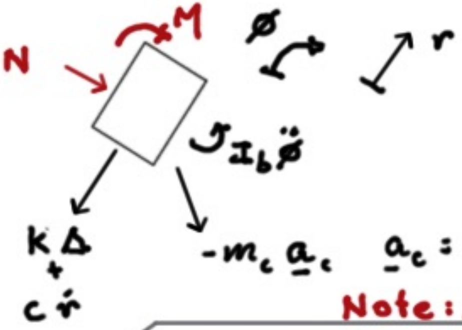
$$= (\dot{r}\dot{\phi} + r\ddot{\phi})\hat{i} + \ddot{r}\hat{k} + \dot{r}\dot{\phi}\hat{i} - r\dot{\phi}^2\hat{k}$$

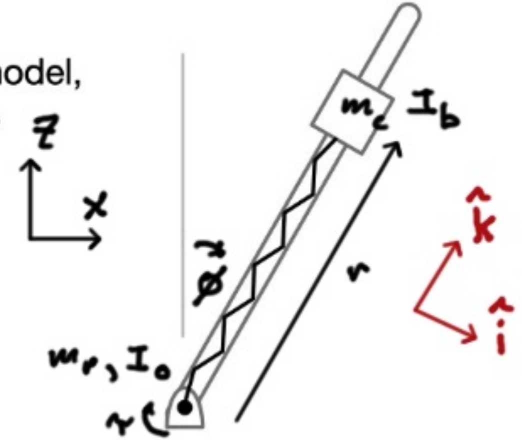
$$\underline{a}_c = (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{i} + (\ddot{r} - r\dot{\phi}^2)\hat{k}$$

Example 4

The internal spring also has some damping. Create a state space model, implement it into MATLAB/Simulink and generate its step response.







$$\underline{a}_c = (2\dot{r}\dot{\phi} + r\ddot{\phi})\hat{i} + (\ddot{r} - r\dot{\phi}^2)\hat{k}$$

Note: $\Delta = r - L_{us}$

i) $N - m_c(2\dot{r}\dot{\phi} + r\ddot{\phi}) = 0$

k) $-k(r - L_{us}) - c\dot{r} - m_c(\ddot{r} - r\dot{\phi}^2) = 0$

ϕ) $M - I_c\ddot{\phi} = 0$

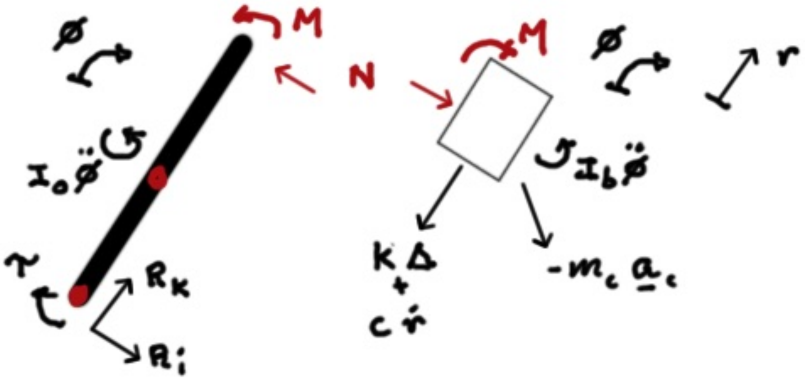
Collar Dynamics

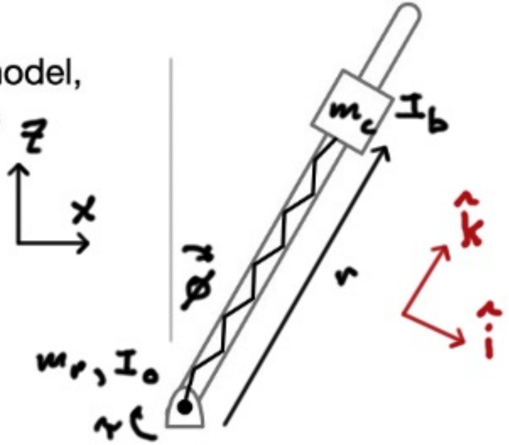
$\Sigma \underline{F} = m\underline{a}_c$

$\Sigma M_{cm} = I_c\ddot{\phi}$

Example 4

The internal spring also has some damping. Create a state space model, implement it into MATLAB/Simulink and generate its step response.





i) $N - m_c(2\dot{r}\dot{\phi} + r\ddot{\phi}) = 0$

k) $-k(r - L_{us}) - c\dot{r} - m_c(\ddot{r} - r\dot{\phi}^2) = 0$

ϕ) $M - I_c\ddot{\phi} = 0$

Rod Dynamics

$\Sigma \underline{F} = m\underline{a}_r \leftarrow$ only if we want R_k, R_i

$\Sigma M_o = I_o\ddot{\phi}$

ΣM_o) $\tau - I_o\ddot{\phi} - M - Nr = 0$

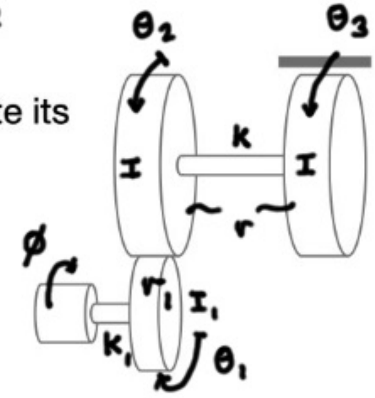
$$\boxed{\begin{aligned} &(I_o + I_c + m_c r^2)\ddot{\phi} - m_c r(2\dot{r}\dot{\phi}) = \tau \quad 1.\\ \text{k)}\quad &m_c\ddot{r} + k(r - L_{us}) + c\dot{r} - m_c r\dot{\phi}^2 = 0 \quad 2.\end{aligned}}$$

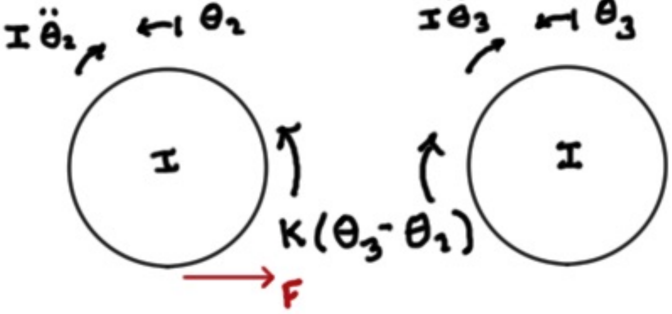
Example 5

3 vars: $\theta_1, \theta_2, \theta_3$ 1 constraint: $r_1 \theta_1 = r \theta_2$

2 DOF

Create a state space model, implement it into MATLAB/Simulink and generate its step response.





ϕ is prescribed: input

$$I_1 \ddot{\theta}_1 - k_1(\phi - \theta_1) - F r_1 = 0 \quad 1$$

$$I \ddot{\theta}_2 - K(\theta_3 - \theta_2) + F r = 0 \quad 2$$

$$I \ddot{\theta}_3 + K(\theta_3 - \theta_2) = 0 \quad 3$$

$\theta_2 = \frac{r_1}{r}\theta_1$ Combine 1 & 2

2. $F = \frac{-1}{r}\left(I \frac{r_1}{r} \ddot{\theta}_1 - K\left(\theta_3 - \frac{r_1}{r}\theta_1\right)\right)$

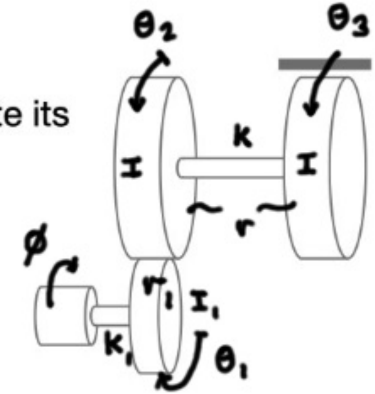
1. $I_1 \ddot{\theta}_1 - K_1(\phi - \theta_1) + \left(\frac{r_1}{r}\right)^2 I \ddot{\theta}_1 - \frac{r_1}{r} K\left(\theta_3 - \frac{r_1}{r}\theta_1\right) = 0$

$$\boxed{\begin{aligned} &\left(I_1 + \left(\tfrac{r_1}{r}\right)^2\right)\ddot{\theta}_1 + \left(K_1 + \left(\tfrac{r_1}{r}\right)^2 K\right)\theta_1 = K_1 \phi \\ &I \ddot{\theta}_3 + K\left(\theta_3 - \tfrac{r_1}{r_2}\theta_1\right) = 0 \end{aligned}}$$

Example 5

$$r_1\theta_1 = r\theta_2$$

Create a state space model, implement it into MATLAB/Simulink and generate its step response.



$$E = \frac{1}{2}k_1(\theta_1-\phi)^2 + \frac{1}{2}k(\theta_3-\theta_2)^2 + \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}I\dot{\theta}_2^2 + \frac{1}{2}I\dot{\theta}_3^2$$

$$= \frac{1}{2}k_1(\theta_1-\phi)^2 + \frac{1}{2}k\left(\theta_3-\frac{r_1}{r}\theta_1\right)^2 + \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}I\cdot\frac{r_1}{r}\dot{\theta}_1^2 + \frac{1}{2}I\dot{\theta}_3^2$$

$$\dot{E} = 0$$

$$\dot{E} = k_1(\theta_1-\phi)(\dot{\theta}_1-\dot{\phi}) + k\left(\theta_3-\frac{r_1}{r_2}\theta_1\right)\left(\dot{\theta}_3-\frac{r_1}{r_2}\dot{\theta}_1\right) + \cdots$$

If it was 1 DOF, $\dot{E}=0 \rightarrow$ DEQ model.

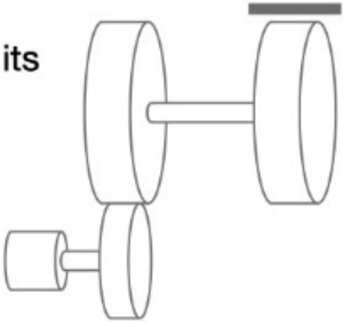
There is an energy approach for DEQ generation: Lagrange's Equation

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i} = Q_i \qquad L \equiv T - V$$

$Q_i \equiv$ force & torque inputs

Example 5

Create a state space model, implement it into MATLAB/Simulink and generate its step response.



Example 5

Create a state space model, implement it into MATLAB/Simulink and generate its step response.