Inertia - Motivation

1. Rotational dynamics analysis of rigid bodies undergoing general motion (e.g. a rocket, satellite, robot, etc.) uses Euler's equation AND mass moment of inertia matrices

$$\sum M_c = I_c\dot{\omega} + \omega \times I_c\omega$$

2. Here, we focus on planar motion with easier rotational dynamics

$$\sum M_c = I_c\dot{\omega}$$

3. For odd shaped bodies you need to be able to find its mass moment of inertia.

Skills

1. Calculate the mass moment of inertia of a homogenous rigid body undergoing planar motion
2. Know the units of mass moment of inertia

Background

1. Consider the RB at right with two reference points, A and B, and the body's volume is discretized at each point on a grid.

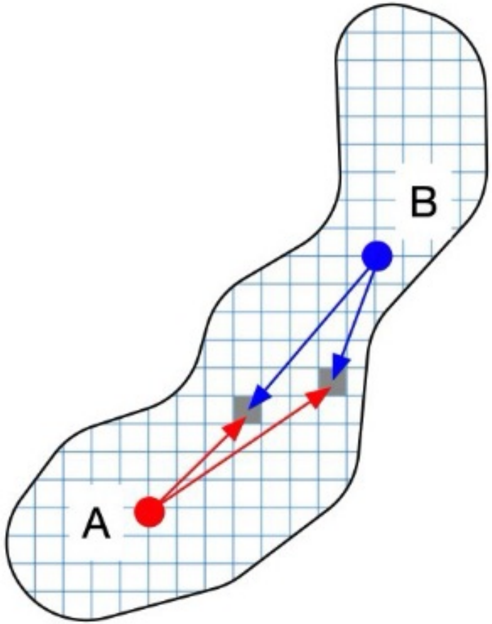
2. The mass moment of inertia of rigid body is

$$I = \int_V \rho(\vec{r}) \cdot r^2 dV$$

where $\rho(\vec{r})$ is the density of each element. If the body is homognous, then ρ is constant and we can pull it outside the integral. Also, r is the perpindicular distance from a reference point to a volume element.

3. Note: The value of I, in kg-m^2, depends on which reference point you use and, in general, the orientation of the reference used to express $\vec{r}$.

4. For general rigid body motion we can have fully populated inertia matrices, or diagonal. Diagonal are easier to work with but require you to use a special orientation for your frame - principle axes.



Parallel Axis Theorem - Planar Motion

1. Consider the RB at right where we know its mass moment of using its center of mass as the reference point, $I_{z,c}$.

2. The parallel axis theorem lets us calculate its mass moment of inertia for a different reference point O

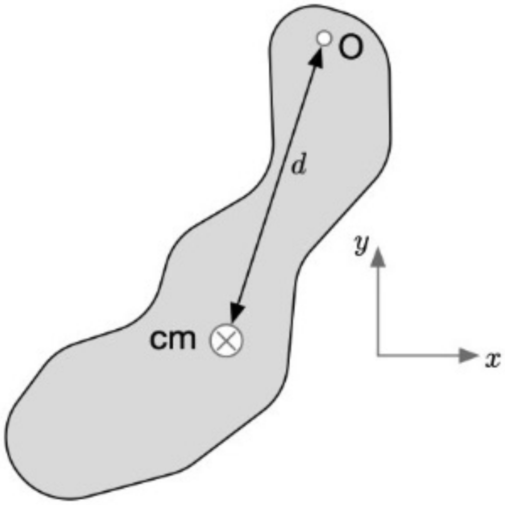
$$I_{z,O} = I_{z,c} + md^2$$

where d is the distance between the points and m is the body's mass.

3. Since we'll be working with planar motion only, let's make the notation easier and dispense with the "z"

$$I_O = I_c + md^2$$

4. **Mass moments of inertia of multiple bodies SUM as long as they are referenced to same point AND their frame orientations are identical**

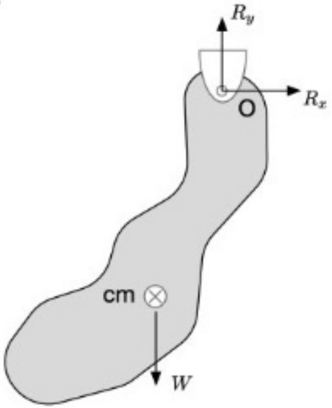


Parallel Axis Theorm - Why Do It?

1. When you have body that is rotating about a fixed point (an axle, a pin, etc.) then using the fixed point as a reference for both energy and dynamic analysis is much much easier.

2. The kinetic energy of rigid body is: $T = \frac{1}{2}mv_c^2 + \frac{1}{2}I_c\omega^2$ where v_c is the magnitude of the velocity of its center of mass and ω is its absolute angular velocity. If we use th fixed rotation point, then, $T = \frac{1}{2}I_o\omega^2$.

3. Consider the FBD of a hanging rod. The rotational equation of motion is either:



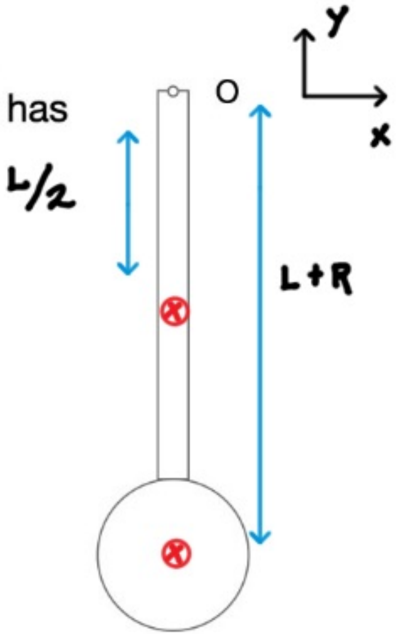
$$\sum M_c = I_c\ddot{\phi} \qquad \sum M_o = I_o\ddot{\phi}$$

the option on the right is better since it has fewer moments to sum.

Example 1

Parallel Axis: $I_o = I_c + md^2$

Consider the object at right that is free to rotate about the pin at O. Calculate its mass moment of inertia about O. The stick-like part is a circular cylinder with radius r and length L. The sphere at the bottom has radius R. Both parts are homogenous but the stick is steel and the sphere is aluminum.



From a table:

rod: $I_c = \frac{1}{12}m(3r^2+L^2)$ sphere: $I_c = \frac{2}{5}mR^2$

rod: $I_o = \frac{1}{12}m(3r^2+L^2)+m\left(\frac{L}{2}\right)^2 = \frac{1}{12}m(3r^2+L^2+3L^2)$

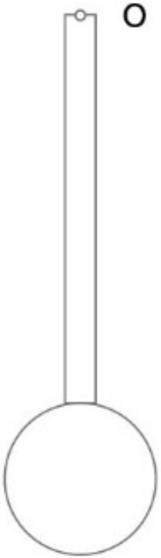
$= \frac{1}{12}m(3r^2+4L^2)$

sphere: $I_o = \frac{2}{5}mR^2 + m(L+R)^2$

total: $I_o = \frac{1}{12}m(3r^2+4L^2) + \frac{2}{5}mR^2 + m(L+R)^2$

Example 1 - continued

Consider the object at right that is free to rotate about the pin at O. Calculate its mass moment of inertia about O. The stick-like part is a circular cylinder with radius r and length L. The sphere at the bottom has radius R. Both parts are homogenous but the stick is steel and the sphere is aluminum.



total: $I_O = \frac{1}{12}m(3r^2+4L^2)+\frac{2}{5}mR^2+m(L+R)^2$

For fun... let R be a parameter that defines the others: $r = s_r R$, $L = s_L R$

$$I_O = \frac{1}{12}m(3s_r^2R^2+4s_L^2R^2)+\frac{2}{5}mR^2+m(s_LR+R)^2$$

$$I_O = \frac{1}{12}mR^2(3s_r^2+4s_L^2+\frac{2}{5}+s_L^2+2s_L+1)$$

$$= \frac{1}{12}mR^2\left(\frac{7}{5}+3s_r^2+7s_L^2\right)$$

$\longrightarrow$ changes in s_L^2 have about twice the effect as s_r^2.

Summary

You should be able to compute the mass moment of inertia of composite body AND check units of your calculations knowing that the units of mass moment of inertia are kg - m^2.

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