Skills

1. Convert complex numbers between rectangular and polar forms
2. Sketch a complex number in the complex plane
3. Correctly calculate a complex number's magnitude and phase angle
4. Perform basic operations on complex numbers in both polar and rectangular forms
5. Use MATLAB to help with complex number operations

Forms

1. **Rectangular:** $z = x + yj$ where both the real part, x, and the imaginary part, y, are real numbers.

2. **Rectangular Complex Conjugate:** $\bar{z} = x - yj$

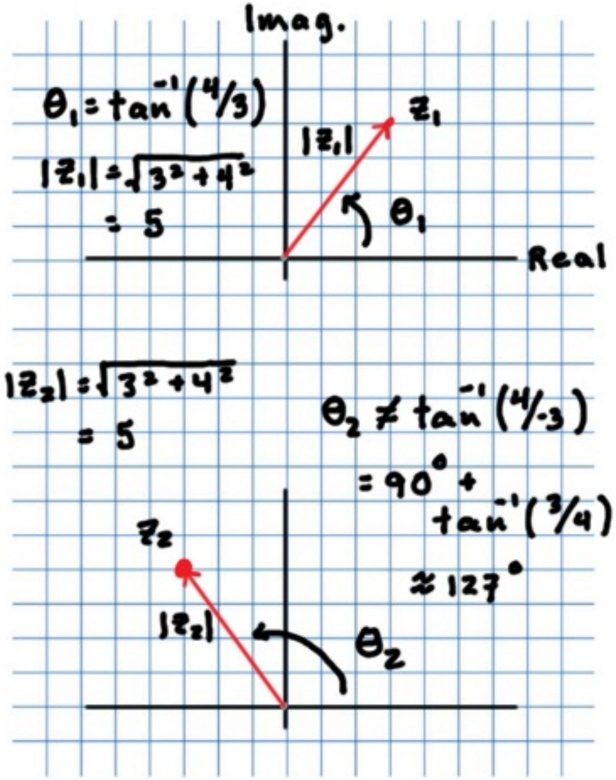
3. **Polar:** $z = |z|e^{j\theta}$ where the phase angle, θ and the magnitude $|z|$ are both real numbers.

4. **Alternate Notation (phasor):** $z = |z|\angle\theta$ where $\angle\theta \equiv e^{j\theta}$

5. **Polar Complex Conjugate:** $\bar{z} = |z|e^{-j\theta}$ or $\bar{z} = |z|\angle(-\theta)$

6. **Relationships:** $|z| = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}\left(\dfrac{y}{x}\right)$ and $e^{j\theta} = \cos\theta + j\sin\theta$

7. Sketch the complex numbers $z_1 = 3 + 4j$ and $z_2 = -3 + 4j$ in the **complex plane**



Euler's Identity

$$e^{j\theta} = \cos\theta + j\sin\theta$$

and

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

With some algebra we can solve for sine and cosine as

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

and

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

This lets you prove just about any trig identity such as:

$$\sin^2(\omega t) = \frac{1}{2}\left[1 - \cos(2\omega t)\right]$$

$$\sin^2(\omega t) = \frac{e^{j\omega t} - e^{-j\omega t}}{2j} \cdot \frac{e^{j\omega t} - e^{-j\omega t}}{2j}$$

$$= \frac{-1}{4}\left(e^{2j\omega t} - 1 - 1 + e^{-2j\omega t}\right)$$

$$= \frac{-1}{2}\left(\frac{e^{2j\omega} - e^{-2j\omega t}}{2} - 1\right)$$

$$= \frac{1}{2}\left(1 - \cos 2\omega t\right)$$

Example 1

Write $z = 3 + 4j$ in polar and phasor form. Then form $z \cdot \bar{z}$ using all three forms.

$z = |z| e^{j\theta}$, $|z| = \sqrt{9+16} = 5$, $\theta = \operatorname{atan}(4/3) \approx .93$, $z = 5e^{.93j}$

$z = |z| \angle \theta = 5 \angle 53.1^\circ$ (deg is accepted).

$z \cdot \bar{z}$

- Polar: $|z| e^{j\theta} \cdot |z| e^{-j\theta} = |z| \cdot |z| e^{j\theta - j\theta} = |z|^2 = 25$

- Phasor: Magnitudes multiply, angles add

$$5 \angle (53.1^\circ) \cdot 5 \angle (-53.1) = 25$$

- Rect: $(3+4j)(3-4j) = 9 + 12j - 12j + 16 = 25$

Example 2

Given the complex numbers $s_1 = 3 + 4j$ and $s_2 = 5 - 12j$ calculate $z_a = s_1 s_2$ and $z_b = \dfrac{s_1}{s_2}$ using rectangular form math.

$$z_a = (3+4j)(5-12j) = 15 - 36j + 20j + 48 = 63 - 16j$$

$$z_b = \frac{3+4j}{5-12j} \cdot \frac{5+12j}{5+12j} = \frac{15 + 36j + 20j - 48}{25 + 144}$$

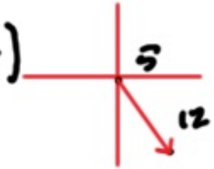
$$= \frac{-33 + 56j}{169}$$

Example 3

Given the complex numbers $s_1 = 3 + 4j$ and $s_2 = 5 - 12j$ calculate $z_a = s_1 s_2$ and $z_b = \dfrac{s_1}{s_2}$ using polar form math.

$$|s_1| = 5 \quad \theta_1 = \tan^{-1}(4/3) = 53.1^\circ \quad |s_2| = 13, \ \theta_2 = -\tan^{-1}(12/5)$$

$$= -67.4^\circ$$



$$s_1 s_2 = (5\angle 53.1^\circ)(13\angle -67.4^\circ) = 65\angle -14.3^\circ$$

$$\frac{s_1}{s_2} = \frac{5}{13}\angle(53.1^\circ + 67.4^\circ) = \frac{5}{13}\angle 120.5^\circ$$

Example 4

Write the complex number $z = -9e^{-\pi/4 j}$ in rectangular form.

$$z = |z|e^{-j\theta} = |z|(\cos\theta - j\sin\theta)$$

$$= -9\left(\cos\frac{\pi}{4} - \sin\frac{\pi}{4}\right) = -6.36 + 6.36j$$

MATLAB

```
>> z = -3 + 4j

>> abs(z)

>> imag(z)

>> real(z)

>> angle(z)

>> atan(-4/3)

>> atan2(4,-3)

>> atan2d(4,-3)

>> -9 * exp(-pi/4 * j)
```

Summary

With a little more practice and time with MATLAB you can…

1. Convert any complex number between rectangular and polar forms and sketch it in the complex plane
2. Calculate its phase angle in several ways
3. Multiply and divide them in any form, and…
4. Manipulate them in MATLAB