Springs and Dampers - Motivation

1. Dynamic systems that have spring behavior, but no physical springs, can be be modeled with springs.

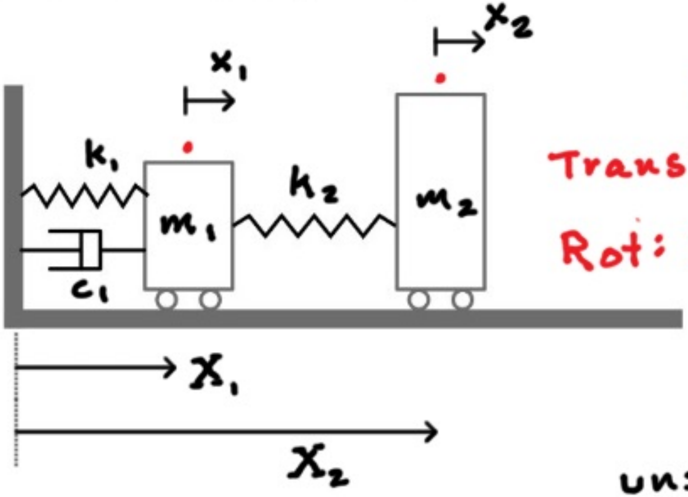
2. Systems with multiple bodies that are connected with a compliant joint, can also be modeled with springs, sometimes in series or in parallel

3. All dynamic systems lose energy over time. Introducing a damping is one way to capture this effect, even if there are no physical dampers.

Skills

1. Given a lumped system with several springs or dampers in series or parallel, create a single equivalent spring or damper
2. Be able to correctly define the variables of motion for a lumped parameter system with springs and dampers

Notation and Symbols



$$X_1 = x_{1,us} + x_1$$

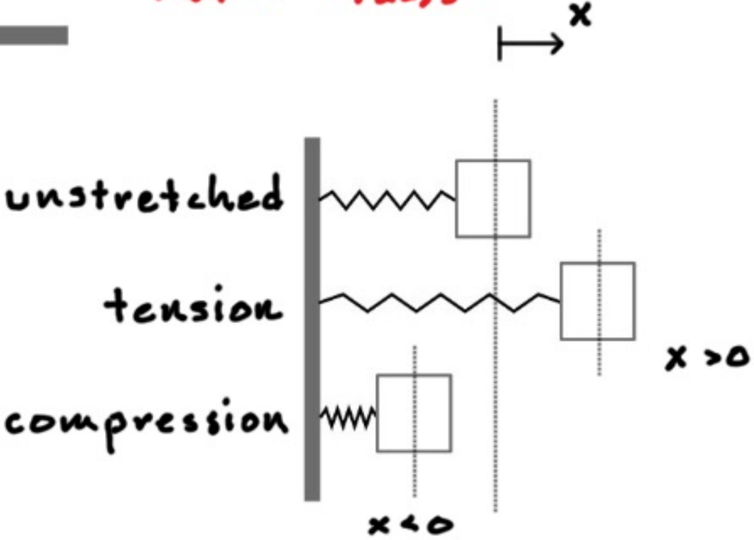
$$X_2 = (x_{1,us} + x_{2,us}) + x_2$$

Units

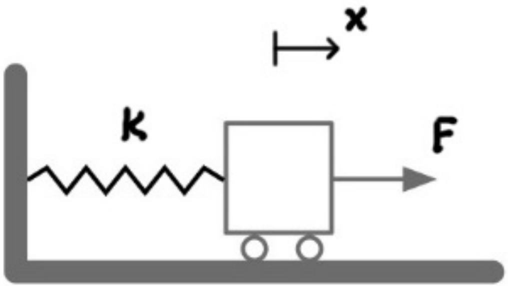
Trans: $k\left(\frac{N}{m}\right), c\left(\frac{N}{m/s}\right)$

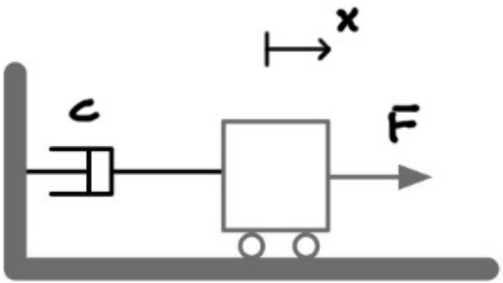
Rot: $K\left(\frac{N\cdot m}{rad}\right), c\left(\frac{N\cdot m}{rad/s}\right)$

Torsional Spring/Damper

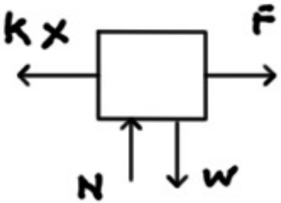


Sign Convention

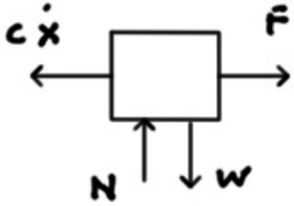




FBD



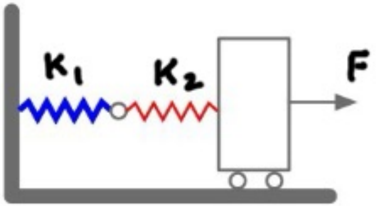
$F_s = -kx$



$F_{damp} = -c\dot{x}$

Springs (Dampers) in Series and Parallel

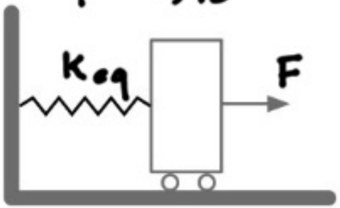
Series



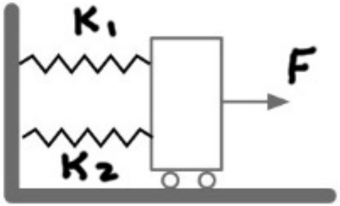
$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} = \frac{K_1 K_2}{K_1 + K_2}$$

E.g. $K_1 = 500\,\frac{N}{m}$ $K_2 = 10\,\frac{N}{m}$

$$K_{eq} = \frac{5000}{510} \approx 9.8\,\frac{N}{m}$$

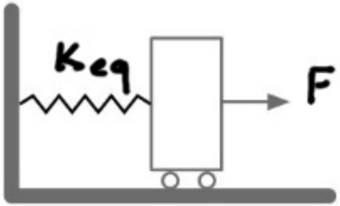


Parallel



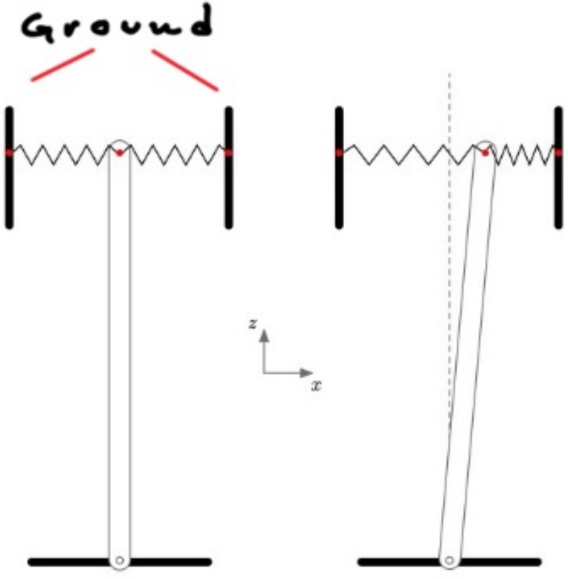
$$K_{eq} = K_1 + K_2$$

E.g. $K_{eq} = 510\,\frac{N}{m}$



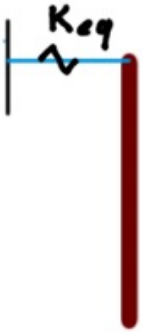
Example 1

Calculate the equivalent spring constant and sketch the new system.



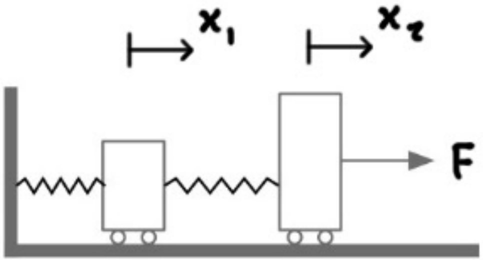
Same as:

Parallel
$K_{eq} = 2K$



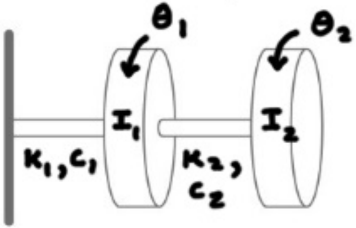
FBDs - Variables of Motion

You can always use a single, inertial reference to define motions, but for systems with springs, we almost always define motion variables as deviations from the equilibrium configuration.



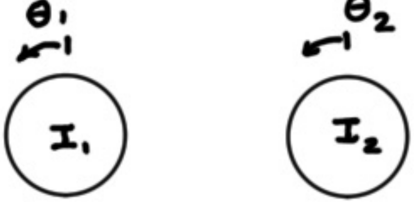
More Notation...

Torsional stiffness, damping in shafts



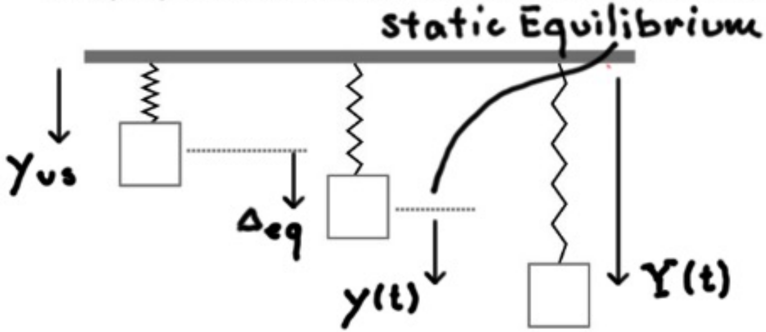
x_1 & x_2 have different, but fixed (inertial) references.

End View



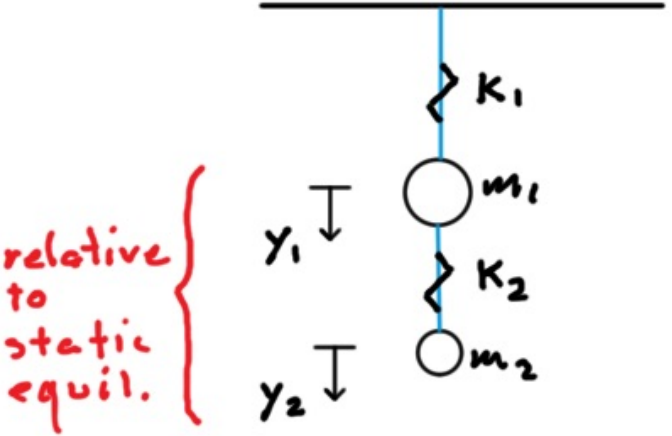
FBDs - Variables of Motion - Hanging Bodies

The common practice of defining motion relative to static equilibrium is the same, however, for hanging bodies static equilirbium doesn’t equate to unstretched as it does for translating bodies.



$$Y = y_{us} + \Delta_{eq} + y$$

2 mass example



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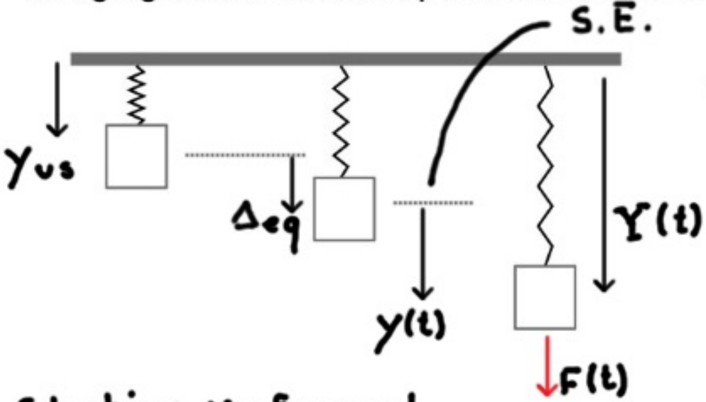
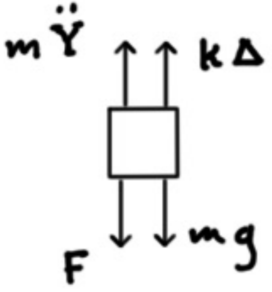


$$-m\ddot{Y} - k(Y - y_{us}) + F + mg = 0 \quad (1)$$

$$Y(t) = y_{us} + \Delta_{eq} + y(t) \quad (2)$$



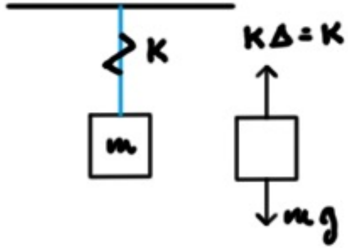
$$\Delta = Y - y_{us}$$

Sub (2) into (1)

$$-m\ddot{y} - k(\Delta_{eq} + y) + F + mg = 0$$

$$m\ddot{y} + k(\Delta_{eq} + y) = F + mg \quad (4)$$

Statics, Unforced



$$K\Delta = K\Delta_{eq}$$

$$\underbrace{K\Delta_{eq} = mg}_{\text{Fact}} \quad (3)$$

Sub (3) into (4)

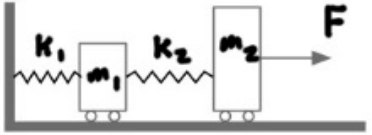
$$\boxed{m\ddot{y} + Ky = F}$$

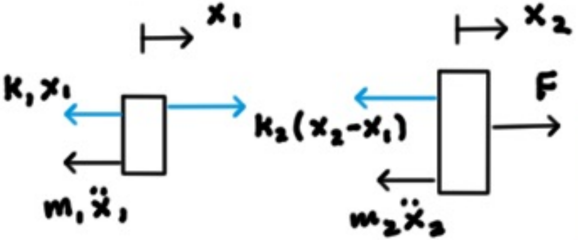
Defing y rel. to Δ_{eq} removes g from DEQ model

FBDs - Spring and Damper Forces of Connected Bodies

Assuming you've started the FBD process and now it's time to annotate them with spring or damper forces…

1. Virtually, apply a different positive displacement to each body
2. For each spring in the displaced configuration, create an expression for the force (or moment) magnitude and direction it exerts on the body it is attached to. If it's attached to two bodies, then apply the same force (or moment) expression, but in the opposite direction.





DEQ Model

$$\boxed{\begin{aligned} m_1\ddot{x}_1 + k_1x_1 - k_2(x_2 - x_1) &= 0 \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) &= F \end{aligned}}$$

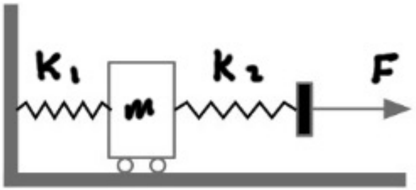
Sanity Check

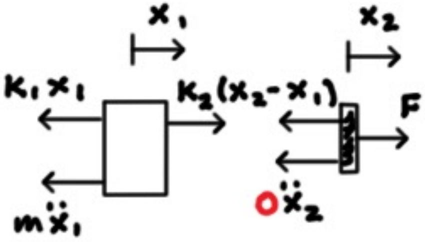
← All x_1 terms +

← All x_2 terms +

FBDs - Massless Bodies and Imposed Motion

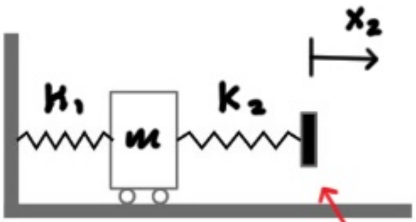
Sometimes motion is enforced on the system, or a force (moment) is applied directly to the spring or damper. The notation takes practice. Treat the massless object like any other FBD and the equations will develop nicely.



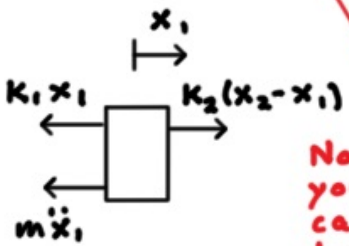


$$m_1\ddot{x}_1 + K_1x_1 - K_2(x_2-x_1) = 0$$

$K_2(x_2-x_1) = F$ So, $\boxed{m_1\ddot{x}_1 + K_1x_1 = F}$



x_2 is an impossed motion, an input.



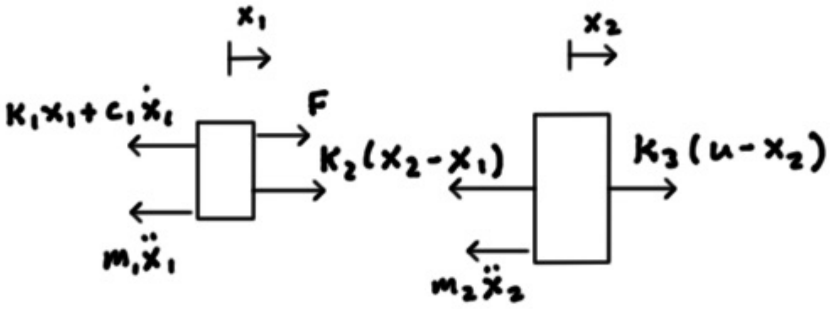
No FBD, unless you need to calc F req'd to cause the motion...

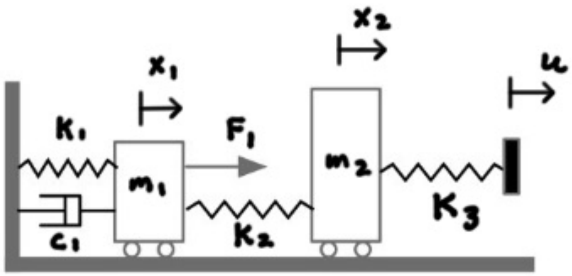
$$m_1\ddot{x}_1 + K_1x_1 - K_2(x_2-x_1) = 0$$

$$\boxed{m_1\ddot{x}_1 + (K_1+K_2)x_1 = K_2x_2}$$

Example 2

Develop the FBDs for the system at right.



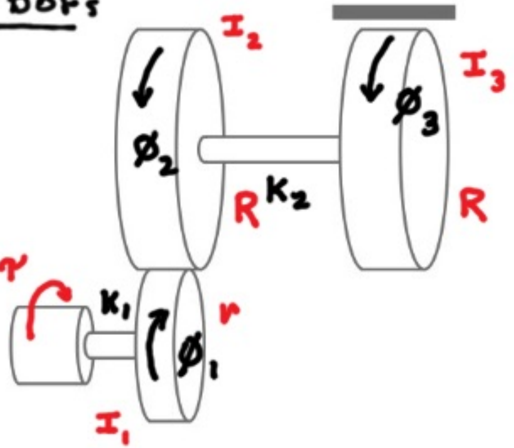


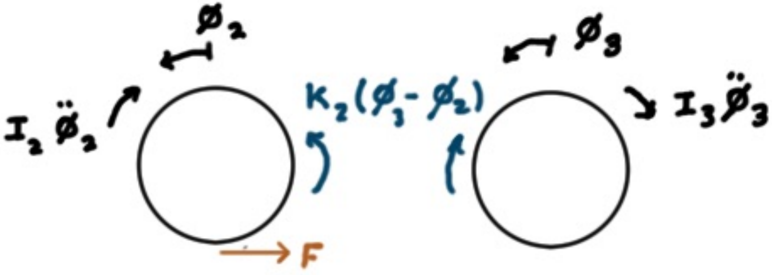
$$
\boxed{\begin{aligned} m_1 \ddot{x}_1 + k_1 x_1 + c_1 \dot{x}_1 - k_2(x_2 - x_1) &= F \\ m_2 \ddot{x}_2 + k_2(x_2 - x_1) - k_3(u - x_3) &= 0 \end{aligned}}
$$

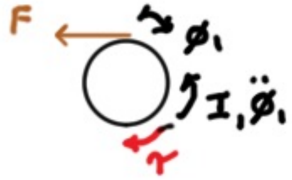
Example 3

$$\left.\begin{array}{l}\text{3 motion vars: } \phi_1, \phi_2, \phi_3 \\ \text{1 constraint: } r\phi_1 = R\phi_2\end{array}\right\} \underline{\text{2 DOFs}}$$

Develop the FBDs for the system at right.







$$-I_1\ddot{\phi}_1 - Fr + \tau = 0 \quad 1. \longrightarrow F = \frac{1}{r}(-I_1\ddot{\phi}_1 + \tau),\ \ddot{\phi}_1 = \frac{R}{r}\ddot{\phi}_2$$

$$-I_2\ddot{\phi}_2 + K_2(\phi_3 - \phi_2) + FR = 0 \quad 2.$$

$$-I_3\ddot{\phi}_3 - K_2(\phi_3 - \phi_2) = 0 \quad 3.$$

$$-I_2\ddot{\phi}_2 + K_2(\phi_3 - \phi_2) + \frac{R}{r}\left(-I_1\frac{R}{r}\ddot{\phi}_2 + \tau\right) = 0$$

$$\boxed{\begin{array}{l}\left[I_2 + (R/r)^2 I_1\right]\ddot{\phi}_2 + K_2(\phi_2 - \phi_3) = \frac{R}{r}\tau \\ I_3\ddot{\phi}_3 + K_2(\phi_3 - \phi_2) = 0\end{array}}$$

Summary

You are now a spring/damper ninja and can create FBDs of just about any translational, rotational dynamic system with springs and dampers.

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