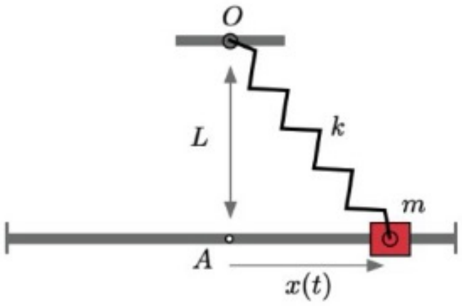
What is this Dynamic Systems course?

Course catalog: "This course deals with the modeling, analysis and control of mixed physics systems. It covers differential equation generation for mechanical, thermal, and electrical systems, their simulation, and methods for analyzing their performance operating in both open and closed loop."



$$m\ddot{x}(t) + kx(t)\left(1 - \frac{L}{\sqrt{L^2 + x(t)^2}}\right) = 0$$

Interpretation: Dynamics meets differential equations…

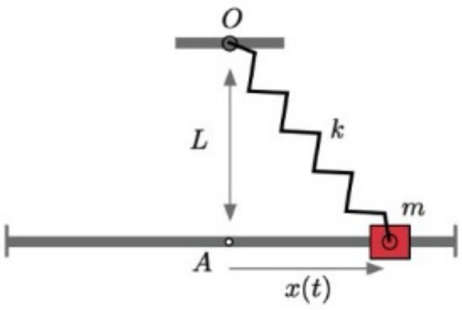
Dynamic System Definition: One or more coupled components (robot arm, motor, water tank) whose "state" (position, velocity, current, temperature), changes in response to external stimuli AND engineering requirements describing how it should behave.

Skills

1. Define and recognize a dynamic system
2. Focused review of dynamics - FBD, conservation of energy, integration of acceleration
3. Introduction to vibration analysis - calculate the natural frequency of a 2nd order, undamped system
4. Describe how a closed loop system differs from an open loop system in appearance and performance using a block diagram
5. See a dynamic system in action

Example: Contrast Between Dynamics and Dynamic Systems

Consider the collar of mass m connected to point O by a spring of stiffness k that slides without friction on a rod. The unstretched length of the spring is L. If the mass is initially released from rest at some distance x0 from A…

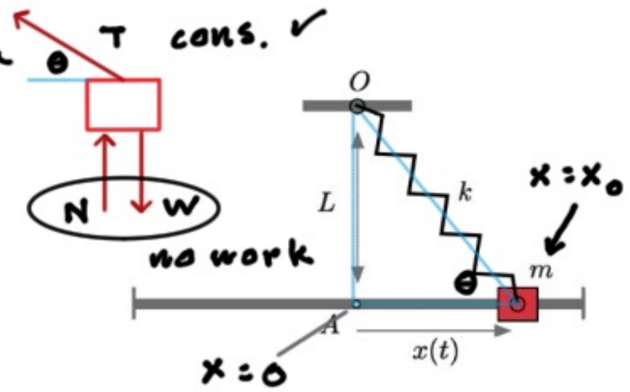


Dynamics Course: … find the collar's velocity as it passes point A.

Dynamic Systems Course: … develop a differential equation model of the system and approximate the collar's vibration frequency (Hz) in its ensuing motion when x0 << L. Next, add a thruster to the collar and develop a stable control system so that it comes to rest at point A (x=0) in 1 second. Size the thruster.

Example: Dynamics Approach

Free Body Diagram

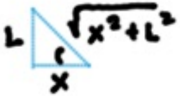


1. Conservation of Total Mechanical Energy
2. Direct Integration of Acceleration Expression

Sys. is conservative : $T_1 + V_1 = T_2 + V_2$

configurations: 1) $x = x_0, v = 0$, 2) $x = 0, v_A = ?$

In general: $T = \frac{1}{2} m v^2$, $V = \frac{1}{2} k \Delta^2$ ← spring stretch $\Delta = ?$



$\Delta = \sqrt{x^2+L^2} - L$

Apply cons. of nrg: $\frac{1}{2} m v_A^2 = \frac{1}{2} k \left(\sqrt{x_0^2+L^2} - L\right)^2$

Simplify: $v_A^2 = \frac{k}{m}\left(\sqrt{x_0^2+L^2} - L\right)^2$, $v_A = \sqrt{\frac{k}{m}}\left(\sqrt{x_0^2+L^2} - L\right)$

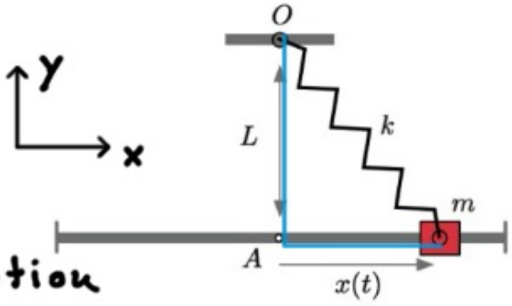
check : Let $x_0 = 0 \rightarrow$ eqn. should indicate no motion. $v_A = \sqrt{\frac{k}{m}}\left(\sqrt{L^2 - L^2} - L\right) = 0$ ✓

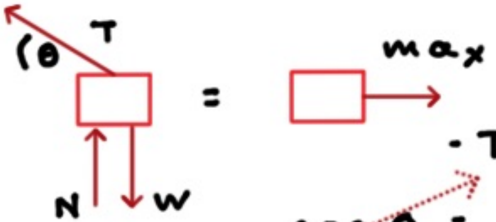
Example: Dynamics Approach

Direct Integration of Acceleration Expression

$a = v\frac{dv}{dx}$, $a(x)dx = v\,dv$, $\int_{x_0}^{0} a(x)dx = \int_{0}^{v} v\,dv$,

$\frac{1}{2}v^2 = \int_{x_0}^{0} a(x)dx$. create $a(x)$ ... FBD

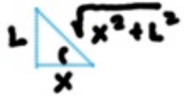




Newton's Law of Motion

$\Sigma F_x = ma_x$, $\Sigma F_y = ma_y = 0$

$-T\cos\theta = ma_x$, $T\sin\theta + N - W = 0$



$\cos\theta = \frac{x}{\sqrt{x^2+L^2}}$, $T = k\Delta = k(\sqrt{x^2+L^2} - L)$

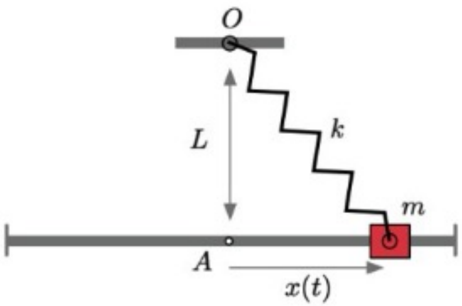
Sub. info $\quad ma_x = -k(\sqrt{x^2+L^2} - L)\frac{x}{\sqrt{x^2+L^2}}$

The $a(x)$ we need is $\div m$! $\quad$ A nasty fn. of x.

Integrate & clever manipulation gives v.

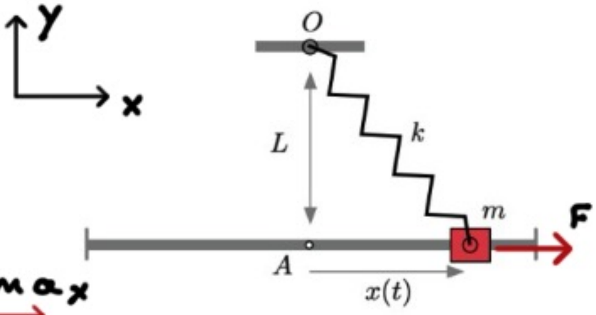
Example: Dynamics Approach

Direct Integration of Acceleration Expression (cont'd)



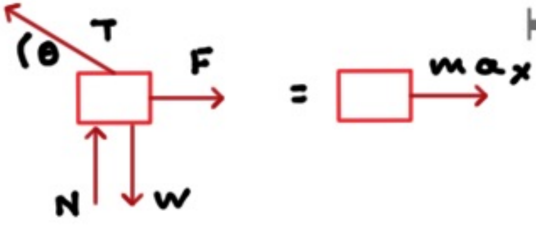
Example: Dynamics Systems Approach and Simulation

Obtain a differential equation model, then analyze it. Similar to previous example (direct integration), but no need to integrate - much easier!



$$m\ddot{x} + kx\left(1 - \frac{L}{\sqrt{L^2 + x^2}}\right) = F$$

Add thruster, F. FBD



$\Sigma F_x = ma_x$

From p.5:

$$ma_x = -k\left(\sqrt{x^2+L^2} - L\right)\frac{x}{\sqrt{x^2+L^2}} + F$$

Call a_x as $\ddot{x}$: $\boxed{m\ddot{x} + kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right) = F}$

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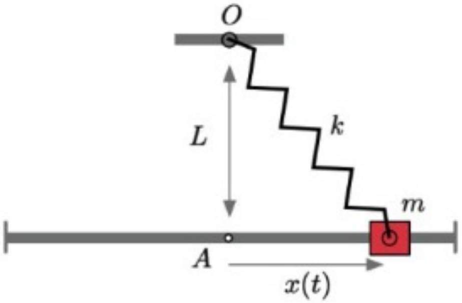
Example: Vibration Analysis

$\boxed{m\ddot{x} + kx\left(1 - \frac{L}{\sqrt{x^2+L^2}}\right) = F}$

Design Goal (from before): … develop a differential equation model of the system and **approximate the collar's vibration frequency (Hz) in its ensuing motion when x0 << L.** Next, add a thruster to the collar and develop a stable control system so that it comes to rest at point A in 1 second. Size the thruster.



For now, $F=0$. "Linearize" the DEQ at $x=x_0$.

$$m\ddot{x} + kx\left(1 - \frac{L}{\sqrt{L^2+x^2}}\right) = F$$

Note: $m\ddot{x} + kx\left(1 - \frac{L}{L(1+(x/L)^2)^{1/2}}\right)$. If $x_0 < L$, then $x_0/L << 1$

and $(x_0/L)^2$ tiny compared to $1 \rightarrow$ neglect!

$m\ddot{x} + kx\left(1 - \frac{L}{L}\right) = 0$ or $m\ddot{x} \approx 0$. No motion…

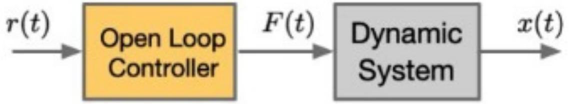
We will have deq models as: $m\ddot{x} + kx = 0$ soon.

where $\boxed{\omega = \sqrt{k/m}}$ $\quad (m\ddot{x} + kx = F)$

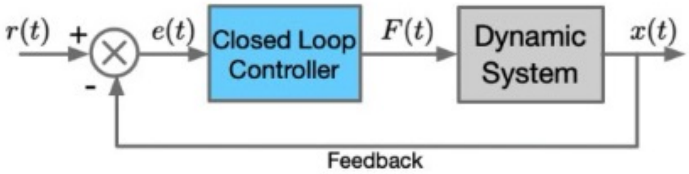
Example: Control Design

Design Goal (from before): … develop a differential equation model of the system and approximate the collar's vibration frequency (Hz) in its ensuing motion when x0 << L. **Next, add a thruster to the collar and develop a stable control system so that it comes to rest at point A (x=0) in 1 second. Size the thruster.**

Open Loop (Feedforward) Control: Use information about the desired (reference) motion and the system parameters (k, m, L, etc.) and create and apply F. If any parameter values are incorrect or there are external disturbances then performance suffers.

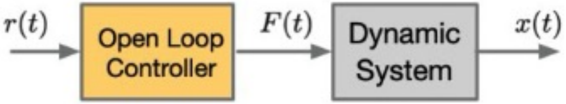


Closed Loop (Feedback) Control: Use measurements of the motion (position, velocity) with a "control law" to automatically compute F to achieve the goal of x=0 in 1 second. The control law is designed based on assumed parameter values. If these are incorrect and/or there are external disturbances, it will likely still work!

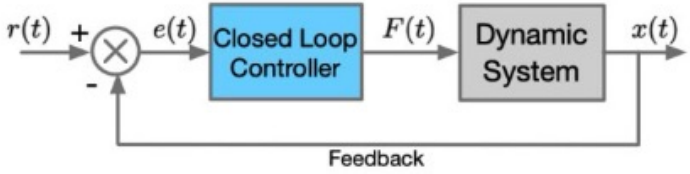


Example: Control System Block Diagrams

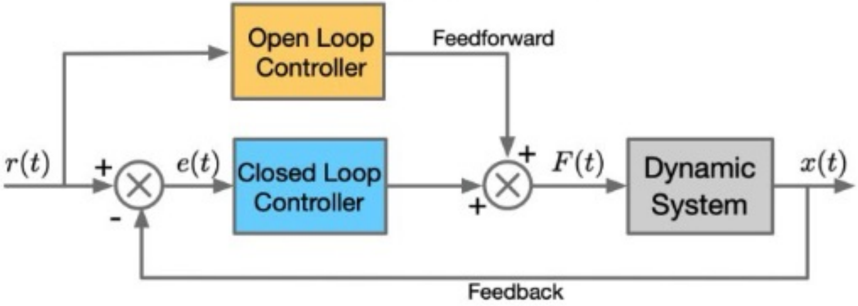
Open Loop (Feedforward) Control



Closed Loop (Feedback) Control



Closed Loop Control With Feedforward



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Summary

- Analysis difference between your previous dynamics course and this one.
- Definition of a dynamic system.
- Your focus will be extracting information from differential equation models and using that information to achieve design requirements.
- Dynamics Review: Free Body Diagram, Principle of Conservation of Total Mechanical Energy and direct integration.
- Determined the vibration frequency of a dynamic system modeled as:

  $m\ddot{x} + kx = 0$, specifically, $\omega = \sqrt{\dfrac{k}{m}}$ in radians/sec.
- Explored the difference in the form and function of open loop and closed loop control systems and their block diagrams.

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