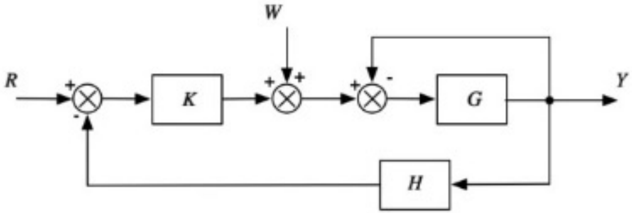
Block Diagrams

Exotic dynamic system models can be described using block diagrams

Each block represents a dynamic system with inputs, outputs, and represented as a transfer function.

Control system analysis and design often starts with a block a block diagram



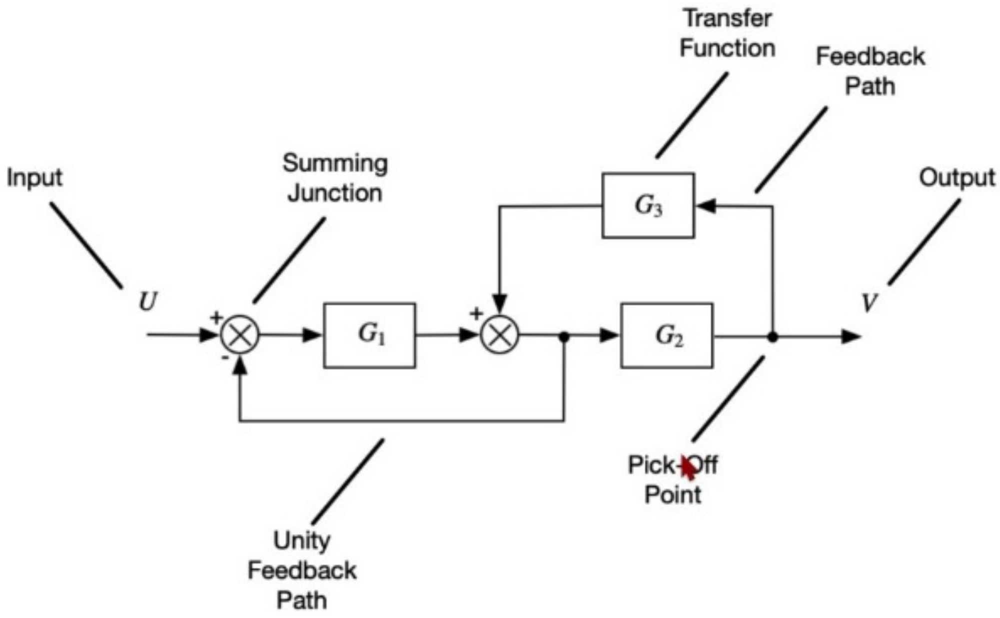
Skills

1. Recognize series and parallel block diagram forms
2. Manipulate blocks connected by summing junctions and pick-off points
3. Recognize the canonical closed-loop block diagram and write its transfer function
4. Given a block diagram, create its overall transfer function, called block diagram reduction
5. Use MATLAB to help simplify block diagrams and create transfer functions

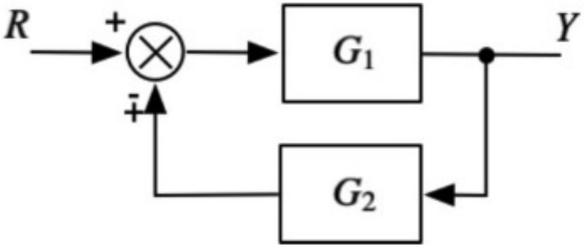
Terminology

1. Summing Junction
2. Pick-off point
3. Canonical Closed-loop Block Diagram
4. Block Diagram Reduction
5. Blocks in Series
6. Blocks in Parallel
7. Reference Input
8. Disturbance Input

Anatomy of a Block Diagram



Canonical Closed-Loop Block Diagram



Case 1: "–"

$(R - YG_2)G_1 = Y$

$RG_1 - YG_1G_2 = Y$

$RG_1 = Y(1 + G_1G_2)$

$$\boxed{\frac{Y}{R} = \frac{G_1}{1 + G_1G_2}}$$

Case 2: "+"

$(R + YG_2)G_1 = Y$

$RG_1 + YG_1G_2 = Y$

$RG_1 = Y(1 - G_1G_2)$

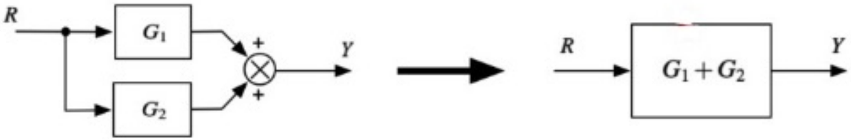
$$\boxed{\frac{Y}{R} = \frac{G_1}{1 - G_1G_2}}$$

Block Diagram Reduction Methods

1. Brute Force - “Walk” from input to output along the primary path and “grow” the transfer function
2. Algebraic “Chess Moves” - Move summing junctions and pick-off points to exploit easy series and parallel simplifications to achieve the closed-loop canonical form.
3. Hybrid (Recommended) - Attach easy simplifications, then use Brute Force

Series and Parallel Connections

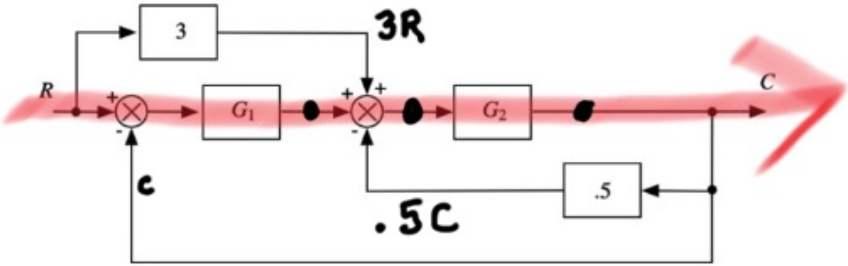
$$\frac{Y}{R} = G_1 G_2$$



$$\frac{Y}{R} = G_1 + G_2$$

Example 1. Brute Force

Form the transfer function for the block diagram at right.



Goal: $\frac{C}{R} = ?$

$$((R - C)G_1 + 3R - .5C)G_2 = C$$

$$(G_1 R - CG_1 + 3R - .5C)G_2 = C$$

$$G_1 G_2 R - G_1 G_2 C + 3G_2 R - .5G_2 C = C$$

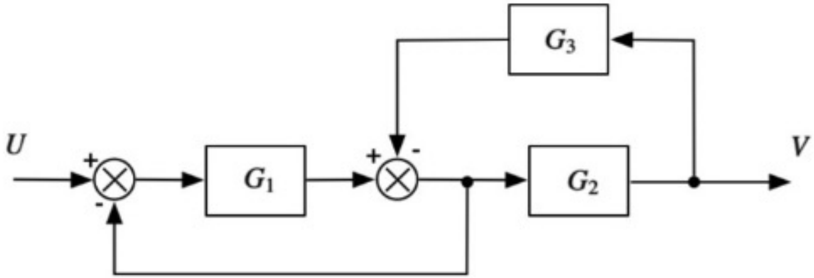
$$G_1 G_2 R + 3G_2 R = C + G_1 G_2 C + .5G_2 C$$

$$\boxed{\frac{C}{R} = \frac{G_2(3 + G_1)}{1 + G_1 G_2 + .5G_2}}$$

Example 2 - Hybrid with Intermediate Variable

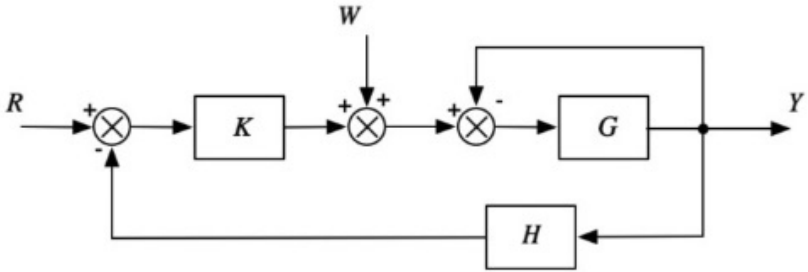
Form the transfer function for the block diagram at right.

Goal: $\frac{V}{U} = ?$



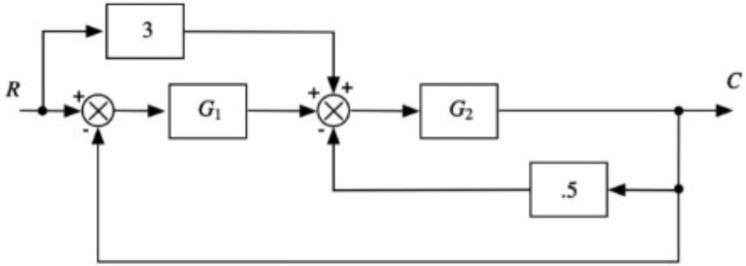
Example 3 - Hybrid with Multiple Inputs

Form the transfer functions for the block diagram at right.



Example 4. MATLAB's tf and feedback commands

Use tf and feedback to create the transfer function for the block diagram of Example 1, where



$$G_1 = \frac{s}{s+4}, G_2 = \frac{7}{s^2+2s+31}$$

```
clearvars; close('all');

G1 = tf([1 0],[1 4]);

G2 = tf([7,[1 2 31]);

H = feedback(G2, 0.5);

Gcl = (3/G1+1) * feedback(G1*H,1);

myPoles = pole(Gcl);

step(Gcl);

stepinfo(Gcl);
```

Summary

With a little more practice, you can

1. Create the overall transfer function from a block diagram using the Brute Force or Hybrid methods
2. Recognize the canonical closed-loop block diagram form and write its transfer function
3. Mainpulate blocks in series and parallel
4. Use MATLAB to help simplify block diagrams